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ABSTRACT

This package for presenters contains an 84-hour professional development course designed to train and support teachers of adult numeracy and mathematics in adult literacy and basic education. The information for presenters provided at the beginning includes material on how the course was developed, content and structure, and lists of resources and references. The curriculum section provides course information such as professional development, rationale, course development, outcomes, structure, assessment, delivery of the course, and module information. The course consists of four modules: exploring practice, mathematics as a human construction, mathematics as a critical tool, and naming theories--implications for practice. The presenter's notes for each module provide descriptive and instructive details (nominal time, brief description, rationale, learning outcomes, assessment requirements, references, and an outline (section correlated to time, development of issues and activities, and math involved). Each section in a module provides a brief description, rationale, aims, preparation, materials needed, and detailed procedure. A resources section that follows each module contains a list of readings, activity sheets, handouts, overhead project transparencies, and references for the module. The final section provides information on curriculum projects and assessment of the numeracy journals kept by course participants. (YLB)

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ED 392 947

Adult NUMERACY Teaching

making meaning in mathematics

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A project commissioned by the
National Staff Development Committee
for Vocational Education and Training
1995

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It was developed jointly by the National Languages and Literacy Institute of Australia, Melbourne and the Centre for Language and Literacy, University of Technology, Sydney.

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Elizabeth Taylor, Sasha Giffard Huckstep and the University of Technology, Sydney: for *Sheep in a hole, Handcuffs, A twist a side* and *Symmetry*
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Jynell Mair and Sue Turner: for materials for *The patchwork connection*
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National Academy Press and Marjorie Senechal: for *Using symmetry*
Routledge Chapman & Hall and Marcia Ascher; Birkhäuser Bosten Inc.; and John G. Neihardt: for *The circle and the line*
The Sydney Morning Herald and Graham Phillips: for *Why your tape measure isn't long enough*
University of Texas Press and Peter Denny: for *What's so great about regular shapes?*
W.H. Freeman & Co. and Ivars Peterson: for *The language of nature*

Module C

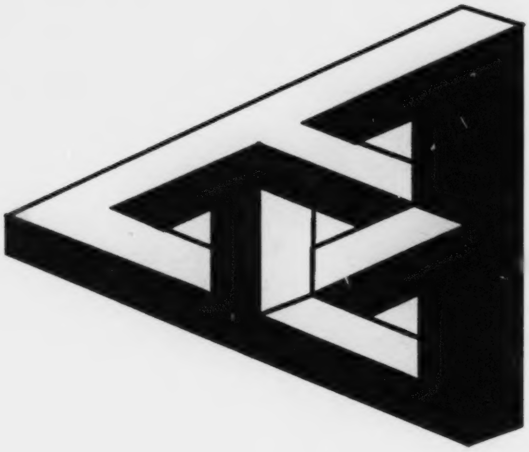
The Mathematical Intelligencer and Ronnie Brown: for *Carpentry: a fable*
Penguin and D.R. Hofstadter: for *On number numbness*
Penguin, P.J. Davis and R.J. Hersh: for *Are we drowning in digits?*
Earthscan Publications/W.W. Norton & Co.: for *State of the world 1994*
Stewart Victor Publishing and M. Newman: for *Domains of learning*
Education Links and Ken Johnston: for *Dealing with difference*
The Falmer Press/Taylor & Francis, J. Evans and M. Harris: for *Work contexts* and *Everyday contexts*
Falmer Press/Taylor & Francis and D. Carraher: for *Street contexts*
Virago in association with the University of London Institute of Education: for *Language contexts*
Flamingo and P. Hoeg: for *Why I like numbers*
For the Learning of Mathematics and Christine Keitel: for *Social practice of mathematics*
Jenny McGuirk and the University of Technology, Sydney: for *The best laid plans*
Cassell, Gila Hanna, Erika Kundiger and Christine Larouche: for *Proportion of female maths teachers*
For the Learning of Mathematics and Lesley Lee: for *Our data do not support...'*
The Sunday Telegraph and Michael Wilkins: for *Inquiry tips single sex classes to help boys*
Department of Employment, Education and Training, A. Lee, A. Chapman and P. Roe: for *Literacy AND numeracy*

Module D

Brooks/Cole Publishing and Marcia Ascher; Oxford University Press and D. Nelson, G.G. Joseph and J. Williams: for *Probability fair: cultural/historical picture*
Hodder & Stoughton and Richard Noss: for *The social shaping of technology*
Longman and Hilary Shuard: for *The impact of technology*
Longman and Hilary Shuard: for *The role of calculators*
National Key Centre for School Science and Mathematics, Curtin University of Technology and Judith Mousley: for *Cups of birdseed*
National Key Centre for School Science and Mathematics, Curtin University of Technology, Gail FitzSimons and Peter Sullivan: for *Constructivism in a straitjacket*
Penguin: for *The school of Barbiana*
Pergamon and E. von Glaserfeld: for *Constructivism in education*
Richard Noss: for *Implementation or critique?*
The Falmer Press and Paul Ernest: for *The impact of beliefs on teaching*
The Shell Centre for Mathematical Education: for *The case for calculators*

Curriculum Projects and Numeracy Journal

Little, Brown & Co., David Fuys and Rosamond Tischler: for *The four operations*
The Falmer Press and Stephen Lerman: for *Mathematics through problem-posing*



Information for presenters

Welcome to presenters

Welcome to *Adult Numeracy Teaching: making meaning in mathematics.*

Adult Numeracy Teaching is an initiative of the National Staff Development Committee for Vocational Education and Training. It is an 84 hour professional development course designed to train and support teachers of adult numeracy and mathematics in Adult Literacy and Basic Education.

We hope that presenters will enjoy using the package. Because there has been little comparable material written in this field we have prepared this course in what may appear excessive detail in order to support presenters who feel they are inexperienced. You may want to present the package as it stands, but please feel free to adapt and add to the material so that it suits your needs.

The course was initially called *Adult Numeracy Teaching*, mainly to parallel the name *Adult Literacy Teaching*, with the intention of finding a more suitable name further down the track. However the course became affectionately called 'ANT' and the name stuck. Its official title became: *Adult Numeracy Teaching: making meaning in mathematics.*

When we started the project, 84 hours seemed an overgenerous estimate of the time we would need. However, we are now only too aware of areas we have skated over or even omitted.

We see this program as a major step in what hopefully will be an evolving process of professional development in adult numeracy. *Adult Numeracy Teaching* is intended to be a continuation and further development of existing professional development packages, such as *Breaking the Maths Barrier* (Marr & Helme 1991) and three NSDC programs: *Numeracy and How We Learn* (Thiering & Barbaro 1992), *Induction Program* (1993) and *Inservice Program Modules 1 to 6*(1993/1995); leading in turn to postgraduate study. *Adult Numeracy Teaching* is therefore targeted at experienced adult literacy, language and numeracy teachers, and we would recommend that inexperienced teachers and tutors be trained first through the above packages before participating in *Adult Numeracy Teaching.*

The primary purpose of *Adult Numeracy Teaching* is to blend theory and practice about teaching and learning adult numeracy within a context of doing and investigating some mathematics, whilst developing a critical appreciation of the place of mathematics in society.

Dave Tout and Betty Johnston

Developing the course

In 1993 the National Adult Literacy and Basic Education Professional Development Reference Committee identified a project, Teaching Mathematics in Adult Literacy and Basic Education, as having high priority. The project to develop the Adult Numeracy Teaching course was awarded jointly to the Centre for Language and Literacy at the University of Technology, Sydney (UTS) and the Adult Basic Education Resource and Information Service (ARIS) at the Victorian Office of the National Languages and Literacy Institute of Australia (NLLIA). The project team was supported by several experienced adult numeracy teachers and trainers from NSW and Victoria.

For further background information see Curriculum, item 2, 'Course Development'.

Initially, there was interstate consultation; a survey was undertaken with state and territory personnel to help identify needs, and a literature search was completed. NSW and Victorian adult numeracy teachers were then employed as consultants. The project team endorsed and refined a model for the course content by early 1994, and considered in detail the aims and learning outcomes, and the content issues and areas. The consultants developed the outline of the materials as a cooperative group rather than by working individually and in isolation. Over subsequent months as the detailed writing continued the National Reference Panel members were kept in touch and the course was piloted in two states, NSW and Victoria, in 1994 and 1995. A Train-the-Trainers program was held in early 1995 for representatives from all states and territories.

For further information on the project team see Acknowledgments, page iv.

Content and structure

The question the team began with was:

What should numeracy teachers know and be able to do after this course?

The following answer was agreed to after some discussion.

After this course a teacher

1 *should have a critical appreciation of the place of mathematics in society and*
2 *should be able to initiate appropriate learning activities by identifying the numeracy needs of students and responding from a variety of approaches to teaching and a range of appropriate mathematical resources and knowledge.*

The two parts of the answer became the two overall aims of the course. For further information on course outcomes and learning outcomes of each module see Curriculum, page 1.

The model of the course structure which was developed consists of three strands:

- two social and pedagogical strands
  - **knowing** about numeracy and
  - **learning** (and teaching) numeracy

- one central strand interwoven with the other two: **doing** mathematics.

The writers also tried to keep in mind:

- the need for the course to act as a model of good practice with particular emphasis on linking theory and practice
- the need to face squarely the fact that to teach numeracy you must know how to do mathematics, so that almost every session must include mathematical activities

- the need to build on and make full use of good existing materials, e.g. *Breaking the Mathematics Barrier, Numeracy and How We Learn* and *Reclaiming Mathematics.*

Using the above model as the basis for the course, we established what the content of the course should be. Draft materials were trialled in both Victoria and NSW. After feedback and evaluations were received, we reorganised the course into its present format and content. The logic behind the flow and structure of the final course is outlined below.

- The theoretical position developed during the course (and named explicitly in the last section) will elicit from the participants, through experience and discussion, a developmental comparison of a transmission type pedagogy, through to a more constructivist approach which incorporates a critical stance—presenting a widening range of alternative approaches.

- The development throughout the course is based on the assumption that the ALBE students of the participants have usually failed in a transmission-type mathematics education and need alternative teaching approaches.

- Most of the issues dealt with will emerge from engagement with specific mathematics.

Course structure

module	issues & investigations	hours
Module A: Exploring practice	**A1: YOU AND NUMERACY** – introductions and maths autobiographies – issues in numeracy	3
	A2: UNLEARNING ABOUT MATHS – unlearning maths: a case study – maths abuse and anxiety	3
	A3: WHY THEORY? – why theory? do we need theories	3
	CURRICULUM PROJECT 1 – Making meaning, Part 1	
	A4: MAKING MEANING WITH SYMBOLS – algebra activities	3
	CURRICULUM PROJECT 1 – Making meaning, Part 2	
	A5: MAKING MEANING FROM COUNTING SYSTEMS	3
	CURRICULUM PROJECT 1 – Making meaning, Part 3 – why count? and how? – operating 'our' number system: a beginning	
	A6: STRATEGIES FOR MAKING MEANING – operating the number system: a continuation – language as a tool – constructing meaning in practice: a review	3
		18 h
Module B: Maths as a human construction	**B1: SHAPING THE WORLD** – shaping our world – filling space – the patchwork connection – mostly Pythagoras	6
	B2: CIRCLES AND CULTURE – problem solving stations – investigating pi	3
	B3: PROBLEM SOLVING – problem solving—problem posing – effects of scale and shape	3
	B4: MODELLING OUR WORLD – measuring the coastline/fractals – maps and models – mindmaps: a review	3
	B5: CURRICULUM PROJECT 2: Exploring maths	9
		24 h

Module C: Mathematics as a critical tool	**C1: BEING CRITICAL ABOUT MATHS** – being critical about number – reflecting on and negotiating your learning	3
	C2: BEING CRITICAL ABOUT LEARNING – looking at learners – street maths, school maths	3
	C3: A NEGOTIATED INTERVAL – a choice of topics	3
	C4: MATHS MYTHS AND REALITIES – why learn maths? why be numerate? – curriculum planning and meeting student needs	3
	C5: EXCAVATING MATHS – who learns? who benefits? a maths excavation – resourcing numeracy – critical literacy, critical numeracy – excavating mathematics	6
	C6: CURRICULUM PROJECT 3: Developing a critical view	9
		27 h
Module D: Naming theories: implications for practice	**D1: AND SO TO THEORY** – naming the theories – for example: technology	3
	D2: IMPLICATIONS FOR TEACHING – case studies: constructivist and critical – a lesson on measurement	3
	D3: IMPLICATIONS FOR ASSESSMENT – assessment alternatives	3
	CURRICULUM PROJECT 4 – Assessment tool, Part 1 **D4: THEORY AND PRACTICE: CLOSING THE GAP**	3
	CURRICULUM PROJECT 4 – Assessment tool, Part 2 – theory into practice	3
	D5: AND SO WHAT IS NUMERACY? – the chance of numeracy – towards numeracy	**15 h**
	TOTAL:	**84 h**

Outline of module content

Module A: Exploring practice

- introduces issues and flags the need for theories
- journeys through a number of crucial mathematical areas including place value, basic operations and algebra
- raises issues of: meaning making, language, mathematics in practice and mathematics in cultural contexts.

Module B: Mathematics as a human construction

- focuses on learning some mathematics, largely to do with space and shape, within an interactive/constructivist environment
- reflects upon this learning experience in the journal
- looks at the role of teacher in all this: how have we been taught, how do we teach, how do we want to teach?
- makes cultural and historical connections
- concludes with a substantial mathematical investigation by participants.

Module C: Mathematics as a critical tool

- involves learning some more mathematics, including statistical and other everyday applications, within a critical framework
- looks at issues of access, needs assessment and negotiation
- reflects upon this learning experience in the journal
- concludes with participants doing a substantial project excavating mathematics from everyday sources and developing appropriate teaching or curriculum materials.

Module D: Naming theories: implications for practice

- pulls out from previous discussion the three theories of transmission, constructivism, and critical constructivism
- applies these theories to teaching, planning a curriculum and assessment
- and comes to the conclusion that:

numeracy is not less than mathematics, but more.

The members of the project team therefore hope that participants will be able to help their students move from:

numeracy as 'doing basic mathematics'
→ numeracy as 'being at home with mathematics'
→ numeracy as 'critical mathematics'.

Adult Numeracy Teaching: the course document

Curriculum

The curriculum for *Adult Numeracy Teaching* has been written to conform with ACTRAC 1994, *User's Guide to Course Design for Competency-Based Curriculum* (2nd edition). See the Curriculum for the following details:

- Qualifications of presenters
- Entry requirements for participants
- Delivery of the course
- Assessment

Modules A, B, C and D

Each module includes all that is needed for delivering the course and is set out in two parts:

- Presenter's notes: an outline of the module and program notes for each session
- Resources: handouts, overhead transparencies, activity sheets, and background readings.

Curriculum Projects and Numeracy Journal

This section gives details of the four Curriculum Projects and the Numeracy Journal entry requirements that make up the assessment tasks for the course.

General information

- **Handouts, overhead transparencies, activity sheets and readings**
  In the Resources sections there are headers, icons and abbreviations to help you identify how each item is to be used.
  P = participants' materials for them to read in advance
  OHT = overhead projector transparencies
  HO = handouts for participants to use in the session and retain
  AS = activity sheets to be used during sessions and returned to the presenter
  Examples:

Talking about maths anxiety	✋	**HO4**
My experiences of maths	▤	**OHT1**
Knowing and believing is seeing		**P4**
At home with algebra	✥	**AS8**

- **Course timetable**
  For the first session prepare a timetable of dates and times for the course. It could be distributed before the course begins or given out at the first meeting.

- **Folders**
  Prepare a set of ring binders for the Group. A large amount of material is collected throughout the course and it is recommended that you provide a thick ring binder for each participant. Include in it copies of any advance readings, course outline, timetables, etc.

- **Readings**
  Many readings have been included in the course materials themselves, but these are often abbreviated excerpts, and most of the longer readings are not provided. The full readings which are listed with an identifier **P** should be collected, photocopied and posted or distributed to the participants in time for them to be read before they are referred to in the relevant session.

Resources and references

A range of resources is essential for the delivery of *Adult Numeracy Teaching*. The details of any resources or readings needed are listed in the presenter's notes for each session. However, there are a number of resources that should be collected if possible before the course begins, especially as some of them may be difficult to acquire. They will be used at different stages before and during the course.

Numeracy in Focus

Numeracy in Focus was specifically written to support the delivery of *Adult Numeracy Teaching*. It contains a number of readings used through the course. A copy for the presenter/coordinator is essential. It is also advisable to ask participants of ANT to purchase a copy, or otherwise make sure they have ready access to a copy. *Numeracy in Focus* is available through either ALIO or ARIS (addresses on next page).

Reclaiming Mathematics

Reclaiming Mathematics by Betty Johnston (1992) is another essential resource for the presenter/coordinator to have. It includes many readings used throughout the course.

Essential hands-on materials

There are also particular hands-on materials needed for activities throughout the course.

- MAB base 10 blocks
- Cuisenaire rods
- rulers
- hinged mirrors (see Section B1)
- straws
- a set of calculators
- string and scissors
- counters
- Centicubes
- grid and/or graph paper

Recommended readings for presenters

The following is a list of essential books and journals for the presenter. They will, as well, be useful references for the course participants.

Ascher, M. 1991, *Ethnomathematics: a multicultural view of mathematical ideas*, Brooks/Cole, California.

Davis, R., Maher, C. & Noddings, N. (eds) 1990, 'Constructivist views on teaching and learning mathematics', *Journal for Research in Mathematics Education*, Monograph No. 4, NCTM, Reston, VA.

*Duckworth, E. 1987, 'Teaching as research', in E. Duckworth, *The Having of Wonderful Ideas and Other Essays on Teaching and Learning*, Teachers College Press, New York.

Frankenstein, M. 1989, *Relearning Mathematics: a different third R—radical maths*, Free Association Books, London.

Harris, M. (ed.) 1991, *Schools, Mathematics and Work*, The Falmer Press, Basingstoke, Hampshire.

Hogben, L. 1936, *Mathematics for the Million*, George Allen & Unwin, London.

Johnston, B. (ed.) 1992, *Reclaiming Mathematics*, UTS, Sydney. [1]

Joseph, G.G. 1991, *The Crest of the Peacock: non-European roots of mathematics*, Tauris, London.

Leder, G. (ed.) 1992, *Assessment and Learning of Mathematics*, ACER, Melbourne.

Malone, P. & Taylor, P. (eds) 1993, *Constructivist Interpretations of Teaching and Learning Mathematics*, National Key Centre for School Science and Mathematics, Curtin University of Technology, Perth.

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[1] Available from School of Adult Education, University of Technology, Sydney, PO Box 123, Broadway NSW 2007. Fax: 02 330 3939

Marr, B. & Helme, S. 1991, *Breaking the Maths Barrier: a kit for building staff development skills in adult numeracy*, DEET, Canberra. [2]
Nelson, D., Joseph, G.G. & Williams, J. 1993, *Multicultural Mathematics: teaching mathematics from a global perspective*, Oxford University Press, Oxford.
Numeracy in Focus, No. 1, 1995, joint production of ALIO & ARIS.
Nunes, T., Schliemann A. & Carraher, D. 1993, *Street Mathematics and School Mathematics*, Cambridge University Press, Cambridge.
Shan, Sharan-Jeet & Bailey, Peter 1991, *Multiple Factors: classroom mathematics for equality* and justice, Trentham Books, Stoke-on-Trent.
Thiering, J. & Barbaro, R. 1992, *Numeracy and how we learn*, TNSDC, Sydney.
Wells, David 1988, *Hidden Connections, Double Meanings: a mathematical exploration*, Cambridge University Press, Cambridge.
Willis, S. (ed.), 1990, *Being Numerate: what counts?*, ACER, Melbourne.

Other recommended resources for the course
During the course it will be almost essential for course participants to have access to the following resources. They are more oriented towards teaching practice than those listed above.

Banwell, C., Saunders, K. & Tahta, D. 1986, *Starting Points: for teaching mathematics in middle and secondary schools*, Tarquin Publications Stradbroke, UK.
Castles, I 1992, *Surviving Statistics: a user's guide to the basics*, Australian Bureau of Statistics.
Goddard, R., Marr, B. & Martin, J. 1991, *Strength in Numbers: a resource book for teaching adult numeracy*, Division of Further Education, Victoria.
Hemmings, R. & Tahta, D. 1984, *Images of infinity*, Leapfrogs, London.
Jacobs, Harold 1982, *Mathematics: a human endeavour*, W.H. Freeman & Co, New York.
Lovitt, C. & Clarke, D. 1988, *Mathematics Curriculum and Teaching Program (MCTP) Activity bank*, Vols 1 & 2, Curriculum Development Centre, Canberra.
Marr, B. & Helme, S. 1987, *Mathematics: a new beginning*, State Training Board, Victoria.
Marr, B., Tout, D. & Anderson, C. 1994, *Numeracy on the Line: language-based activities for adults*, National Automotive Industry Training Board, Victoria.
Pappas, Theoni 1989, The *Joy of Mathematics: discovering mathematics all around you*, Wide World Publishing, California.
Perl, Teri 1978, *Math Equals: biographies of women mathematicians and related activities*. Addison-Wesley, California.

Note:
References in the text which are marked with an asterisk are available in Johnston, B. (ed.) 1992, *Reclaiming Mathematics*, University of Technology, Sydney.

For further information about availability of these resources please contact either ALIO, the Adult Literacy Information Office (NSW), or ARIS, the Adult Basic Education Resource and Information Service (Victoria) at the addresses below. Both ARIS and ALIO supported the pilots of *Adult Numeracy Teaching* and purchased resources explicitly to support ongoing delivery of the course.

ALIO
Adult Literacy Information Office
6–8 Holden Street
Ashfield NSW 2131
Phone: 02 716 3666
Fax: 02 716 3699

ARIS
Adult Basic Education Resource and Information Service
GPO Box 372 F
Melbourne VIC 3001
Phone: 03 9614 0255
Fax: 03 9629 4708

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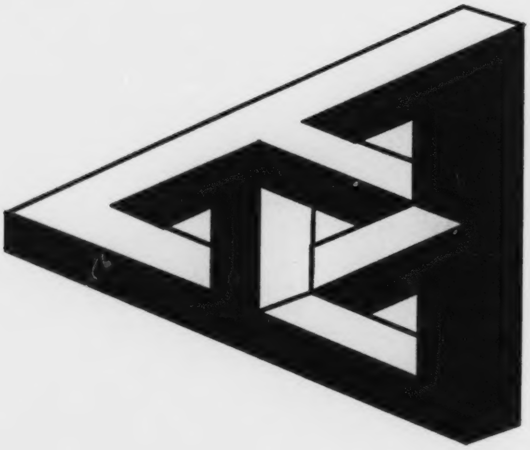
[2] Available from the National Languages and Literacy Institute of Australia, Melbourne Office, GPO Box 372F, Melbourne VIC 3001 (Fax: 03 9629 4708) and from the Adult Literacy Information Office, 6–8 Holden St, Ashfield, NSW 2131 (Fax: 02 716 3699)

Module C: Mathematics as a critical tool	**C1 - BEING CRITICAL ABOUT MATHS** - excavating maths from the real world	3
	C2 - BEING CRITICAL ABOUT LEARNING - Who are the learners? - What are their needs? - curriculum planning	6
	C3 - A NEGOTIATED INTERVAL - critique of maths and its uses - graphs in the media and other negotiated topics	3
	C4 - EXCAVATING MATHS - fractions etc: a sample excavation - maths in context - literacy and numeracy interface - resources for teaching: packages, concrete materials, people, organisations, contexts	6
	CURRICULUM PROJECT 3: Maths Excavation From the excavation of some maths embedded in a context, develop a critical teaching activity - planning - exploring - presenting	9 **27 h**
Module D: Naming theories: implications for practice	**D1 - AND SO TO THEORY** Naming the theories - summaries of previous discussions - transmission, constructivism, and critical practices	3
	D2 - IMPLICATIONS FOR TEACHING - case studies - measurement	3
	D3 - IMPLICATIONS FOR ASSESSMENT - assessment alternatives	3
	D4 – THEORY AND PRACTICE: CLOSING THE GAP CURRICULUM PROJECT 4 - Assessment tool – theory into practice	3
	D5 - SO WHAT THEN IS NUMERACY? – chance and probability – towards numeracy	3 **15 h**
	TOTAL:	**84 h**



Curriculum

Course information

1 Course name, qualification and ASF level

Course name	Adult Numeracy Teaching: making meaning in mathematics
ASF level	Level 6
Nominal duration	84 hours

2 Course development

Background needs of the ALBE field

The National Adult Literacy and Basic Education Professional Development Reference Committee makes recommendations to the National Staff Development Committee for Vocational Education and Training (NSDC) on the planning and development of professional development in the field of Adult Literacy and Basic Education. The Australian Language and Literacy Policy (ALLP) in 1991 identified a skills shortage amongst Adult Literacy and Basic Education (ALBE) teachers. The ALLP made funds available to promote the professional development of ALBE personnel in TAFE. Following this, the NSDC funded a project to identify resource gaps and to investigate the professional development needs of ALBE staff.

The result was the identification of a need for:
- a more systematic approach nationally to ALBE professional development
- opportunities for staff currently working in the area to upgrade the quality of teaching and management of ALBE programs with TAFE
- professional development for newly recruited teachers of ALBE and
- a framework for future professional development programs.

A National Framework for Professional Development of Adult Literacy and Basic Education Personnel was published by the NSDC in 1994, following a draft Framework issued in 1992. The aim of the Framework was:

> *To ensure that there is a sufficiently trained human resource base within the national vocational education and training sector to deliver quality adult literacy and basic education programs.*

The Framework provides a plan for a nationally coordinated approach to planning, development and implementation of professional development programs for the field. This plan aims to equip ALBE personnel to deal with the major features of the National Training Reform Agenda such as competency based training, flexible delivery and a national system for the recognition of training. It also aims to ensure that teachers are able to deliver high quality language and literacy programs and that, as a result, language and literacy education can take a central role in the Training Reform Agenda. The National ALBE Professional Development Reference Committee identified this project, Teaching Mathematics in Adult Literacy and Basic Education, as having high priority.

Rationale

Increasingly ALBE teachers have been facing the challenge of teaching some aspects of mathematics in what have traditionally been literacy programs. Many such teachers feel inadequate for the task due to their earlier mathematics experiences, and the formal systems of symbols, passwords, processes, routines and rules which accompanied this. It is therefore not surprising that many ALBE teachers approach teaching any mathematics with a lack of confidence in terms of their personal numeracy and in their ability to teach it in a variety of ALBE contexts including the workplace.

In addition to this group of teachers there are other ALBE teachers, well-trained in teaching mathematics, who may need to refocus their approach in terms of teaching mathematics to adults, supporting vocational students, providing mathematics programs in the workplace and updating their theoretical and methodological approaches.

This professional development package therefore aims to give ALBE teachers a broad understanding of the content and structure of mathematics and how it is applied in modern life. It also aims to develop teachers' confidence in their own use of mathematics and in the theories, methodologies and communication processes appropriate for teaching numeracy in ALBE.

Course development

There were a number of stages in the development of the final package. There was an initial consultation and survey undertaken with State and Territory personnel to identify needs and gaps, and a literature search was completed.

A group of experienced NSW and Victorian adult numeracy teachers and trainers were employed as writers and consultants. There were two meetings of the project team held in Sydney in late 1993 and early 1994. This group was able to endorse and refine a model for looking at the content of the course, and considered in detail the objectives and outcomes of the course and the areas and issues that needed to be addressed. The consultants developed the outline of the materials in a cooperative manner rather than writing individually and in isolation, preferring to work together as a group.

The question the team began with was: *what should numeracy teachers know and be able to do after this course?*

The answer proposed after some discussion was:
After this course a teacher
1 should have a critical appreciation of the place of mathematics in society;
2 should be able to initiate appropriate learning activities by
    – identifying the numeracy needs of students and
    – responding from a variety of approaches to teaching and a range of appropriate mathematical resources and knowledge.
This answer became the basis of the course outcomes.

3 Course outcomes

General outcomes

Teachers will gain a critical appreciation of the place of mathematics in society and be able to initiate appropriate learning activities by identifying the numeracy needs of students and responding with a variety of approaches, a breadth of knowledge and appropriate mathematical resources.

Specific outcomes

- Participants will identify and analyse concepts of mathematics and numeracy and how people learn mathematics; and identify their own assumptions about what mathematics is, how they and others learn it, how they feel about it and why, and how this affects the teaching and learning of mathematics in a range of ALBE contexts.
- Participants will identify a range of learning theories for mathematics and the relationship of those theories to teaching practice.
- Participants will examine the role that mathematics plays in conveying information, and the interplay between mathematics, language, context and the political, social and cultural contexts within which the mathematics arises.
- Participants will develop an appreciation of the social and human construction of mathematics; develop a critical stance on appropriate and inappropriate use of mathematics; identify and use mathematics from different cultures in teaching numeracy; identify and analyse historical, social and political contexts of numeracy; and help students construct mathematical meaning not only through conceptual engagement but also through social, historical and cultural understanding.
- Participants will develop an understanding that numeracy is not a subset of mathematics; that it is a critical awareness which
  - enables individuals to bridge the gap between mathematics and the diverse realities of their life and
  - is reflected by learners in their social practice.
- Participants will become familiar with specific areas of mathematics and available teaching and learning strategies and resources for these areas; and determine whether they are useful for appropriate areas of numeracy learning.
- Participants will undertake curriculum planning activities in mathematics areas, applicable to their teaching context, to meet the needs of their students including:
  - developing guidelines for good assessment
  - giving consideration to alternative methods as they evolve and
  - placing the issue of assessment in a more theoretical framework.

Competency standards

There are no established competency standards endorsed by the NTB for teachers of adult numeracy or basic mathematics in Australia. However there has been an attempt to map this program against competency statements developed by the Department of Employment, Education and Training's International Literacy Year (ILY) project, 'What is a Competent Adult Basic Education Teacher?' In 1990 DEET set up this ILY project; the University of Technology, Sydney conducted the research. An interim report, *What is a Competent ABE Teacher?*, was published in 1992 and in 1993 they published their final report, *The ABE Profession and Competence: Promoting best practice*. Their work provides the only framework currently available for describing the competencies of ALBE personnel.

The results of the research provide a set of competency statements which describe teachers who have had a few years experience in the field. *Adult Numeracy Teaching* relates to many of the competencies—some in detail, others only very briefly.

The particular competency statements relevant to this program are found in Chapter 4, pages 24 to 30, of *The ABE Profession and Competence: Promoting best practice*. The learning outcomes were mapped to the following Units of Competence:

Unit 1: Adult learning and teaching approaches and practices
Unit 2: Selection and placement of students
Unit 3: Managing learning situations

Unit 4: Monitoring learning
Unit 5: Community communication and consultation
Unit 7: Professional development and training

General competencies

Participants will make personal gains in the following key competency areas.

Collecting, analysing and organising ideas and information
They will be able to:

- identify issues in numeracy and mathematics education
- identify and explain the concept of adult numeracy and its relationship to mathematics

Using mathematical ideas and techniques; solving problems
They will be able to:

- identify and analyse concepts of mathematics and numeracy and their relationship to the learning and teaching of adult numeracy
- identify and describe theories relevant to the teaching of adult numeracy
- identify and explain the need for theories of learning mathematics, and their implications for teaching
- identify and analyse how mathematics is a human and social construction
- explain how mathematics is used or misused to influence opinions and decision-making processes in terms of political, social, work, cultural, gender and racial issues.

Planning and organising activities; working with others and with teams
They will be able to:

- analyse and apply specific areas of mathematics, and identify teaching and learning strategies and resources for these specific areas
- identify the implications of theories of learning mathematics to curriculum, teaching and assessment practices in adult numeracy
- plan and develop a numeracy curriculum activity for a particular context and student group.

Recognition given to the course

There have been no formal arrangements made for recognition of the course. Participants may choose to present the Statement of Attainment (or Statement of Achievement) to support a request for Advanced Standing towards courses offered by tertiary and post-secondary institutions. Participants who intend to seek advanced standing are advised to keep all written assignments and other assessment tasks in case they are required for submission to the institution of their choice.

Licensing and regulatory requirements

not applicable

4 Course structure

Outline

The course consists of four compulsory modules to be undertaken in the given order.

A Exploring practice
B Mathematics as a human construction
C Mathematics as a critical tool
D Naming theories: implications for practice

Requirements to receive the qualification

The program includes concurrent and post-course assessment of participants' skills. Those who attend the full program and complete the assessment tasks to the required standard will receive a Statement of Attainment, provided that the course has been accredited in their State or Territory. Otherwise the award will be a Statement of Achievement. The coordinator or presenter will be responsible for assessing whether participants have satisfactorily completed the assessment tasks.

The coordinator will ensure that a statement of their award is sent to each successful participant and will also handle requests for Recognition of Prior Learning.

Exit points None except by completion of all four modules.

On the job training

It is a requirement that participants be in a position to teach numeracy to adults. It is anticipated that some of the learning activities will require application in ALBE contexts and that further learning will occur by applying the course content in this way.

Customisation

The course lends itself to customisation for particular teaching situations or contexts. For example: workplace, TAFE vocational application, SkillShare, community based programs, correctional education.

Entry requirements

Essential

- teaching qualifications
- adult teaching experience
- demonstrated interest in teaching adult numeracy (e.g. current teaching of adult numeracy; past attendance at adult numeracy professional development activities)
- currently teaching adult literacy, language or numeracy

Recognition of prior learning

Participants who wish to have prior learning recognised instead of completing all four modules may apply to the presenter for assessment of current competence against the learning outcomes identified in the program.

5 Assessment

Assessment approach

Participants will be expected to:

- maintain a journal reflecting on the course content and teaching implications
- complete two short curriculum projects, requiring some out-of-class work and oral reporting back
- complete two longer curriculum projects, requiring some out-of-class work and a class presentation. One of these projects is to be written up as a 2500 word essay incorporating discussion of both theory and practice
- participate actively in course activities and discussions.

6 Delivery of the course

Delivery modes

The course has been designed for approximately 60 hours of face to face delivery to participant groups, plus approximately 24 hours of incorporated project work.

The delivery mode is flexible and open to the needs and contexts of each group. Each module is divided into a number of 3 hour sessions, which the presenter/coordinator can therefore utilise in a variety of ways to deliver the course.

It is recommended, however, that the delivery be spread out over a minimum of 3 months with combinations of blocks of course time (half days, full days or several consecutive days). This enables participants to have sufficient time to practice outcomes of the course within their teaching context and to reflect upon the course. Any form of flexible delivery should allow a minimum of one full day face to face per module because the content promotes a hands-on interactive approach to the teaching of adult numeracy. It is therefore essential that the delivery of the course enables the course itself to model this approach. The whole course could be delivered face to face, by the use of small groups of locally based participants with a mentor, or via video conferencing and/or teleconferencing. The course is not designed for self-study.

Resources

The package for *Adult Numeracy Teaching* contains ten sections arranged as follows:

Curriculum:	the competency based curriculum
Modules A–D:	each containing teaching notes for the presenter
Resources A–D:	resources for the presenter and participants, including activity sheets, handouts, overhead transparencies and readings
Projects:	information on Curriculum Projects and the Numeracy Journal and how they should be assessed.

Qualifications of presenter/coordinator

The Minimum Competency Statements of Presenters are the same as for other National Staff Development Committee professional development packages.

Presenters should:
- be well-informed about policies and theoretical debates in the field of numeracy and mathematics education related to ALBE, and abreast of current issues;
- be knowledgeable and fully up to date about broader issues, policies and current events in political, social, industrial and educational fields as they impinge on the field of ALBE;
- be actively involved in continuing professional education and making a contribution to the field; and
- have a proven ability in delivering professional development programs, workshops or seminars.

It is also recommended that one of the presenters/coordinators for each delivery of *Adult Numeracy Teaching* should be a trained mathematics teacher.

7 Articulation and credit transfer

The course is designed as a professional development program for teachers and is not designed as a formal teaching qualification. Opportunities will occur for articulation from this course to a variety of university courses, including the opportunity for some credit transfer.

Individual states and territories, or organisations, may choose to organise formal articulation and credit transfer agreements. Where no formal agreements exist, individual participants will need to negotiate credit with individual universities. (See section 3 above.)

Career pathways

The course will improve the teaching skills of ALBE numeracy teachers. It also intends to widen the skills of literacy and language teachers to enable them to address the learning needs of their students more adequately.

8 Ongoing monitoring and evaluation

State and territory systems will be responsible for monitoring and evaluating the course to maintain quality and relevance.

Module information

Module A: Exploring practice

1 **Nominal duration:** 18 hours face to face

2 **Module purposes**

Participants will identify and analyse for a range of ABE contexts
- concepts of mathematics and numeracy
- how mathematics is learnt
- their own assumptions and feelings about mathematics
- how these affect the learning and teaching of mathematics

Participants will
- analyse and apply specific areas of mathematics
- identify the teaching and learning strategies and resources for these areas
- evaluate their usefulness in a numeracy teaching and learning context.

3 **Prerequisites:** No course prerequisites

4 **Relationship to competency statements**

This module is related to the following elements of competency:

Element 1.2	Applies knowledge of theories of mathematics learning and teaching to develop adult numeracy skills.
Element 1.3	Applies knowledge of theories of learning, including learning relevant to adults in any ABE situations.
Element 1.4	Uses a variety of learning and teaching strategies to pursue literacy and numeracy goals for personal, social, educational and vocational purposes.
Element 4.4	Continually reflects on and adjusts own practice.
Element 6.2	Represents ABE both within the teachers' workplace and in the wider community.
Element 7.1	Is informed about current issues, policies and theoretical debates in the field of ABE.
Element 7.2	Is actively involved in continuing professional development in the field of ABE.

5 **Content**

- Issues in numeracy and mathematics education
- Mathematics abuse and anxiety
- Mathematical and teaching topics
- Why we need theories of learning and teaching
- Perceptions of mathematics
- Making meaning in mathematics
- Language and mathematics
- Using language to enhance the learning of mathematics
- Cultural differences in mathematics

6 Assessment strategy

Assessment method

1 Maintain a journal of reflections on the content of the module and any teaching implications.
2 Complete Curriculum Project 1 (CP1) – Making meaning.

Conditions of assessment

1 The journal which comprises personal reflections on the course is to be done out of course time, consisting of 2 brief informal entries for the module. Detailed guidelines are provided in the teaching resources.
2 CP1 to include a mixture of practical trials and course discussion.

7 Learning outcome details

Learning outcomes

A1 Identify significant issues in numeracy and mathematics education.
A2 Identify and analyse concepts of mathematics and numeracy and their relationship to the learning and teaching of adult numeracy.
A3 Identify and explain the need for theories of learning mathematics, and their implications for teaching.
A4 Analyse and apply specific areas of mathematics.
A5 Identify teaching and learning strategies and resources for the specific areas analysed in A4.

Assessment criteria

A1 Name a range of key issues facing the teaching of adult numeracy in Australia.
A2 Identify two contrasting concepts of the learning of mathematics and their implications for teaching.
A3 Identify causes and signs of mathematics anxiety and detail strategies for overcoming mathematics anxiety.
A4 Explain how theories of learning mathematics can make teaching more effective.
A5 Identify mismatches between the participant's own theories and their teaching practices.
A6 Identify strategies for narrowing the gap between participant's own theories and practices.
A7 Apply knowledge for the teaching of selected mathematical concepts.
A8 Describe the relationship between mathematics learning, language and literacy.

8 Delivery of the module

Delivery strategy

18 hours face to face and 3 hours project work
For flexible delivery see Curriculum, page 7.

Resource requirements

A teaching room set up for whole group and small group work.

Materials

Detailed lists of materials needed for each session are included with the teaching notes for each module, including readings. Participants will themselves need access outside of course time to an adult education teaching/learning situation where they can work with adult numeracy students.

Useful learning resources for Module A are:
- *Adult Numeracy Teaching*: Presenters' Notes
- *Adult Numeracy Teaching*: Participants' readings (P)
- *Numeracy in Focus,* No. 1
- *Breaking the Maths Barrier*
- *Reclaiming Mathematics*
- *Numeracy and How We Learn*

Module B: Mathematics as a human construction

1 Nominal duration: 24 hours (15 hours face to face, 9 hours project work)

2 Module purposes

Participants will develop knowledge of the social and human construction of mathematics.
Participants will develop teaching strategies that will help their students construct mathematical meaning not only through conceptual engagement, but also through social, historical and cultural understanding.
Participants will analyse and apply specific areas of mathematics identify related teaching and learning strategies, and resources and evaluate their usefulness in an adult numeracy context.

3 Prerequisite: Completion of Module A

4 Relationship to competency statements

This module relates to the following elements:

Element 1.2	Applies knowledge of theories of mathematics learning and teaching to develop adult numeracy skills.
Element 1.3	Applies knowledge of theories of learning, including learning relevant to adults in any ABE situations.
Element 1.4	Uses a variety of learning and teaching strategies to pursue literacy and numeracy goals for personal, social, educational and vocational purposes.
Element 4.4	Continually reflects on and adjusts own practice.
Element 7.1	Is informed about current issues, policies and theoretical debates in the field of ABE.
Element 7.2	Is actively involved in continuing professional development in the field of ABE.

5 Content

- Mathematics as a human construction
- Problem solving in mathematics
- Using mathematics from different cultures
- The mathematics, historical origins and cultural connections of selected topics: classification of geometric shapes; angles, shape and symmetry; regular polygons; space, shape and symmetry; Pythagoras theorem; circles and pi; lengths, scales and ratios; topology; measurement; fractals; algebra, relationships and patterns; and tessellations and mosaics.

6 Assessment strategy

Assessment method

1 Maintain a written journal reflecting on the content of the module and any teaching implications.
2 Complete Curriculum Project 2 (CP2) – Exploring mathematics.

Conditions of assessment

1 For the journal, see Module A.
2 A contract for the content of CP2 is to be negotiated between the participants and the presenter. CP2 is a mathematical investigation of a topic of the participants' choice. It will include planning, exploring the topic, and a presentation to the whole group. It may include the writing of an essay. CP2 is preferably to be done as a team activity.

7 Learning outcome details

Learning outcomes

B1	Identify and analyse how mathematics is a human and social construction
B2	Analyse and apply specific areas of mathematics
B3	Identify teaching and learning strategies and resources for the specific areas in B2.

Assessment criteria

B1	Identify and explain how concepts of space and shape are a social and human construction
B2	Describe how particular cultural needs affect concepts of shape and space or some other mathematical concept
B3	Report on an investigation of a mathematical problem
B4	Apply knowledge for the teaching or investigation of selected mathematical concepts
B5	Demonstrate knowledge of problem solving strategies

8 Delivery of the module

Delivery strategy

15 hours face to face and 9 hours project work
Project work is to be done as a team if possible, in the workshop or an outside location.

Resource requirements: See Module A.

Module C: Mathematics as a critical tool

1 Nominal duration: 27 hours (18 hours face to face, 9 hours project work)

2 Module purposes

Participants will develop an understanding that numeracy is a critical awareness which enables them to bridge the gap between mathematics and the political, social, cultural, work and training contexts of their lives.
Participants will analyse the role that mathematics plays in conveying information, including the interplay between mathematics, language and content.

Participants will analyse and apply specific areas of mathematics; develop curriculum activities for particular contexts; and identify appropriate teaching and learning strategies and resources.

3 Prerequisites: Completion of Modules A and B

4 Relationship to competency statements

This module relates to the following elements:

Element 1.2	Applies knowledge of theories of mathematics learning and teaching to develop adult numeracy skills.
Element 1.3	Applies knowledge of theories of learning, including learning relevant to adults in any ABE situations.
Element 1.4	Uses a variety of learning and teaching strategies to pursue literacy and numeracy goals for personal, social, educational and vocational purposes.
Element 2.1	Interviews, assesses, places or refers students.
Element 2.2	In selecting students, requirements of government and organisation policies are balanced with the learning needs of individuals.
Element 3.1	Uses knowledge of curriculum theories and curriculum documents to develop and implement a program/curriculum compatible with individual, group and program needs.
Element 3.2	Manages time, space and resources to maximise educational outcomes.
Element 3.3	Adapts curriculum in the light of changing circumstances and changing student needs.
Element 4.1	Uses knowledge of current theories of language, mathematics and learning to select and evaluate appropriate assessment methods.
Element 4.2	Modifies students' programs as a result of continual monitoring.
Element 4.4	Continually reflects on and adjusts own practice.
Element 6.3	Plans and implements ABE programs.
Element 7.1	Is informed about current issues, policies and theoretical debates in the field of ABE.
Element 7.2	Is actively involved in continuing professional development in the field of ABE.

5 Content

- Critical mathematics and numeracy:
- Integration of literacy and numeracy
- Contexts of learning mathematics:
- Curriculum development
- Mathematical and teaching topics:

6 Assessment strategies

Assessment method

1. Maintain a journal reflecting on the content of the module and any teaching implications.
2. Complete Curriculum Project 3 (CP3) – Mathematics excavation

Conditions of assessment

1. For the journal, see Module A.
2. A contract for CP3 is to be negotiated between the participants and the presenter.

CP3 will involve identifying the mathematics embedded in a context of the participant's choice and developing a critical teaching activity or curriculum outline for that context, also related to their work context.
It will include planning, exploring the topic, and a presentation to the course group. CP3 is preferably to be done as a team activity. It may also include the writing of an essay.

7 Learning outcome details

Learning outcomes

C1 Explain how mathematics is used or misused to influence opinions and decision-making processes in terms of political, social, work, cultural, gender and racial issues.
C2 Plan and develop a numeracy curriculum activity for a particular context and student group.
C3 Analyse and apply specific areas of mathematics.
C4 Identify teaching and learning strategies and resources for specific areas in C2 and C3.

Assessment criteria

C1 Describe examples of how mathematics can be used to disempower adults.
C2 Describe numeracy activities that can be used to explore mathematics critically.
C3 Identify learner needs.
C4 Identify how mathematics is used and applied in different contexts.
C5 Identify relevant state and national policies and strategies that influence ABE curriculum.
C6 Identify and analyse the mathematics embedded in selected contexts.
C7 Develop, for a particular context and group of students, a curriculum activity.
C8 Apply knowledge for the teaching, investigation and curriculum development of selected mathematical topics.
C9 Demonstrate knowledge of issues in the integration of literacy and numeracy.

8 Delivery of the module

Delivery strategy
18 hours face to face and 9 hours project work
Project work to be undertaken as a team if possible in the course workshop or another outside location.

Resource requirements: As for Module A

Module D: Naming theories: implications for practice

1 Nominal duration: 15 hours face to face

2 Module purposes

Participants will identify a range of learning theories for mathematics and their relationship to numeracy teaching practice.
Participants will analyse and apply specific areas of mathematics, identify related teaching and learning strategies and resources and evaluate their usefulness in a numeracy context.

3 Prerequisites: Completion of Modules A, B and C

4 Relationship to competency statements

This module relates to the following elements:

Element 1.2	Applies knowledge of theories of mathematics learning and teaching to develop adult numeracy skills.
Element 1.3	Applies knowledge of theories of learning, including learning relevant to adults in any ABE situations.
Element 1.4	Uses a variety of learning and teaching strategies to pursue literacy and numeracy goals for personal, social, educational and vocational purposes.
Element 2.1	Interviews, assesses, places or refers students.
Element 2.2	In selecting students, requirements of government and organisation policies are balanced with the learning needs of individuals.
Element 3.1	Uses knowledge of curriculum theories and curriculum documents to develop and implement a program/curriculum compatible with individual, group and program needs.
Element 4.1	Uses knowledge of current theories of language, mathematics and learning to select and evaluate appropriate assessment methods.
Element 4.2	Modifies students' programs as a result of continual monitoring.
Element 4.4	Continually reflects on and adjusts own practice.
Element 6.2	Represents ABE both within the teachers' workplace and in the wider community.
Element 6.3	Plans and implements ABE programs.
Element 7.1	Is informed about current issues, policies and theoretical debates in the field of ABE.
Element 7.2	Is actively involved in continuing professional development in the field of ABE.

5 Content

- Looking at theories of learning and teaching mathematics: transmission, constructivism, critical constructivism
- Implications of theories for practice
- What is numeracy?
- Mathematical and teaching topics in selected areas

6 Assessment strategy

Assessment method

1 Maintain a journal reflecting on the module content and teaching implications.
2 Complete Curriculum Project 4 (CP4) – Assessment tool.

Conditions of assessment

1 The journal is to be personal reflections on the course to be done out of workshop time, consisting of: one brief, informal entry for this module and one summative entry reflecting on the whole course and the question, 'What is numeracy?'
2 CP4 is a small group or individual activity, which will include course discussions, and trialling in a teaching situation. It will be based on developing an assessment task for a teaching/learning activity.

7 Learning outcome details

Learning outcomes

D1 Identify and describe theories relevant to the teaching of adult numeracy.
D2 Identify the implications of the theories to curriculum, teaching and assessment practices in adult numeracy.
D3 Identify and explain the concept of adult numeracy and its relationship to mathematics.
D4 Analyse and apply specific areas of mathematics.
D5 Identify teaching and learning strategies and resources for the specific areas in D4.

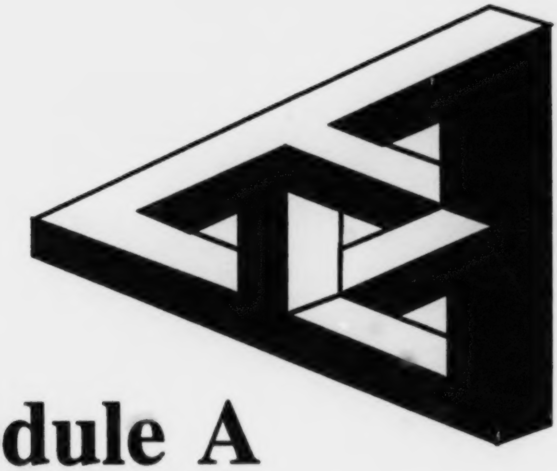
Assessment criteria

D1 Describe transmission, constructivism and critical constructivism theories of the learning of mathematics.
D2 Identify and describe the implications of the theories for developing adult numeracy curriculum.
D3 Describe a range of alternative assessment strategies for adult numeracy.
D4 Report on the participant's view and concept of adult numeracy and mathematics.
D5 Explain how technology, including calculators, can be used in a numeracy teaching/learning context.
D6 Apply knowledge for the teaching of selected mathematical topics.

8 Delivery of the module

Delivery strategy
Curriculum Project 4 (CP4) to be undertaken as a team, if possible, within the course workshop or another outside location.

Resource requirements
As for Modules A, B and C



Module A
Exploring practice

Presenter's notes

Module A: Exploring practice

Nominal Time: 18 hours

Brief description

By exploring practice participants are introduced to the need for theoretical and other tools for constructing meaning in mathematics teaching and learning as they journey through some crucial mathematical areas.

Rationale

The course begins by raising issues about how people learn mathematics in an effort to make it clear that taken-for-granted practices and assumptions are actually problematic. The need for theory is discussed, but particular theories are not explored in detail until Module D. By then participants will have had chances throughout the course to query and construct different theories. In this module participants will consider the genesis of maths anxiety and its connections with lack of meaning. A variety of tools for making meaning: language, concrete models and social contexts, will be introduced as participants explore perimeter and area, place value, basic operations, fractions and algebra. The accompanying Curriculum Project I, *Making meaning*, will encourage participants to examine these issues in greater depth in relation to the four basic operations: addition, multiplication, subtraction and division.

Learning outcomes

Participants will be able to:
- identify significant issues in numeracy and mathematics education
- identify and analyse concepts of mathematics and numeracy and their relationship to the learning and teaching of adult numeracy
- identify and explain the need for theories of learning mathematics, and their implications for teaching
- analyse and apply specific areas of mathematics
- identify teaching and learning strategies and resources for the specific areas analysed above.

Assessment requirements

1 Maintain a journal of reflections on the content of Module A and any teaching implications.
The journal, which consists of personal reflections on the course, is to be done out of course time, with at least 2 brief informal entries for this module. Detailed guidelines are provided in the Curriculum Projects and Numeracy Journal section.

2 Complete Curriculum Project 1(CP 1) – Making meaning.
CP 1 is to include a mixture of practical trials and course discussion.

References

A bibliographical list of references for this module is given at the end of Resources A [HO17].

Module A outline

section	*time*	*development of issues and activities*	*maths involved*
A1 – You and numeracy			
A1.1 Introduction and maths autobiographies	1.5 h	– *icebreaker* – *mathematical autobiographies*	
A1.2 Issues in numeracy	1.5 h	– *looking at posters to raise issues about numeracy education* – *discussing outline, expectations of the course*	
A2 – Unlearning about maths			
A2.1 Unlearning maths: a case study	1.5 h	– *looking at boundaries* – *discussion* – *talking about the case study*	area, perimeter and their relationship
A2.2 Maths abuse and anxiety	1.5 h	– *cooperative problem solving* – *maths anxiety: how do you know if you've got it?* – *anxiety and meaninglessness* – *meaning, patterns and the 9x table*	cooperative logic patterns: 9x table
A3 – Why theory?			
A3.1 Why do we need theories?	2 h	– *case study reflection* – *What theories are we already working from?* – *using metaphors and pictures* – *different theories, different questions* – *so why theory?*	review area and perimeter methods of subtraction
A3.2 CURRICULUM PROJECT 1 – MAKING MEANING	1 h	Curriculum Project 1 (CP1) – Part 1 *Exploring the four operations*	four basic operations
A4 – Making meaning with symbols			
A4.1 Algebra activities	2 h	– *issues surrounding algebra* – *algebra through domestic situations* – *algebra through patterns* – *algebra through physical representation* – *review*	algebra and relationships and pattern
A 4.2 CURRICULUM PROJECT 1 – MAKING MEANING	1 h	Curriculum Project 1 – Part 2 *Reflection on student observations and modelling the four operations*	four operations

A5 – Making meaning from counting systems			
A5.1 CURRICULUM PROJECT 1 – MAKING MEANING	1 h	Curriculum Project 1 – Part 3 – *different cultures and different ways of doing maths*	four operations
A5.2 Why count—and how?	1 h	– *the word for five ...* – *looking at other number systems* – *why do we count?*	
A5.3 Operating 'our' number system: a beginning	1 h	– *exploring 'our' Hindu–Arabic numeration system*	place value and number systems
A6 – Strategies for making meaning			
A6.1 Operating the number system: a continuation	1.5 h	– *building meaning – how do we do it?* – *multiple embodiments* – *and what about little numbers?*	place value: more on operations and numbers < 1
A6.2 Language as a tool	1 h	– *Lesley's story* – *exploring fractions* – *talk, play, write, argue, negotiate, reflect*	division fractions
A6.3 Constructing meaning in practice: a review	30 min		

A1 You and numeracy

A1.1 Introduction and maths autobiographies

Brief description

After an introduction to the course, participants talk to each other about their maths autobiographies, reflecting on their own past experiences with mathematics, and the positive and negative influences on their learning and their attitudes to mathematics.

Rationale

This activity is an ice-breaker. It will also start to identify issues related to the teaching of mathematics and numeracy. Individuals will be introduced to the Group. The presenter and participants should get a picture of the work done by the participants, their workplaces and their mathematical background. Discussion of the impact of past mathematical experiences should help identify those experiences which formed positive attitudes to maths and those which hindered progress. Effective teaching strategies in numeracy education will become more explicit.

Aims

This session aims to:
- introduce participants to each other
- foster an awareness that negative experiences in mathematics are not uncommon
- use the experience of participants as learners to focus on what they would like to be as teachers.

Preparation

Presenter
You will need to be fairly well acquainted with the flow of the entire course before you begin, and with the specific administrative requirements of your particular program.

Familiarise yourself with journal activities and guidelines.

Time: 1.5 hours

Materials needed

Handouts/OHTs/paper
The Numeracy Journal [NJ1] from the Curriculum Projects and Numeracy Journal section
OHP, clear OHT and marker

A1.1 Introduction and maths autobiographies

Detailed procedure

1 Welcome

2 Introduce the idea of the journal.
Refer participants to *The Numeracy Journal* [NJ1].
During this course you will be expected to keep a fairly regular journal. It can be in a separate exercise book, or loose-leaf in the Course folder. You will be hearing more about it next time. The first entry in the journal will be your answers to these questions.
Have participants respond to the first set of questions [OHT1].

3 In pairs, as far as possible unfamiliar to each other, ask participants to tell each other their mathematical autobiographies.
You should focus on CRITICAL INCIDENTS or episodes that impacted on your own learning, teaching or attitudes to maths. Positive and negative influences should be noted.

4 Pairs of participants then briefly introduce their partners (name, job and workplace) and report on any central or critical incidents in the other person's autobiography.
Write the qualities, conditions and issues arising from these autobiographies and incidents on the board or OHP. Relate them to aspects of good and bad teaching, or other factors.
You could tell your own story briefly.

5 Working with the Group try to categorise and compare the qualities, conditions and issues.
Discuss as a Group conditions and qualities that influence people's reactions to maths, and the implications and issues for good or bad numeracy teaching.
Highlight the good ones:

- encouraging the use of good strategies
- discussing the effectiveness of particular strategies
- teasing out the principles behind them
- looking at
    - how we learn
    - the way we remember and /or store information
    - how we use it outside school
    - its relevance.

Elicit the factors that made some of us enjoy our experience of it, some of us feel indifferent and others hate it.

A1.2 Issues in numeracy

Brief description

By analysing quotations and cartoons participants raise issues about adult numeracy and share their feelings and attitudes towards topical issues in the numeracy context.

Rationale

This session begins by raising some of the issues in numeracy education. It does this in a relaxed and informal way which also acts as a follow on icebreaker from the first more individual activity. Small group work is used in an attempt to establish a collaborative environment in the Group and to honour the experiences bought to the Group by individuals.

Aims

This session aims to:

- act as an icebreaker, i.e. get participants talking and getting to know each other;
- use the participants' experience to raise issues in adult numeracy;
- identify issues in numeracy and mathematics education and analyse concepts of mathematics and numeracy and their relationship to the learning and teaching of adult numeracy; and
- talk explicitly about the program: the expectations of both the presenter and the participants, and the flow and philosophy of the course.

Preparation

Presenter
Be clear about the expectations of this section for you as presenter as well as for the participants before you begin this session, e.g. time commitment. Fix the posters to the wall with reasonable space in between.

Time: 1.5 hours

Materials needed

Concrete materials
5 to 7 posters AS1–7;
(Blow them up to A3 size.)
butcher's paper
thick pen for each group
blue tac
Handouts
participants' ring-binders, with any background course material, readings, notes etc., including *Outline of the ANT Course* [HO1]

A1.2 Issues in numeracy

Detailed procedure

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1 Ask participants to walk around the room and read all the posters.

Before they start, ask them, as they move about, to decide on a poster they would like to consider in more depth. (15 min)

2 Have participants gather around their chosen poster in groups of 4 or 5. If one poster seems to be left out, point this out and ask if anyone is willing to take it on.

Ask each small group to choose a spokesperson to report back to the whole Group.

On butcher's paper each group should record:

- a title for their poster
- issues arising from the poster (these could be recorded in words or illustrations)
- questions it raises that the course might do well to address.

3 As each spokesperson reports to the Group, use the opportunity to highlight issues, draw connections and indicate how the course addresses these.

The quotes were originally collected under these headings:

AS1	What is maths?
AS2	anxiety and dislike
AS3	mathematical delight
AS4	ways of teaching it
AS5	its meaningfulness—or not
AS6	its social origins
AS7	access and equity

These issues may arise, but quite different categories could emerge and should be welcomed.

4 Overview and administration

Talk about the philosophy, aims, structure, content and expectations of the course.

More specifically, begin to marry and adapt your own expectations to the expectations of the participants.

Assessment and time commitments should be clearly spelt out.

Give out the ring-binders.

A2 Unlearning about maths

A2.1 Unlearning maths: a case study

Brief description

The participants engage in a question about the relationship between areas and boundaries and examine some of the assumptions they bring to the course about the nature of maths and its learning and teaching.

Rationale and aims

Coming at the end of the initial session which has included a lot of discussion, this session gives the participants their first chance to engage in a mathematical problem. The aim, however, is not necessarily for the participants to demonstrate successful problem-solving, so much as to

- observe themselves during the process
- note how they went about the task and how they felt about it and
- try to tease out from this evidence the assumptions they bring to the course about maths and its learning and teaching.

Preparation

Presenter
Read
Johnston (1992)

Time: 1 hour

Materials needed

Concrete materials
match sticks
a ball of string and scissors to cut it
grid paper (not too fine)
Handouts
Learning and teaching mathematics: [HO2]
Looking at boundaries [HO3]

Reference

Johnston, B. 1992, 'Teachers making meaning in maths: or, what does it mean to learn maths?' in B. Southwell et al., Proceedings of the 15th Annual Conference of the Mathematics Education Research Group of Australasia, MERGA, Sydney.

A2.1 Unlearning maths: a case study

Detailed procedure

This session uses both small group and Group discussion and is divided into four parts:
- **Looking at boundaries** (30 min approx.)
- **Discussion** (30 min approx.)

There will be **Reflections** on this activity in Session A3.1.

Looking at boundaries

1 Introduce the activity by describing the case study, handing out *Learning and teaching mathematics ... case study notes ...* [HO2] and going through them.
Emphasise that the activity is a case study of the participants themselves, so as well as actually participating they should be watching themselves, perhaps noting down points of interest.

2 Before you hand out *Looking at boundaries* [HO3] say something like:
I am going to ask you a question about area. I want you to give a quick intuitive answer. You don't have to tell anyone else, but please commit yourself one way or the other. It makes it more interesting if you have some sort of stake in it.
Then ask the central question which is in bold in Q1.

3 Wait for a minute or two, and hand out *Looking at boundaries* [HO3].
Don't discuss the issue at this point.

4 Go straight on to Q2. Do the same here. Get participants to write down T or F.

5 In small groups of 3 or 4, ask participants to come to a consensus, convincing each other. It is the valuable task of those who do not follow the argument to say so, and the task of those who think they are right to find ways of passing on their conviction to others.

6 Go on to Q3.
Because the point of the exercise is self-observation, it does not matter in fact whether the participants finds the problem easy or hard: both reactions are grist to the mill at this stage, both positions have something to offer in a small group discussion.
Your task during this time is to circulate through the groups, allay excessive anxiety, ask clarifying questions, listen to those who are finding it difficult to express ideas or puzzlement, suggest to any group whose members (rightly) agree that they find alternative

methods to demonstrate their conviction, and offer materials such as matches, string or squared paper to those who want them.

Could you draw a diagram or give an example of that?

Would any of these materials help?

Some string? is one piece enough? two bits? the same length?

Is there another way you could demonstrate that?

When all the small groups have come to some conclusion, explore the question in the large Group by using the different strategies that have arisen.

Discussion

1 There will probably emerge a variety of ways of finding an answer to the question through using

- diagrams
- particular examples ('If the fence is to be 12 m long, what gardens might I have?')
- concrete materials, e.g. string or a belt
- formulas or algebra, if they emerge.

The aim is to come to the realisation that:

the length of a boundary is not enough to tell us how much area there is inside it.

For most people, the most convincing evidence seems to be the demonstration that with a belt you can make shapes from those with large area (more circular), to those with progressively less area (longer, or more indented), to ones with almost as little area as you nominate (by making the oval as thin as you like).

Points that may arise during the discussion

- It is likely that most people will initially answer YES to the main question, and gradually change, as they respond to the statements in Q2, and listen to others' arguments.
- It is not only people with little understanding of formal maths that make the wrong assumptions here. Often those with quite advanced maths are imprisoned by their use of formulas.
- Some people will be shaken and disturbed, at least initially, by being wrong. It is important that they see they are not alone, and important that a variety of solutions is presented so they have more chance of being convinced.
- Language has different functions. Mathematicians would probably say that Lee's statement in Q2 was false, because it is not always true, but most people using everyday meanings would say it was not necessarily true, because it is sometimes true, sometimes false. We need to know this when we choose our words.
- Real world constraints often interfere with the neatness of a model.

What about a garden on the top of a hill, like a cone? The enclosure is no longer 2-D and so we have a greater area inside the boundary!
Or, mathematically, it seems fine to say that Soheila's statement is false, but surely if you know the area of your garden then you will know the shape and therefore the dimensions. What about a tree in the garden that you need to fence around (a useful counter example to Lee's statement)?

- This is all very interesting but how is it relevant? Some real world examples include:
  making a hen-house—a square (if keeping to right angles) or a circle gives most interior space for limited materials;
  for the same reason, cheaper houses probably have more square rooms (participants could check—do they, in fact ?);
  a long thin building will need more windows than a more square one with the same space inside.
- Many people only think about the question in terms of rectangles. Why? For some it is because they learnt about area initially in terms of the formula for rectangles.
- Are squares rectangles? Yes, but why? Who says so? The mathematicians, basically, have organised their sets and subsets this way. So what is an oblong? This is definitely a more everyday word for the rectangle that has one side longer than another.
- Usually about 70% or more are intuitively wrong in their response to this activity. Why so many? For some people the concepts of area and perimeter have only a hazy existence outside formulas. Often these distantly remembered formulas were learnt at the same time, and remain somehow connected. If one is big, so is the other. For these people, it is eye-opening to realise that 'area' refers to something in the real world … something that it is allied to ideas of covering and skin.
- Did learning occur during this activity? did teaching? what is teaching? is teaching *telling?*

2 Talking about the case study

Remind participants that before next time they are expected to carry out the activity with another adult (using Q4) and write up in their journal brief reflections on the whole process.

3 Follow up: reflections

During the next session the participants in small groups discuss:

- what assumptions were brought to light in this case study about maths
- how they and others learn it
- how they feel about it and why
- whether these assumptions need to be challenged or can be challenged
- what implications there are for teaching maths to adults.

A short report can be made from each small group to the whole Group.

A2.2 Maths abuse and anxiety

Brief description

Participants identify signs and causes of maths anxiety, work out strategies for lessening it and tease out to what extent the problem is individual or systemic.

Rationale/aims

Many participants will come to the course with some feelings of maths anxiety, and such feelings are widespread amongst their students. This session allows those feelings to be analysed. To do this, participants will engage in collaborative problem-solving situations and identify how they differ from more traditional ones, discuss signs and possible causes of maths anxiety, consider the importance of meaning in mathematics and develop strategies that would promote a better learning environment. By the end of the session, participants should also begin to critique the notion of maths anxiety as an individual problem.

Preparation

Presenter
Read
Buerk (1985) and Frankenstein (1989)
Organise cooperative logic problems

Participants
Read
Math anxiety defined [P1] or
Mathematics anxiety [P2]

Time: 1.5 hours

Materials needed

Concrete materials
at least 2 cooperative logic problems from *Mathematics: A New Beginning/Breaking the Maths Barrier/Get It Together*
7 cups and 20 counters for each group

Handouts
Making meaning [HO5]
Looking for patterns [HO6]
Talking about maths anxiety [HO4]
Math anxiety defined [P1]
Mathematics anxiety [P2]

OHTs/paper etc.
Making meaning [OHT2]

References

* Boomer, G. 1986, 'From catechism to communication: language, learning and mathematics', *Australian Mathematics Teacher*, Vol. 42, pp. 2–7.
* Buerk, D. 1985, 'The voices of women making meaning in maths', *Journal of Education*, Vol. 167, No. 3, pp. 59–70.
Erickson, T. (ed.) 1989, *Get It Together*, EQUALS, Lawrence Hall of Science, University of California.
Frankenstein, M. 1989, *Relearning Mathematics: a different third R—radical mathematics*, Free Association Press, London.
Jacobs, H. 1982, *Mathematics: a human endeavor*, W H Freeman & Co, New York
Johnston, B. 1992, 'Mathematics: an abstracted discourse', Paper presented at the Sixth International Congress of Mathematics Education, Quebec.
* Oxreider, C. and Ray, J. 1982, *Your Number's Up*, Addison-Wesley, USA
* Webber, V. 1988, 'Maths as a subversive activity', *Education Links*, 32, 6–9.

* available in Johnston, B. (ed.) 1992, *Reclaiming Mathematics*, UTS, Sydney

A2.2 Maths abuse and anxiety

Detailed procedure

This section consists of four parts:

– **Cooperative problem-solving activity and discussion**	(45 min)
– **Maths anxiety: How do you know if you've got it?**	(20 min)
– **Anxiety and meaninglessness**	(10 min)
– **Meaning, patterns and the 9x table**	(15 min)

Cooperative problem solving

This section is an activity in small groups followed by a Group discussion.

1 Participants form groups of approximately 4. Introduce to them the concept of the cooperative logic activity. Give each group a set of cards. Each person has information to share with the group in turn. Together the group tries to solve the given problem.

2 Go around the groups encouraging them to focus on the process involved in coming to a solution. What strategies are they enlisting? Which previously given rules are they struggling with? Are these rules still appropriate in this context?

3 Quickly extract as many general problem-solving strategies as possible, e.g.
- read and understand all the information
- trial-and-error or guess-and-check
- modelling with concrete materials
- drawing diagrams
- organising information into charts/tables
- looking for patterns
- trying alternative approaches.

4 Brainstorm how this type of cooperative activity differs from traditional maths activities:
- 'cheating' is not applicable as there is nothing to gain
- ideas bounce off one other
- it is a language rich environment
- it is less competitive
- it depends on the cooperation of each member of the group.

5 Ask participants how they feel about this style of teaching/learning compared with traditional teaching. This can lead into contrasting the two scenarios of learning—transmission versus constructivism, without necessarily spelling them out.

Maths anxiety: How do you know if you've got it?

1 Use the previous discussion to raise the issue of 'maths anxiety', which will have appeared somewhere in the discussion on bad teaching practices.
Brainstorm the signs of maths anxiety, e.g. tension, guilt, panic, headaches, avoidance.
Use *Talking about maths anxiety* [HO4] if inspiration runs dry, and refer participants to *Math anxiety defined* [P1] or *Mathematics anxiety: Misconceptions about learning maths* [P2] for further reading.

2 Discussion the possible causes of maths anxiety, e.g.

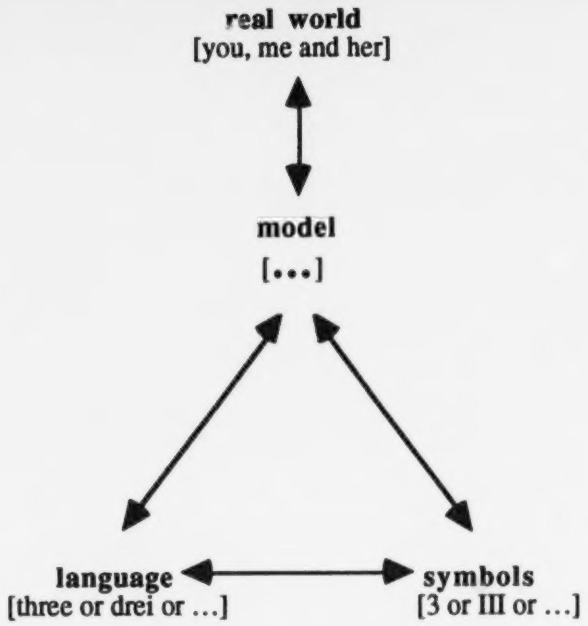
- dislike of school
- fear of a particular teacher
- uncomfortable learning methods
- pressure to be 'clever'
- emphasis on product rather than process
- myths about the importance of maths
- gaps in schooling
- definitions of masculinity/femininity
- lack of use since leaving school
- introduction of calculators.

Anxiety and meaninglessness

1 Consider the following definition of anxiety in *Making meaning* [OHT2, HO5]
Anxiety is
'the sense of disintegration which occurs when a meaning-making organism finds itself unable to make meaning'. (Buerk, 1985, p. 67)

2 Discuss the importance of meaning in mathematics.
Point out that a number of the causes that we have just listed—though not all of them—would result in a sense of the meaninglessness of much mathematical activity.
What do we mean by 'meaning'? Making meaning involves making connections—a variety of connections.

3 Discuss the diagram on OHT2.
The more arrows that connect, the more meaningful a particular concept becomes. Most of us were lucky if even the two bottom categories connected. Rarely did we connect them to any model, and still more rarely to the real world.
Show participants how the diagram can illustrate possible connections for establishing a strong concept of the number 'three'.



Meaning, patterns and the 9x table

1 One level of meaning is in the connections we can make between the numbers in something like the 9 times table ... it is the level of pattern-making.
Ask participants about any patterns that they know exist within the 9 times table.
Introduce any that have not been covered by the participants.
See *Looking for patterns* [HO6] for details.

2 Indicate the two aspects of meaning involved:
- a sense that patterns are ways of connecting and give a power of prediction, a non-arbitrariness
- the possibility of asking and investigating the question:
  Why *this* pattern? Why not some other pattern?

3 If there is time, or as the basis of a possible journal task, get participants to look at questions starting with *what would happen if ...?* [See HO6]

A3 Why theory?

A3.1 Why do we need theories?

Brief description

Participants begin to unravel the assumptions they work from, and to consider the implications of theories, implicit or explicit, for their practice.

Rationale/aims

This session points to the importance of theory for informed practice. In Section A2.1, *Unlearning to teach numeracy*, participants began making explicit the assumptions that shape their teaching. In this section, participants consider the relevance of theory and use a variety of approaches to tease out their assumptions further, in preparation for naming and analysing related theories later in the course.

Preparation

Presenter
Read
Johnston (1995), Siemon (1989) and Allan (1994)
Participants
Read
Which map shall I use? [P3] and
Knowing and believing is seeing [P4]

Time: 1 hour

Materials needed

Concrete materials
at least 2 cooperative logic problems from *Mathematics: A New Beginning* or *Breaking the Maths Barrier* or *Get it Together*
7 cups and 20 counters for each group
Handouts
Metaphors [HO7]
Which map shall I use? [P3]
Knowing and believing is seeing [P4]
OHTs/paper etc.
The present state of maths.[OHT3]

References

Allan, L. 1994, *Reflection and Teaching: cooperative workshops to explore your experience*, Adult Literacy Information Office, TAFE NSW, pp. 27–28.
*Cobb, P. 1988, 'The tension between theories of learning and instruction in mathematics education', *Educational Psychologist*, Vol. 23, No. 2.
Davis, R., Maher, C . & Noddings, N. (eds) 1990, 'Constructivist views on teaching and learning mathematics', *Journal for Research in Mathematics Education*, Monograph No. 4, NCTM, Reston, VA.
Johnston, B. 1995 'Which map shall I use?' *Numeracy in Focus*, No. 1, pp. 33–37.
Siemon, D. 1989, 'Knowing and believing is seeing: a constructivist's perspective of change' in *School Mathematics: the challenge to change*, N.F. Ellerton & M.A. Clements, Deakin University Press.
Willis, S. 1989, *'Real girls don't do maths': gender and the construction of privilege*, Deakin University Press, p. 72.

A3.1 Why do we need theories?

Detailed procedure

This session is a small group and whole Group discussion session, in five parts:

- **Looking at boundaries: reflections** (30 min)
- **What theories are we already working from?** (30 min)
- **Using metaphors and pictures** (30 min)
- **So, why theory?** (15 min)
- **Different theories, questions** (15 min)

Looking at boundaries: reflections

Ask participants, in small groups, to talk about the results of the journal activity related to the *Looking at boundaries* case study and discuss the activity as illustration of these points:

- how they feel about maths and why
- what assumptions were brought to light in this case study about maths and about how they and others learn it
- whether these assumptions need to be, or can be challenged
- what implications there are for teaching maths to adults.

A short report can be made from each small group to the whole Group.

What theories are we already working from?

1 Start with the quote, 'I taught them but they didn't learn.'...

Anyone who teaches has probably faced the situation epitomised by this teacher.

We need to ask:

What is knowledge—especially, but not exclusively, mathematical knowledge?

How do people learn?

And so, what is teaching?

Let's look at some maths we might have learnt and try to tease out some of our assumptions about these three: students, knowledge and teachers.

2 Boundaries

Participants have already spent time in A2.1 looking at area and perimeter.

Get the Group to distinguish between ways this might have been taught:

- probably, for most people, most of the time, as formulas: $A = l \times b$, $A = \pi r^2$
- in session A2.1, through confronting assumptions, developing conceptual understanding, but still in a teaching/learning environment
- how else? ... perhaps from use in real life contexts, e.g. building a hen-house.

3 Teaching subtraction: Ask the participants to try this one: 73 – 46

4 **On the board, collect the various ways participants have done the subtraction, and others they might have used.** *These should include the following (and if not, add them yourself)*:

- by rote … borrow and pay back
- by shuffling numbers … decomposition
- by 'street maths' … shopkeeper's subtraction
- by estimation

Discussion points that may emerge

- Most of us were probably taught it by the borrow–pay back method. Look at this method. Ask participants for reasons why it worked.
  Emphasise that for many people there are no reasons, or the reasons make a commercial or moral sense ('if you borrow you must pay back') but not a mathematical sense.
  At some point, perhaps later, show or get a participant to show why in fact there are reasons why it works—equal additions.
- Some participants, particularly those under 35 years, will do it by decomposition or grouping. Many will recognise it as being the way their children do it.
  Do an example, get participants to make up and do another one and to explain to each other how they have done it. Stress the idea of 'making meaning'.
- Remind participants about how change is counted out in shops (or used to be, before the machine did it all) or on buses, by complementary addition:

$$\begin{array}{rl} 46 + 4 & \Rightarrow 50 \\ + 10 & \Rightarrow 60 \\ + 10 & \Rightarrow 70 \\ + 3 & \Rightarrow 73 \quad \text{so} \quad 10 + 10 + 4 + 3 = 27 \end{array}$$

=> means 'implies'

  This also makes sense, and allows users to make whatever jumps they are happy with.
- Talk about estimation. When would an approximate answer be appropriate?

Using metaphors and pictures to make assumptions explicit

1 Start to analyse some of these preceding approaches to learning and teaching by asking participants to look at the borrow–pay back method of subtraction.
Point out that the method and the rules in this situation are 'givens' and ask:
What does the teacher do here? What does the student do?
(The teacher tells, the student receives.)
What would be a good picture or diagram or metaphor for this kind of learning situation?

2 Get participants to work in pairs to consider other subtraction methods that have emerged and the work in the earlier session on Boundaries.
Ask again: *What does the teacher do in each situation? What does the student do?*
What would be a good picture or diagram or metaphor for this kind of learning situation?

Depending on the situation it may be that the teacher tells, facilitates or provokes while the student receives, discovers or constructs.

Again, depending on the situation, metaphors might include:

a coin-in-the-slot machine, building or construction, a flower blooming.

3 Now ask participants to think about their own teaching. One way of becoming more conscious of our own beliefs is explored in the activity *Metaphors* [HO7] described by Laurinda Allan, an ABE teacher in Sydney. Use Steps 2 and 3 of this activity to help participants develop metaphors for their roles as teachers.

Participants consider the following questions:

Q1 What do you do when you teach?

Q2 What roles do you use when you teach?

Q3 What metaphors do you associate with these roles?

4 Share the metaphors amongst the Group. Get participants to discuss these questions:

Which of the metaphors best matches your experiences as a student of maths? as a numeracy or maths teacher?

Does the same model match your experiences in other areas of knowledge: literacy, science, history ...?

5 At the end of the activity, ask participants to answer *More questions about metaphors* on the back of *Metaphors* [HO7], as a possible activity to work on for their journals.

So, why theory?

1 Theory often seems irrelevant. To avoid this it is important to reflect on just how theory and practice affect each other.

In a Group discussion get participants to summarise what they think are the key points of the reading *Which map shall I use?* [P3]. Include these points:

- we all have theories, explicit or implicit, whether we admit it or not
- different theories allow or disallow different questions and explanations, and lead to or exclude different actions
- making our assumptions and theories explicit gives us a chance to choose which to hang on to, which to discard.

2 In small groups reconsider the question of maths anxiety discussed in Session A2.2.

Say:

Think about a case of maths anxiety, yours or someone else's. What was the cause of it? Can you work out what assumptions lie beneath your explanation?

It may help to ask, *Do other subjects engender such anxiety, e.g. history anxiety, art anxiety, biology anxiety?*

3 **Discuss the last question still in small groups. These points may arise:**
Perhaps sometimes…reading, writing or spelling anxiety…but mostly not.
The patterns of maths anxiety among individuals are too widespread to be explained as the fault of those individuals. If we see, as Buerk (1985, p. 67) [HO5/OHT2] suggests, that maths anxiety is ' the sense of disintegration that occurs when [people cannot] make meaning', then we have to ask: What is it that prevents so many people from making meaning? Institutions and their practices must play a part.

4 When the small groups report back, try to make explicit the two general and contrasting positions that are held and untangle them:
- it is the fault of the individual
- it is the fault of society.

Different theories, different questions: maths anxiety, or maths abuse?

1 Mention that earlier we argued that it is important to recognise that different theories allow or disallow different questions and explanations, and lead to or exclude different actions.
Ask now:
What questions, explanations and actions might arise from these two very different theories about maths anxiety?

2 Explain that if we shift the focus of explanation from personal inadequacy to the way that maths and maths teaching are socially constructed then we stop placing our students in the position of victim and start to critique the system: there is a shift in our assumptions. We are using a different theory that allows us to ask different questions and to seek different courses of action.
Ask: *If we change our description of the problem from maths anxiety to maths abuse, what questions will we ask? What action can we take?*

Examples: focusing on changing
- what we teach, why we teach it, how we teach, the place of exams...
  rather than remediating or blaming individuals for 'not having a mathematical mind'.

3 Show the quote from Australian educator Garth Boomer, *The present state of maths...* [OHT3]. What assumptions lie beneath his claims?

4 Finish this activity and discussion by raising these questions for ongoing reflection:
- What are the assumptions beneath what you are doing?
- What are the implications for practice, for action?
- What are the implications for how you relate to your students?

A3.2 Curriculum Project 1 – Making meaning Part 1

Time required: 3 hours over 2 or 3 weeks. This part takes 1 hour.

Brief description

Participants explore the meanings that they and their students have for the four basic operations: addition, multiplication, subtraction and division.

Rationale/aims

'Making meaning' is the first of four Curriculum Projects in the course. It is a short project, giving participants an opportunity to analyse how they and their students use language and models to make meaning.

By the end of the project the participants should have developed some activities for use with their students, and should have:

- an extended repertoire of calculation methods
- an appreciation of the validity of different methods and
- familiarity with a variety of materials and language that can model different situations.

Preparation

Presenter
Familiarise yourself with Curriculum Project 1.

Time
1 hour initially, followed by two more hours in later sessions

Materials needed

Concrete materials
(for each small group of 4 or 5)
30 counters (15 each of 2 colours)
small set of Cuisenaire rods
metric tape measure to use as a number line
money
Also, if wanted, MAB blocks, straws or matches

Handouts
Curriculum project 1: Making meaning [CP1–1] and *The four operations* [CP1–2] both from **Curriculum Projects and Numeracy Journal** section
OHT *Making meaning* [OHT2]

Procedure

In order for participants to talk to their students, there needs to be a gap of at least a few days between Part I, the first hour, and Parts II and III. These later two Parts could be programmed on the same day, but would probably be more useful if they also were separated by some days.

Detailed instructions for this part of the Curriculum Project will be found in the **Curriculum Projects and Numeracy Journal** section.

A4 – Making meaning with symbols

A4.1 Algebra activities

Brief description

Another level of meaning lies in having concrete models for symbols used in, for example, algebra. In this session participants use non-threatening examples in which everyday knowledge is translated into algebraic language and visual patterns are expressed as algebraic generalisations.

Rationale/aims

Algebra is a branch of mathematics that typically causes anxiety. Because symbolic representation will come up in activities throughout the course it is important that participants are introduced to this idea in a relaxed fashion in these early stages.

The session also models how to work with different levels in a student group by having extension material available, and by not having too many questions on a worksheet. Advanced students can be given more complex problems to tackle.

Preparation

Presenter
Photocopy Activity Sheets onto card and cut so that there is only one question per card. Make one for each pair.

Time: 2 hours

Materials needed

Handouts/OHTs/paper
Activity Sheets AS8 to AS11
Squaring with matches [HO8]
Choose a number [HO9]
Making meaning [OHT2]
Concrete materials
Matches, cups, counters or jelly beans, enough for each small group.

A4.1 Algebra activities

Detailed procedure

This session involves group discussion and 3 activities:

– **Issues surrounding algebra**	(15 min)
– **Algebra through domestic situations**	(30 min)
– **Algebra through patterns**	(30 min)
– **Algebra through physical representation**	(30 min)
– **Review**	(15 min)

Issues surrounding algebra

1 Discuss the whole issue of algebra with participants. Ask participants questions like:

Think for a minute ... who can remember ...

their first experiences of algebra at school?

ANY experiences of algebra at school?

How was it introduced?

How was it explained?

Raise and discuss issues as they arise such as:

- Was algebra meaningful?
- What methods were used to teach it?
- Was there any hands on materials used?
- Was it related to everyday or real situations?
- Did it cause maths anxiety?

2 Refer back to the model for meaning making on *Making meaning* [OHT2] and discuss where and in what ways the traditional teaching of algebra matched this model.

3 Introduce the aim of this section which is to look at a few of the many ways of giving meaning to algebra:

- through familiar formulas used in domestic situations
- through patterns
- through physical representation.

Algebra through domestic situations

1 One introduction to algebra, especially to simple formulas, can be through familiar rules … in this case domestic situations. This is particularly valuable in a women's course. Do an example or two together.

Making tea:

Who remembers the expression, 'one for each person and one for the pot'?

Questions:
- What's it about?
- What does it tell you?
- Can you write it in words first?
- How can we then translate it into symbols?
- What would each of these symbols represent?

Try to encourage people to use meaningful symbols—not **x** or **y**.

number of teaspoons of tea = number of people plus one extra for the pot
=> $t = p + 1$
that is, $t = p + 1$

2 Get the group to think of other examples.

Here's another example of what might emerge:

When cooking rice using the evaporation method, how many cups of water are needed?

A possible answer may be:

2 cups of water for each cup of rice means
cups of water = double the number of cups of rice
=> $w = 2 \times r$
that is $w = 2r$

Some other possibilities are:

- number of potatoes related to the number of people for a dinner
- money needed for some adults to go to the movies related to the number of people.

Try to get the examples for formulas from participants' input, but write them as a Group. In each case tell participants to substitute some sample numbers to check if the answers, using their formulas, make sense.

3 Give out algebra cards from AS8 and AS9, *At home with algebra* numbered 1 to 4—one at a time—and ask participants to try them in pairs. Ask them to write a formula to represent the situation illustrated in each example using first words and then symbols. Give out more only as they finish each one.

Note: Question 4 can provoke discussion about conventions for writing symbolic notation, especially for division which often causes confusion.

For example the following are equivalent modes of writing the same thing:

$C = (F - 30) \div 2$ or

$C = \frac{1}{2}(F - 30)$ or

$C = \frac{F - 30}{2}$

4 **In addition, use the session to discuss how the format of the activity attempts to alleviate anxiety by not giving out worksheets with lots of problems on them.**
They are handed out one at a time, with others available only when participants (or students) are ready for further challenge.
Extension activities can be used as well.

Algebra through patterns

Another way of introducing algebra is to use patterns and number relationships.

1 Give out *Squaring with matches* [HO8], and ask participants to work out how many matches are needed for each of the shapes. Ask them to look for any patterns in their answers and see if they can generalise the answers for the tenth and one hundredth figures without working them out.

2 Ask if they can write out a rule in their own words which generalises how many matches are needed. Then get them to discuss how to write it in symbolic form. Encourage all participants, including any maths teachers, to write a description in words first, and secondly represent it in meaningful symbols.

3 Discuss as a group and highlight how patterns were used to develop the relationship and to describe the relationship in words before writing it in symbolic form.
Ask:
Were all the final relationships the same?
How did each of you think it through?
The same thought processes? Different processes—same results?
Discuss any proposal from participants if they come up with different ways of getting the result. An obvious one might be:

> Start with one match, and then for each square you need to add three more matches.
> (Use m = number of matches and p = number of squares in pattern)
> number of matches = one match plus three times the number of squares
> => $m = 1 + 3 \times p$
> => $m = 1 + 3p$

Note: As you progress through this session remember to point out and discuss the various conventions of writing algebra, such as with the conversion from Fahrenheit to Celsius in the previous section. In this case you could discuss the convention of leaving out the x sign in multiplication. It is highly likely that many participants themselves will be anxious about algebra, and all participants should be sensitive to this throughout the session.

4 Give out Activity Sheets AS10 to AS12 with *Writing your own rules Numbers 1 to 5,* one at a time, for participants to work on in pairs. Use Numbers 4 and 5 only for those participants who finish the first three activities.

Remind any maths teachers who feel comfortable with algebra themselves to think about how and when they might use such activities with their own ABE students.

Could they adapt the activities? Can they think of other examples?

Algebra through physical representation

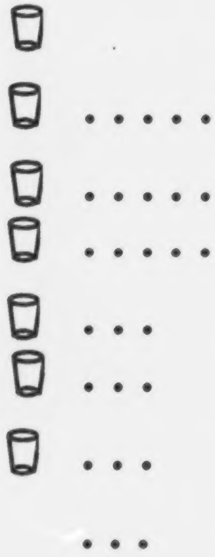
1 Explain that another level of meaning lies in having concrete models for the symbols used in algebra. Adapt the activity in the Harold Jacobs' extract *Choose a number* [HO9] to demonstrate this. *Before using the box and circle as signs, start with cups and counters to represent, more concretely, the unknown and known numbers.*

2 Ask the participants to:

choose a number ... add 5 ... double the result ... subtract four ...
divide by two ... subtract the number they first thought of

Tell them that you know the answer they have is 3. Magic! Now to unravel the mystery!

3 Demonstrate how this must always be true, using cups with counters or jelly beans, as below. A cup represents the number a participant has thought of (and nobody else knows what this is) and a counter represents 1.



choose a number

add 5

double the result

subtract four

divide by two

subtract the number first thought of
and the answer is 3!

4 Return to the instructions and draw them with boxes and circles, describing them as shorthand for the concrete materials.

5 Get participants to work on *Choose a number* [HO9] in pairs or small groups, using cups and counters as well.

Review

1 Revisit the model of making meaning. In the algebra activities covered, what parts of the model have we visited? What connections have been made? How does this compare with our earlier comments on the introduction and teaching of algebra in a traditional setting? Anyway you can connect algebra to models, to the real world and to language (rather than leaving it unconnected) will be an improvement on traditional practice and will help students to make some meaning from algebra.

2 Ask:
How do you now feel about algebra?
How does this relate to the needs of your students?
Have any of you introduced or taught algebra in other meaningful ways?
Discuss any comments and feedback.

You may like to give participants some references for resources that encourage and support this style of approach to the teaching of algebra.
Some examples are:

- *Access to Algebra* by Ian Lowe
- *Some Beginnings in Algebra* by Beth Marr, 1995, Northern Metropolitan College of TAFE (additional material to *Mathematics: A New Beginning*)
- algebra blocks and tiles.

A4.2 Curriculum Project 1 – Making meaning Part 2

Time

This is the second part of CP1 and takes 1 hour.

Detailed procedure

Detailed instructions for this part of the Curriculum Project will be found in the **Curriculum Projects and Numeracy Journal** section.

A5 – Making meaning from counting systems

A5.1 Curriculum Project 1 – Making meaning Part 3

Time

This is the third part of CP1 and takes 1 hour.

Detailed procedure

Detailed instructions for this part of the Curriculum Project will be found in the **Curriculum Projects and Numeracy Journal** section.

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A5.2 Why count? and how?

Brief description

Participants begin to explore the structure of 'our' Hindu-Arabic number system and consider teaching strategies that enhance the making of meaning in mathematics.

Rationale/aims

Lack of understanding of how place value works in our number system underlies much of the failure that adults have experienced at school.

This session and the next aim to give participants time to become familiar with the fundamental ideas of number base and place value. This is done initially by giving participants a glimpse of another number system and thus of the idea that things could be different, that number systems, like mathematics more generally, are responses to particular cultural needs and material circumstances.

Preparation

Presenter and Participants
Read
Kearins (1991) [P5]

Time: 1 hour

Materials needed

Concrete materials
books and extracts describing other numeration systems, e.g. Hogben (1936), Shan & Bailey (1991)
Handouts and paper
Filling the gaps [HO10]
The word for five [HO11]
Why count? [HO12]
Number experience [P5]
OHTs
Counting [OHT4]

References

*Duckworth, E. 1987, 'Teaching as research', in E. Duckworth, *The Having of Wonderful Ideas and Other Essays on Teaching and Learning*, Teachers College Press, New York.

*Kearins, J. 1991, 'Number experience and performance in Australian Aboriginal and Western children', in *Language in mathematical education*, K. Durkin & B. Shire (eds), Open University Press, London.

A5.2 Why count? and how?

Detailed procedure

This section consists of an introductory excursion, a view of some other number systems, and a discussion on why people count:

– **The word for five....**	(15 min)
– **Looking at other number systems**	(30 min)
– **Why do we count?**	(15 min)

Tell participants that the point of this session and the next is for them to work on some mathematics and, while doing so, to consider what general factors might help them or their students to make sense of what they are doing.

The word for five....

1 Before telling participants that the session is about counting systems hand out *Filling the gaps* [HO10] and ask them how they would fill in the gaps on the dotted lines.
If some people do this quickly, you could ask them to develop a code for writing the words in shorthand.
When everybody has had a chance to fill in the gaps, check that they all have the same answers, and ask: *What do you think these words are?*

2 After some discussion of possibilities, tell participants that this is an Aboriginal numeration system, belonging to the Gumatj people, one of many different Aboriginal numeration systems. Show how it works by using fingers and hands:
one, two, three, four, one hand, one hand and one, one hand and two etc.

3 Explain the system as being a base-5 system. Ask:
Why do you think they counted in lots of five?
What do you think the word 'rulu' might mean? (possibly 'hand' or 'pile')
What base do we use? Why? (ten = 2 hands)
What other counting systems do you know of?
What bases do they use?(feet and inches, Babylonian base 60, hours/minutes)

4 Give out and read *The word for five* [HO9] and discuss the analogous use of the word 'foot' in our measuring system—used here until recently and still used in the USA.
Get participants to develop a code or shorthand way to write these long words.
There are many possibilities.
The most formal mathematical one would be to write something like this:
$1, 2, 3, 4, 10_5, 11_5, 12_5, 13_5, 14_5, 20_5, 21_5$ and so on
where the subscript 5 indicates that we are using 5 as the counting base, i.e. base 5.

Looking at other number systems

1 Share out books and materials so that participants can investigate other number systems, and how they are constructed, e.g. Babylonian, Egyptian, Chinese, Mayan, Roman.

2 Ask: *What bases do they have? Why?*
ten for fingers; twenty for fingers and toes
sixty as a fraction of 360, the (approximate) number of days in a year

3 Ask: *Why are they written the way they are? How were they written?*
Babylonian: stamped with a triangular stick on clay
Roman: adapted from finger signs, e.g. 3 as III showing 3 fingers held up
5 as V representing the whole hand held up.

4 Ask: *Do they all have place value? That is, does it matter to the value of the whole number WHERE (in what PLACE) each symbol is located?*
Mayan, late Roman, Hindu-Arabic: 'yes',
early Roman: 'no'
(when 9 was written xiiii, and ix meant the same as xi, both represented what we call 11)

5 Ask: *How many symbols does each system need to count to 20? 100? 1000? infinity?*
Roman: 3, 5, 7, infinity
Babylonian: 2, 2, ?, ?
Mayan: 3, 3, 3, 3
Hindu-Arabic: 10, 10, 10, 10

6 *Why is the Hindu-Arabic system so good? Does it have disadvantages?*
Points that may arise:
disadvantages – it needs a lot of symbols even to count to low numbers like 20
– you have to know basic number combinations to work it properly.
advantages – no conceivable number needs more than ten digits
– very large numbers are written with (comparatively) few digits
– calculations are very efficient in terms of space and effort

Why do we count?

1 Get participants to discuss in small groups the questions on *Counting* [OHT4]:

- *What do we count?*
- *What don't we count?*
- *Why do we count?*
- *Do all societies count?*

After a few minutes, hand out *Why count?* [HO12] to read and stimulate discussion.

2 With the large Group, discuss the issues involved and conclude with the point that societies develop number systems—and mathematics generally—that meet their needs. Describe the article 'Number experience' by Kearins [P5] as an example.

A5.3 Operating 'our' number system: a beginning

Brief description

Participants explore the structure of 'our' Hindu-Arabic number system in operating numbers equal to or larger than one.

Rationale

Lack of understanding of how place value works in our number system underlies much of the failure that adults have experienced at school. In the last session, participants considered briefly the question of why we count. In this session they go on to begin to explore the structure of the Hindu-Arabic system by using concrete material to model numbers and operations on numbers larger than one.

Preparation

Presenter
Read and prepare materials for Activity 4.3 *Modelling place value: addition and subtraction* from *Breaking the Maths Barrier*, pp. 179–186.

Time: 1 hour

Materials needed

Concrete materials
straws, rubber bands, etc.
(materials as used for Activity 4.3 in *Breaking the Maths Barrier*, pp. 179–186)
Handouts
Using straw models [P9]

References

Goddard, R., Marr, B. & Martin, J. 1991, *Strength in Numbers,* Adult Community and Further Education, Victoria.
Marr, B. & Helme, S. 1987, *Mathematics: A New Beginning,* Northern Metropolitan College of TAFE, Victoria.
Marr, B. & Helme, S. 1991, *Breaking the Maths Barrier,* DEET.
Marr, B., Tout, D. & Anderson, C. 1994, *Numeracy on the Line,* National Automotive Industry Training Board.

A5.3 Operating 'our' number system: a beginning

Detailed procedure

Exploring 'our' Hindu-Arabic numeration system

1 Mention briefly that 'our' numeration system is known as the 'Hindu-Arabic' system, because of its historical origins.

2 Ask participants to complete Activity 4.3 *Modelling place value: addition and subtraction* [P9] in small groups.

Participants who are already familiar with the activity and/or the use of straws could be asked to develop a new game along the lines of Game 36.

3 At the completion of the activity ask participants to share their thoughts on the use of the straws, and on any issues that arose about place value.

If other methods of teaching about place value come up, discuss them briefly, but let participants know that the next session will be looking further at this issue.

A6 Strategies for making meaning

A6.1 Operating the number system: a continuation

Brief description

Participants continue to explore the structure of 'our' Hindu-Arabic number system and consider teaching strategies that enhance the making of meaning in mathematics.

Rationale/aims

In the last session, participants worked with one concrete material to model place value and operations on whole numbers. In this session, participants use a wider variety of materials and also consider decimal fractions. They conclude by trying to tease out teaching strategies that have helped in making meaningful the maths learnt in this session.

Preparation

Presenter and Participants
Read
Duckworth 1986 [P6]

Time: 1.5 hours

Materials needed

Concrete materials
Base 10 Multi-Arithmetic Blocks
(1 set per group of 4)
coloured counters
(five colours, 20 of each)
several different abacuses
Handouts
Multiple embodiments [HO13]
Coloured counters [AS12]
Dienes blocks.... MAB blocks [AS13]
The abacus [AS14]
What about little numbers? [HO14]
Teaching as research [P6].

References

* Duckworth, E. 1987, 'Teaching as research', in E. Duckworth, *'The Having of Wonderful Ideas and Other Essays on Teaching and Learning'*, Teachers College Press, New York.

Goddard, R., Marr, B. & Martin, J. 1991, *Strength in Numbers*, Adult Community and Further Education Board, Victoria.

Marr, B. & Helme, S. 1987, *Mathematics: A New Beginning*, Northern Metropolitan College of TAFE, Victoria.

Marr, B., Tout, D. & Anderson, C. 1994, *Numeracy on the Line*, National Automotive Industry Training Board.

Thiering, J., Hatherly, S. & McLeod, J. 1992, *Teaching Vocational Mathematics*, NCVER, Adelaide.

A6.1 Operating the number system: a continuation

Detailed procedure

This session consists of a class activity, a small group activity and a Group discussion:

• **Building meaning—how do we do it?**	(15 min)
• **Multiple embodiments**	(30 min)
• **And what about little numbers?**	(45 min)

Building meaning—how do we do it?

1 Discuss *Teaching as research* [P6], asking participants:

What two elements of teaching does Eleanor Duckworth describe in detail? Why does she think they are important? Do you agree?

2 Brainstorm these questions:

What strategies do you use to do this?

What strategies have been used here in the course that have worked for you?

What strategies don't work for you or your students?

Some of the points that may arise include:

- taking into consideration the physical needs and learning styles of students
- organising the student group and teaching room—comparing probable effects of formal and informal settings
- engaging the learners by using intriguing or relevant material
- making connections to historical origins and/or social issues
- using a variety of materials including concrete and pictorial materials
- employing a variety of teaching approaches
- teaching meta-strategies, e.g. how to solve problems

Multiple embodiments

1 Put out on tables materials that model place value, including counters, abacuses and MAB base 10 blocks (and other bases if available).

2 Get participants to go around the room using the MAB blocks and at least one other type of material to try out the questions on the accompanying worksheets:

Coloured counters [AS12]

Dienes blocks.... MAB blocks [AS13]

The abacus [AS14]

Your role is to circulate, asking questions that help participants to model appropriately.

3 Conclude with a Group discussion, showing how the addition would be done with the three materials. Extract from the participants the differences between the three sorts of materials, including the straws.

- Bundled straws are excellent to begin with, making a clear model of tens and units, showing that one bundle is in fact ten units. They are not so good when more places are necessary with larger numbers.
- MAB blocks are slightly more abstract, in that 'trading' rather than bundling is necessary, and they make an excellent model for large and small numbers. They are the next stage towards treating ten as a single object. The 'long' can still be seen to be worth 10 units, but it cannot be decomposed.
- Coloured counters are even more abstract, in that the 'ten' is now replaced by one object that is identical in size and shape—only colour is different. Like coins, one is arbitrarily said to be 'worth' ten of another. The relationship between the counters is no longer visible, but must be remembered.
- The traditional abacus is quite abstract, in that the beads are identical in size, shape and colour and are distinguished only by their position or *place*. They are not so good for developing the concept of place value, but are excellent for reinforcing it and for carrying out more complex operations.

4 Ask: *Would you use a variety of different materials for teaching place value and the four operations? Or would you stick to one type of material?*

Hand out *Multiple embodiments* [HO13] and discuss.

And what about little numbers?

1 Use one set of materials—possibly the MAB blocks are best—to show the upwards growth of decimal numbers, starting with a unit and increasing by a factor of 10 each time.

2 When you reach the large cube, 1000, ask: *Need we stop here?*

Show how you can then visualise a 'large long', a 'large flat', a 'very large cube' (1 m^3 or 1 metre cubed, equivalent to 1 000 000 units or 'small cubes'), a 'huge long', a 'huge flat'—and so on for ever. Point out that we have been collecting bundles of ten.

3 Ask: *What happens if we break everything down into ten bits instead?*

Start with the large cube, break it into ten flats, break a flat into ten longs, a long into ten units and ask: *What now? Do we have to stop here?*

Show how you can visualise the small cube being sliced into ten 'small flats', each small flat being sliced into ten 'small longs', each small long being sliced into ten 'tiny cubes' (...sawdust!)

– and need we stop there?

Point out that theoretically, if not practically, the system goes on for ever in both directions reaching very, very large numbers—as large as you like—and very, very small numbers—as small as you need.

3 Using the large cube to represent 1, get participants to use the MAB blocks to make 0.1, 0.01, 0.001...

4 Give participants *What about little numbers?* [HO14] and get them to do the first question, using MAB blocks.
If there is time, ask them to complete the worksheet.
Otherwise, they might like to consider the other questions in their journal.

5 To conclude the activity, make sure that participants are aware of the variety of resources available to explore decimal fractions further, with their students, in particular:

Numeracy on the Line and

Maths: a new beginning.

There is also material on decimal concepts in a resource for teachers, *Teaching Vocational Mathematics*, Thiering et al. 1992, NCVER (especially chapter 5).

A6.2 Language as a tool

Brief description

Participants consider how language can be used to make meaning in mathematics, and review what other meaning making tools have been discussed in Module A.

Rationale/aims

Language permeates our lives, at home or at work, and in this section we consider some ways that we can use it to enhance the process of learning to be numerate. The emphasis here is on language *for* numeracy. Later in the course we will look at how numeracy is implicated in literacy. This section engages participants in an analysis of some ways in which the use of talk and a variety of ways of representing concepts can help the learning process.

Preparation

Presenter and Participants
Read
Boomer (1986) [P8] and
Lesley's story [P7]

Time: 1 hour

Materials needed

Concrete materials
some old and current textbooks with definitions of fractions
Handouts
From catechism to communication: language, learning and mathematics.[P8]
Lesley's story [P7]
Murdering the innocents [HO15]
Talking about fractions [HO16]

References

* Boomer, G. 1986, 'From catechism to communication: language, learning and mathematics', *Australian Mathematics Teacher*, Vol. 42, pp. 2–7.
Tout, D. 1995, 'Leftovers: some lessons learnt', *Numeracy in Focus*, No. 1, pp. 20–23.

A6.2 Language as a tool

Detailed procedure

This session begins with a story, and is followed by an activity designed so that several of the principles spelt out by Boomer in *From catechism to communication: language, learning and mathematics* [P8] can emerge in the discussion following the activity.

– **Lesley's story**	(15 min)
– **Exploring fractions**	(30 min)
– **Talk, play, write, argue, negotiate, reflect**	(15 min)

Lesley's story

Discuss the division problem in *Lesley's story* [P7], the way Lesley did it and particularly the implications of language use.

Exploring fractions

1 Divide the participants into small groups and ask them to come up with a definition of a 'fraction', using everyday language.

2 Share the definitions as a large Group and then return the discussion to the small groups who will refine the definitions.
The role of the Presenter here is to circulate among the groups 'asking sticky questions to encourage them to be more explicit' (Boomer).

3 Get the groups to find some other definitions of 'fraction', e.g. in Frankenstein, and traditional textbooks, and compare their own with them.
Suggest that they rewrite any definitions from the texts if they feel they are too formal.

4 Next, get participants within their small groups to show each other how they would teach fraction multiplication, using as the example $\frac{1}{2} \times \frac{3}{7}$.
After a few minutes give participants *Talking about fractions* [HO16] to work through.

Talk, play, write, argue, negotiate, reflect

1 Participants have already read *From catechism to communication: language, learning and mathematics* [P8]. Refer to the use of Vygotsky's theory in the article, and ask:
What is the core of what Boomer (using Vygotsky) is arguing?
Extract these points:
- the teacher must find a variety of ways to connect the learner's 'spontaneous' concepts to more 'formal' concepts

- understanding of new ideas is enhanced by wide use of language, and a variety of representations of the idea.

2 Discuss:

Which of Boomer's 8 principles did we use in earlier activities?

Points might include:

- *transformation:* We transformed the symbolic process of multiplication into a diagram illustrating a concrete situation.
- *translation* : We converted some of the definitions of a fraction into our own words; we tried to translate other people's ways of multiplying fractions and relate them to our own;

  in *Talking about fractions* we translated the formal symbols and words (x and multiplication) into everyday words like 'lots of' allowing connections to be made between formal, school language and more familiar 'spontaneous' understandings.
- *guessing/hypothesising* : in Lesley's story, the learner's explanations helped the teacher to start from where the learner was.
- *withholding, and asking questions* : while the participants made up their definitions, the presenter asked sticky questions.
- *collaboration and formulating in talk and writing* : participants shared their ideas on a number of occasions, and the talking allowed negotiation of meaning to occur, e.g. for the concept of 'fraction'.

3 Ask:

Which of these techniques did you find helpful yourself?
Which do you think your students might benefit from?

Possible journal activity

Suggest that participants might use this activity as a springboard for a journal entry. They could take some concept that their students have trouble with, e.g. ratio, percentage, division, square root … and try applying Boomer's principles to develop some activities that would help students connect formal and 'spontaneous' understandings.

4 If there is time, you might finish by reading *Murdering the innocents* [HO15] as an example of what happens when the split between 'spontaneous' and formal definitions is extreme.

A6.3 Constructing meaning in practice: a review

Brief description

Participants review issues and strategies involved in constructing meaning in mathematics that have been discussed in Module A.

Rationale/aims

In this session the participants review the issues and strategies which are involved in constructing meaning in mathematics and which have been discussed so far.

Preparation

Presenter

Participants

Time: 30 min

Materials needed

Concrete materials
Making meaning [HO5 from A2.2]
Metaphors [HO7 from A3.1]
OHT
Making meaning [OHT2 from A2.2]

A6.3 Constructing meaning in practice: a review

Detailed procedure

1 Returning to *Lesley's Story* (Session A6.2) point out that

- students usually *do* have some meaning for what they do
- lack of meaning leads to anxiety
- it is our job as teachers to help students build structure and meaning.

2 Ask: *In the last few sessions, what teaching strategies were used to make the concept of place value and base 10 more meaningful? What other strategies might we have used?*

Other strategies could include:

- discussion of the way that people from a different culture count
- discussion of the purposes of counting
- use of a variety of concrete materials to model concepts
- connecting appropriate language to the modelling process
- recording the modelling process appropriately
- making connections to place value used in students' lives
- encouraging talk
- using a variety of learning situations: individual, small group, large Group.

3 Discuss the fact that these could all be summarised by the phrase 'making connections':

- connections to concrete materials
- connections to language and symbols
- connections to cultural, historical and everyday contexts.

Use the OHT *Making meaning* [OHT2 from A2.2] to illustrate this.

Possible journal activity

Choose another topic, e.g. percentages, graphs, probability, and focus on a particular student group and consider how to use some of the above strategies to begin thinking about how to plan a teaching session.

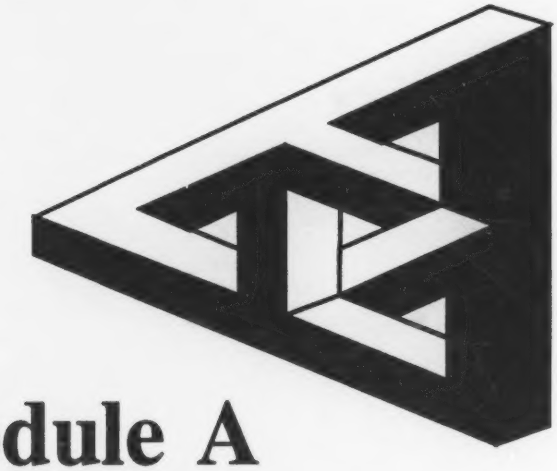
Discuss which strategies you have found most useful and helpful for yourself and/or for your students.

4 In pairs, ask the participants to discuss the metaphors they developed in session A3.1.

Do the metaphors describe – *the way you see yourself currently teaching?*
– *the way you would like to teach?*

In what way are the things you do, the roles you perform and the metaphors you use different in a) and b)? (See *Metaphors* [HO7] in A3.1.)

5 Refer participants to *References for Module A* [HO 17].



Module A
Exploring practice

Resources

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Key:

AS = Activity sheets to be used and returned to the presenter ✥
HO = Materials for participants to use in the session and retain ✋
OHT = Overhead projector transparencies ▤

List of readings

P = Participants' materials for them to read in advance

Make copies of the following readings and distribute them before the course begins or when indicated in Module A Presenter's Notes. (There are no readings supplied in Module A Resources.)

P1 * Math anxiety defined
'Math anxiety defined: what is it and how do I know if I have it?', chapter 1 in C. Oxreider & J. Ray 1982, *Your number's up*, Addison-Wesley, USA.

P2 Mathematics anxiety
'Mathematics anxiety: misconceptions about learning mathematics', chapter 1 in M. Frankenstein 1989, *Relearning Mathematics: a different third R—radical mathematics*, Free Association Press, London.

P4 Knowing and believing is seeing
Siemon, D. 1989, 'Knowing and believing is seeing: a constructivist's perspective of change' in N.F. Ellerton & M.A. Clements 1989, *School Mathematics: the challenge to change*, Deakin University, Geelong.

P5 Number experience
Kearins, J. 1991, 'Number experience and performance in Australian Aboriginal and Western children', in K. Durkin & B. Shire (eds), *Language in Mathematical Education*, Open University Press, London.

P6 * Teaching as research
'Teaching as research', chapter 10 in E. Duckworth 1987, *The Having of Wonderful Ideas and Other Essays on Teaching and Learning*, Teachers College Press, New York.

P8 * From catechism to communication: language, learning and mathematics
Boomer, G. 1986, From catechism to communication: language, learning and mathematics, Address to the Australian Association of Mathematics Teachers, Brisbane, pp. 2–7;

- reproduced in *Inservice Program for ALBE Personnel,* Module 5, 'Language in ALBE teaching and learning', pp. 170–177
- and in *Numeracy and How We Learn*, pp. 12–19.

P9 Using straw models
Activity 4.3 'Modelling place value: addition and subtraction' in S. Helme & B. Marr 1991, *Breaking the Maths Barrier*, pp. 179–186.

* These extracts are available in Johnston, B. (ed.) 1992, *Reclaiming Mathematics*, DEET.

The following readings do not have to be supplied as they are available in *Numeracy in Focus*, which participants are advised to purchase from ALIO or ARIS before the course begins.

P3 Which map shall I use?
Johnston, B. 1995, 'Which map shall I use?' *Numeracy in Focus*, No. 1, pp. 33–37.

P7 Lesley's story
Tout, D. 1995, 'Leftovers–some lessons learnt', *Numeracy in Focus*, No. 1, pp. 20–23.

My Experiences of Maths OHT1

What I remember most about maths …

My maths teachers were …

When I have to use maths nowadays …

When I think about being a
numeracy teacher …

A good numeracy teacher would …

Outline of the ANT Course

HO1
page 1

Adult Numeracy Teaching: making meaning in mathematics

Welcome

Welcome to *Adult Numeracy Teaching: making meaning in mathematics*. It is an 84 hour training course designed to train and support teachers in the teaching of adult numeracy and mathematics in the Adult Literacy and Basic Education classroom.

Adult Numeracy Teaching (ANT) is an initiative of the National Staff Development Committee for Vocational Education and Training, Melbourne.

The primary aim of ANT is to blend theory and practice about teaching and learning adult numeracy within a context of doing and investigating some mathematics, whilst developing a critical appreciation of the place of mathematics in society. The mathematics content taught and explored will be familiar to some participants, and less so to others. The focus however will always be on understanding how the material might best be learnt by Adult Basic Education students.

The course has been developed by the Centre for Language and Literacy at the University of Technology, Sydney (UTS) in conjunction with the Adult Basic Education Resource and Information Service (ARIS) at the Victorian Office of the National Languages and Literacy Institute of Australia (NLLIA), and a team of experienced adult numeracy teachers and trainers from NSW and Victoria.

Assessment

Throughout the course you will be expected to:

- maintain a written journal reflecting on the content of the course and teaching implications;
- complete an action trial applying material from the course in a classroom (Curriculum Project 1);
- investigate an aspect of mathematics either for your own mathematical development or for application in the classroom (Curriculum Project 2);
- design and develop teaching or curriculum materials (Curriculum Project 3);
- design and develop an assessment task (Curriculum Project 4); and
- generally contribute to course activities and discussions.

Time is given in the course for working on some of the assessment tasks, including giving presentations to the group.

HO1
page 2

Course structure

module	issues & investigations	hours
Module A: Exploring practice	**A1: YOU AND NUMERACY** – introductions and maths autobiographies – issues in numeracy	3
	A2: UNLEARNING ABOUT MATHS – unlearning maths: a case study – maths abuse and anxiety	3
	A3: WHY THEORY? – why theory? do we need theories	3
	CURRICULUM PROJECT 1 – Making meaning, Part 1	
	A4: MAKING MEANING WITH SYMBOLS – algebra activities	3
	CURRICULUM PROJECT 1 – Making meaning, Part 2	
	A5: MAKING MEANING FROM COUNTING SYSTEMS	3
	CURRICULUM PROJECT 1 – Making meaning, Part 3 – why count? and how? – operating 'our' number system: a beginning	
	A6: STRATEGIES FOR MAKING MEANING – operating the number system: a continuation – language as a tool – constructing meaning in practice: a review	3
		18 h
Module B: Maths as a human construction	**B1: SHAPING THE WORLD** – shaping our world – filling space – the patchwork connection – mostly Pythagoras	6
	B2: CIRCLES AND CULTURE – problem solving stations – investigating pi	3
	B3: PROBLEM SOLVING – problem solving—problem posing – effects of scale and shape	3
	B4: MODELLING OUR WORLD – measuring the coastline/fractals – maps and models – mindmaps: a review	3
	B5: CURRICULUM PROJECT 2: Exploring maths	9
		24 h

HO1
page 3

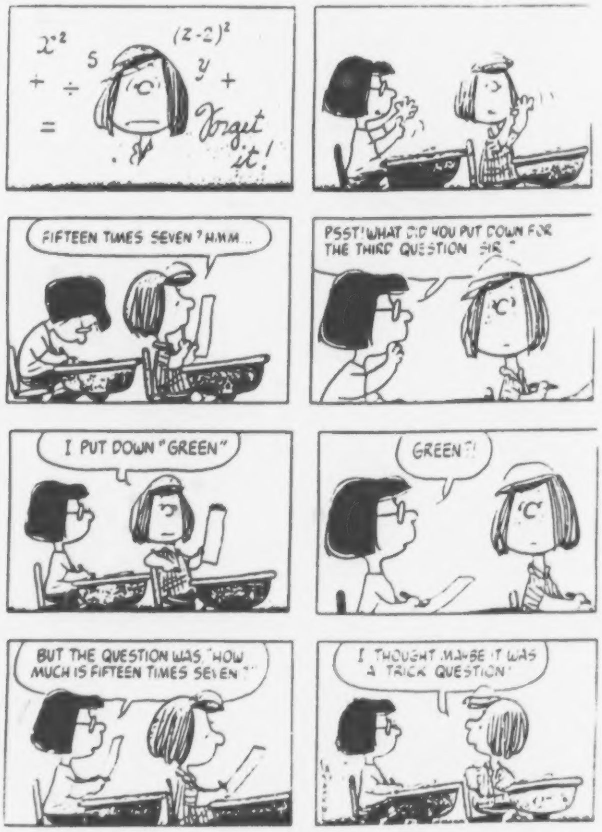
Course structure

Module C: Mathematics as a critical tool	**C1: BEING CRITICAL ABOUT MATHS** – being critical about number – reflecting on and negotiating your learning	3
	C2: BEING CRITICAL ABOUT LEARNING – looking at learners – street maths, school maths	3
	C3: A NEGOTIATED INTERVAL – a choice of topics	3
	C4: MATHS MYTHS AND REALITIES – why learn maths? why be numerate? – curriculum planning and meeting student needs	3
	C5: EXCAVATING MATHS – who learns? who benefits? a maths excavation – resourcing numeracy – critical literacy, critical numeracy – excavating mathematics	6
	C6: CURRICULUM PROJECT 3: Developing a critical view	9 **27 h**
Module D: Naming theories: implications for practice	**D1: AND SO TO THEORY** – naming the theories – for example: technology	3
	D2: IMPLICATIONS FOR TEACHING – case studies: constructivist and critical – a lesson on measurement	3
	D3: IMPLICATIONS FOR ASSESSMENT – assessment alternatives	3
	CURRICULUM PROJECT 4 – Assessment tool, Part 1 **D4: THEORY AND PRACTICE: CLOSING THE GAP**	3
	CURRICULUM PROJECT 4 – Assessment tool, Part 2 – theory into practice	3
	D5: AND SO WHAT IS NUMERACY? – the chance of numeracy – towards numeracy	**15 h**
TOTAL:		**84 h**

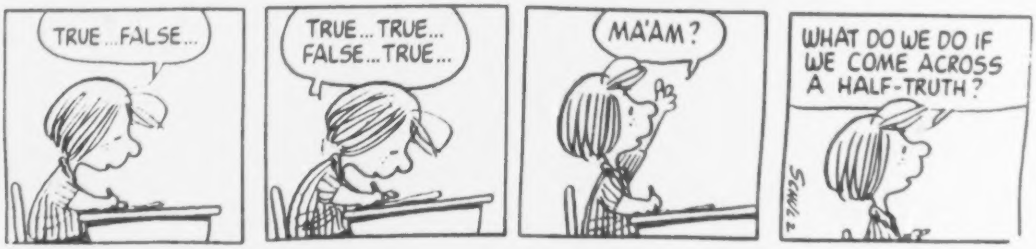
Poster

✣ AS1

So, what is maths? I can't get very far with the answer. I have heard maths spoken of as a language. I can't imagine it. Language is personal. Language carries thought forward, discovers thought, creates thought. Whatever it is I know as maths doesn't do those things.

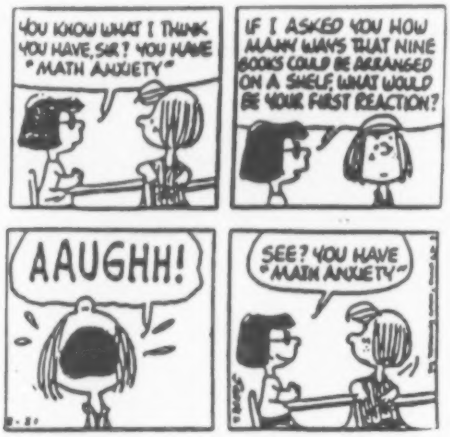

x^{2}
5
$(z-2)^2$
y
+ ÷ +
=
Forget it!
FIFTEEN TIMES SEVEN? HMM...
PSST! WHAT DID YOU PUT DOWN FOR THE THIRD QUESTION, SIR?
I PUT DOWN "GREEN"
GREEN?!
BUT THE QUESTION WAS, "HOW MUCH IS FIFTEEN TIMES SEVEN?"
I THOUGHT MAYBE IT WAS A TRICK QUESTION

Many adults, while not certain of what maths really is, feel that the subject is simply beyond their reach ... They individually believe that they can't 'do' maths. Perhaps they have been told they don't have a 'mathematical mind', or that only 'smart' people can understand it. They are made to feel inferior in a system which falsely equates maths performance with the more elusive concept of overall intelligence.


TRUE...FALSE...
TRUE...TRUE... FALSE...TRUE...
MA'AM?
WHAT DO WE DO IF WE COME ACROSS A HALF-TRUTH?
SCHULZ

Poster ✣ AS2

On the eighth day, God created mathematics. He took stainless steel, and rolled it out thin, and he made it into a fence, forty cubits high, and infinite cubits long. And on this fence, in fair capitals, he did print rules, theorems, axioms, and pointed reminders: 'Invert and multiply'; 'The square on the hypotenuse is three decibels louder than the sound of one hand clapping'; 'Always do what is in the parentheses first'. And when he finished, he said, 'On one side of this fence will reside those who are good at maths, and on the other will remain those who are bad at maths, and woe unto them, for they shall weep and gnash their teeth.'


YOU KNOW WHAT I THINK YOU HAVE, SIR? YOU HAVE "MATH ANXIETY"
IF I ASKED YOU HOW MANY WAYS THAT NINE BOOKS COULD BE ARRANGED ON A SHELF, WHAT WOULD BE YOUR FIRST REACTION?
AAUGHH!
SEE? YOU HAVE "MATH ANXIETY"

A Poem

Jan Elnar Nordgreen

to an excellent math teacher

you taught me to
factorise any expression
away from my face of curiosity

you taught me to
manipulate linear equations
making the left side of my brain equal to the right side

you taught me to
find the right solution
to problems I did not have

you taught me to
integrate functions
but not those most in need of integration

you taught me to
draw a given line
between my own feelings and mathematics

you were an excellent maths teacher
I just another slow learner

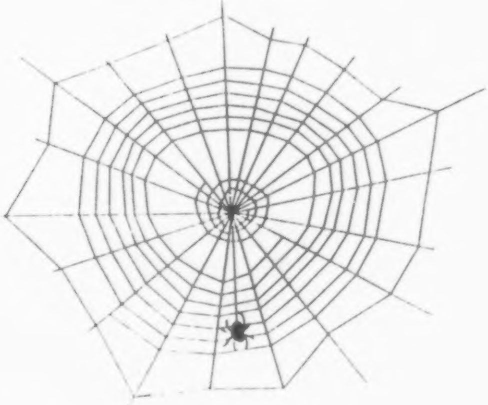
Poster ✣ AS3

No-one, I hope, imagines that Shakespeare is so highly treasured because he had assiduously practised handwriting, and surely not because of his effortlessly accurate spelling … Yet somehow in school mathematics it is as if we have chosen to discard or ignore all of the challenges and excitement and thought that make up the essential part of mathematics, and to teach instead only a few rules for writing some symbols on paper. We behave like a teacher of literature who has students work only on punctuation.

> If anyone asked me what makes me truly happy, I would say: numbers … And do you know why? … Because the number system is like human life. First you have the natural numbers. The ones that are whole and positive. The numbers of the small child. But human consciousness expands. The child discovers longing, and do you know what the mathematical expression is for longing? … The negative numbers. The formalisation of the feeling that you are missing something. And human consciousness expands and grows even more, and the child discovers the in-between spaces …

Mathematics

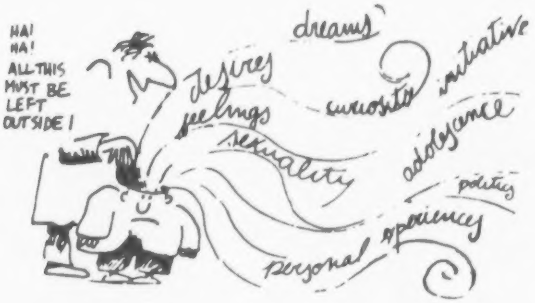
	the word has been
	known
to conjure up	
love	*hate*
delight	*despair*
recreation	*anxiety*
joy	*fear*
Mathematics	
is a variety of	
subjects	
and ideas	
restricted only	
by our imagination	*by our imagination*
Mathematics touches	
	so much of our lives
of our world –	*of our universe –*
art,	*nature,*
music,	*science,*
history,	*architecture,*
economics,	*literature.*
Mathematics is	
everywhere,	*a pastime,*
a recreation,	*conundrums,*
games,	*puzzles,*
problems,	*solutions,*
a way of thinking	*a way of thinking*

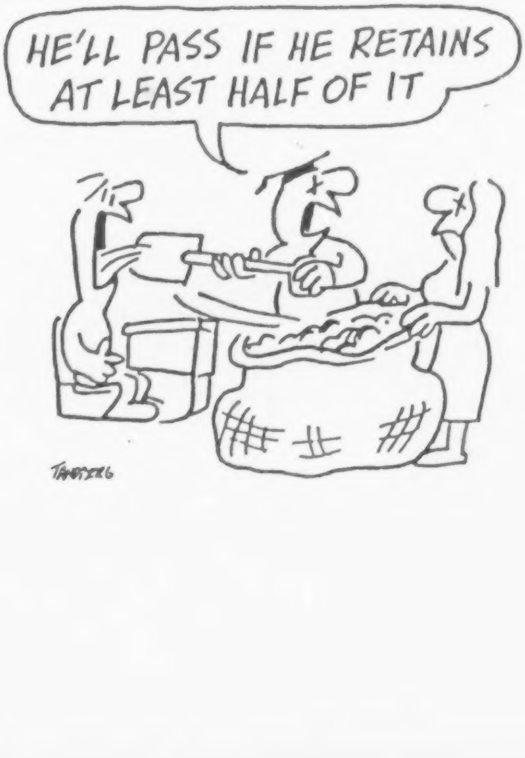


Theoni Pappas © 1991

Poster ✣ AS4

I have found that the vitality and immediacy of the classroom constantly reminds me that while I might be a cog in the education system, I am at the front line of the learning process and therefore in a position of power. My methodology can be either an expression of my power, or a means for my students to gain power over the world …

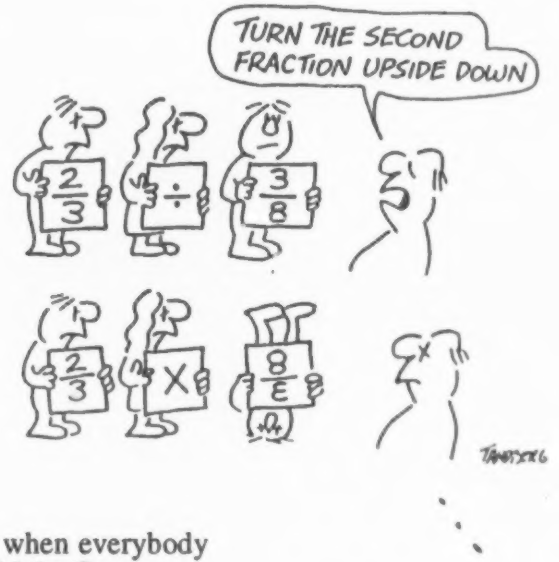

HA! HA! ALL THIS MUST BE LEFT OUTSIDE!
dreams
desires
feelings
curiosity
initiative
sexuality
adolescence
politics
personal experience


HE'LL PASS IF HE RETAINS AT LEAST HALF OF IT
TANDBERG

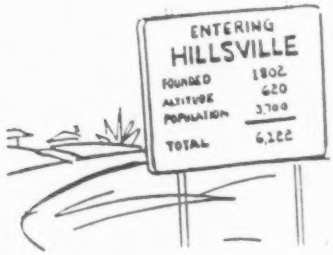
For so long, learners in mathematics classrooms have been socialised to believe that their own experience, concerns, curiosity and purposes are not important. Mathematics is seen as being devoid of meaning, bearing no relevance either to their everyday experience, or to the pertinent issues of their societies. Learning mathematics for these students partakes more of the nature of obedience than of understanding.

Poster ✣ AS5

In a class discussion on subtraction, a student was talking about borrowing and paying back. I asked him why he did that.
He said, 'That's the rule.' I asked him why the rule said that.
'It just does,' he replied. 'It's the rule I was taught.'
'But why?' I asked again. He looked at me very seriously and asked,
'You mean there's a reason?'


TURN THE SECOND FRACTION UPSIDE DOWN
2/3
÷
3/8
2/3
X
8/3
TANDBERG

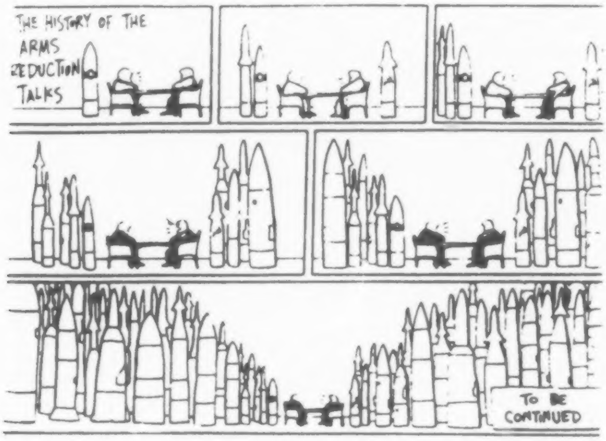
'… how can you possibly award prizes when everybody missed the target?' said Alice. 'Well,' said the Queen, 'some missed more than others and we have a fine normal distribution of misses which means we can forget about the target.'


ENTERING
HILLSVILLE
FOUNDED 1802
ALTITUDE 620
POPULATION 3,700
TOTAL 6,122

Teacher: *'Do you know what volume means?'*
Student: *'Yes.'*
Teacher: *'Could you explain to me what it means?'*
Student: *'Yes, it's what is on the knob on the TV set.'*

Poster ✣ AS6

The history of mathematics is like some pastiche of school history. It's all kings and queens except in our case it's just kings: Euclid, Fibonacci, Descartes, Pascal, Newton. Where is the social history of mathematics? These people did not just jump out of a bath tub or from a gaming table and start having amazing insights. They created mathematics which echoed the concerns of the society around them. The patrons of Galileo … were in the business of wiping out fellow human beings with the help of more accurate siege gun trajectories. There is blood on the face of mathematics.

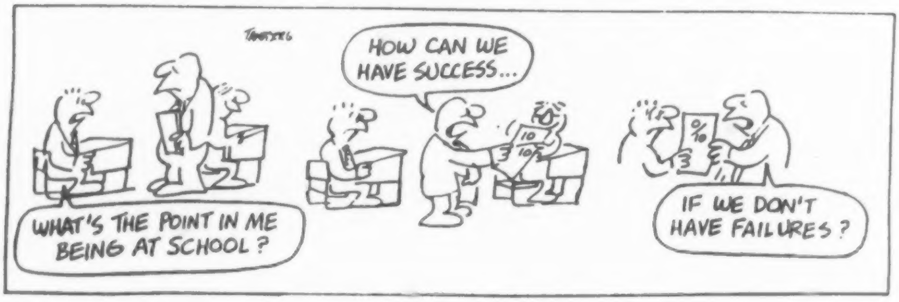

THE HISTORY OF THE ARMS REDUCTION TALKS
TO BE CONTINUED

Though scientific histories of the past two hundred years
have ignored or have been reluctant to admit the association,
mathematics has drawn inspiration and nourishment
from business, from religion, from law, from war, from politics,
from ethics, from gambling, from metaphysics,
from mysticism, from ritual, from play
(look at what a mathematical thing the children's game of hopscotch is),
and not just from a 'sanitised' physical science
approved by positivism.

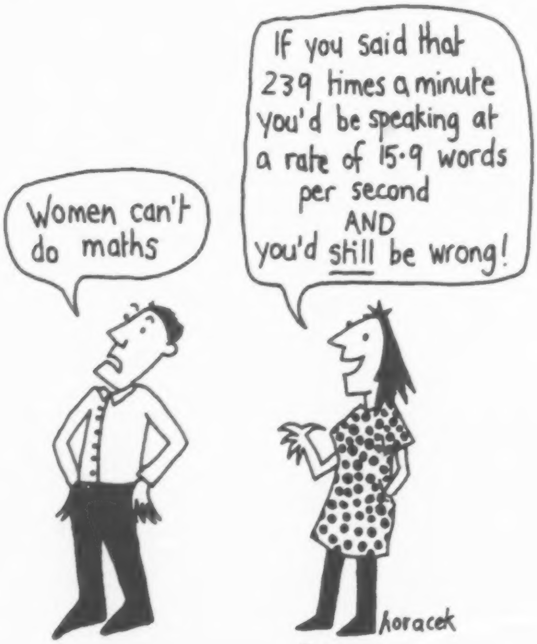
Poster

✣ AS7

Not to put too fine a point on it,
the present state of mathematics is a threat to democracy.
Too many are denied full access to it, too many fail it
and too many come to rely on those few who have been initiated.
Human dignity is undermined by
the submerged guilt about inadequacy that
resides with so many of our citizens.


WHAT'S THE POINT IN ME BEING AT SCHOOL?
HOW CAN WE HAVE SUCCESS...
IF WE DON'T HAVE FAILURES?

The mere fact that the student is learning maths skills is not necessarily evidence that educating is occurring, since it is possible to learn in ways that hinder the exercise of the intellect, emotion, imagination, judgment and action …


Women can't do maths
If you said that 239 times a minute you'd be speaking at a rate of 15·9 words per second AND you'd still be wrong!
horacek

Learning and teaching maths HO2

Learning and teaching mathematics
... case study notes ...

As teachers, you bring to your teaching assumptions about content and pedagogy in whatever subject you teach. In an effort to tease out some of these assumptions in relation to maths, and to examine them, we would like you to participate in a case study: of yourself, predominantly, and of one other adult, as well.

The purpose of the case-study is not for you to be either a model learner or a model teacher at this point; in fact discords will probably be more productive than harmonies. What we do want you to do, through a discussion of your experiences in this case study as *learner* and *teacher*, is to reflect on what mathematics is, how you and others learn it, how you feel about it and why, and what implications this has for teaching maths to adults. Can you tease out your assumptions? other peoples'? can you trace their origins? do they stand firm as you examine them closely?

Write up your reflections briefly in your journal, so that in one of the next sessions you can share your problems and insights with the rest of the group.

Looking at boundaries

HO3
page 1

1 The central question we are looking at is:

> ***If you have a fixed length of fencing with which to fence around a garden, does it matter what shape the garden is? Will the garden have the same total amount of area for planting no matter what shape you make it?***

Without going into mathematical justifications what do you think the answer is? Now go straight to question 2. We will return to this one later.

2 Let's look here at some related questions.

Let's suppose everyone in the class has some fencing, not necessarily the same length, and is going to use it make a fenced-in garden.

The following discussion takes place. First decide quickly by yourself whether you agree or disagree with the statements. Then try to come to a consensus within your group. Be sure that everyone in the group is convinced by the group decisions.

Lee:	*If I use more fencing than you then my garden will be bigger than yours.*
Soheila:	*If I know what the area of my garden is, then I can work out how much fencing I'll need.*
Chris:	*If you make a rectangular garden, I can always make another one with the same length of fencing but with a larger area.*

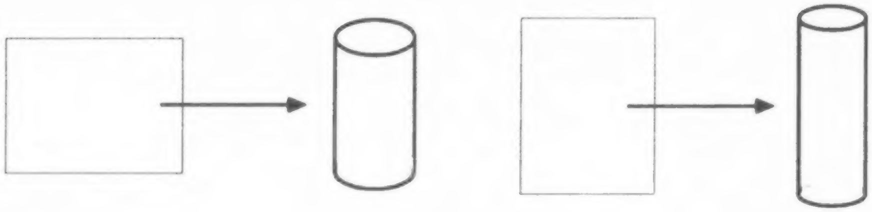
3 So, let's go back to the original question.

What do you think now? Why?
Can you convince others? (What does convincing involve? Is it the same as proving?)
Does your answer make *sense* to you? (What sort of sense? mathematical? real-world?)

4 And now, before the next session, we would like you to use this activity, and what you have learnt in it, to *teach* these concepts to some other adult—preferably not an "expert" at maths. During one of the next sessions there will be time to discuss with others in the class your reflections on this whole process of learning and teaching, in an effort to tease out some of your assumptions in relation to maths.

and where to now?

If you want to go on with another related question, take 2 identical sheets of paper and roll them to make two different cylinders.



Are they the same size? (How is this a related question? How is it similar, or different? What could 'size' mean?)

Talking about maths anxiety HO4

On the eighth day, God created mathematics. He took stainless steel, and rolled it out thin, and he made it into a fence, forty cubits high, and infinite cubits long. And on this fence, in fair capitals, he did print rules, theorems, axioms, and pointed reminders: "Invert and Multiply," "The square on the hypotenuse is three decibels louder than the sound of one hand clapping," "Always do what is in the parentheses first." And when he finished, he said, "On one side of this fence will reside those who are good at maths, and on the other will remain those who are bad at maths, and woe unto them, for they shall weep and gnash their teeth."

Buerk 1985, p. 128

I remember "learning" to divide fractions. We were taught this rule: "invert the second fraction, then multiply." We were told it was very easy because we already "knew" how to multiply fractions, and dividing had just one extra step. I couldn't do it. The teacher "explained" again, but couldn't understand my confusion … I pretended to be "coming down with something", to cover my deficiency. Maths was my favourite subject, but I felt then that I would never really understand it again.

Webber 1988, p. 69

… My memories of maths at school, was that it was a construction that other people made for me and of me, but that I never shared myself with ... and because I feel like I was mimicking and that things were not coming from within then I have always had a very deep feeling of being found out one day, that one day the axe would fall down.

Marie, an ABE teacher *

... that's the only course that I've done that has in any sense come from me and touched me and it marked quite a profound personal shift ... my first experience of that—of knowing in general and quite specifically in maths—was with the maths anxiety workshop and the handshake problem ... and this whole revelation, that it has a basis in concrete experience; that is if I understand the reality of it then I don't need to know the rules because I can just get there again ... I could get back to the formula!

Marie, an ABE teacher *

* in Johnston 1992

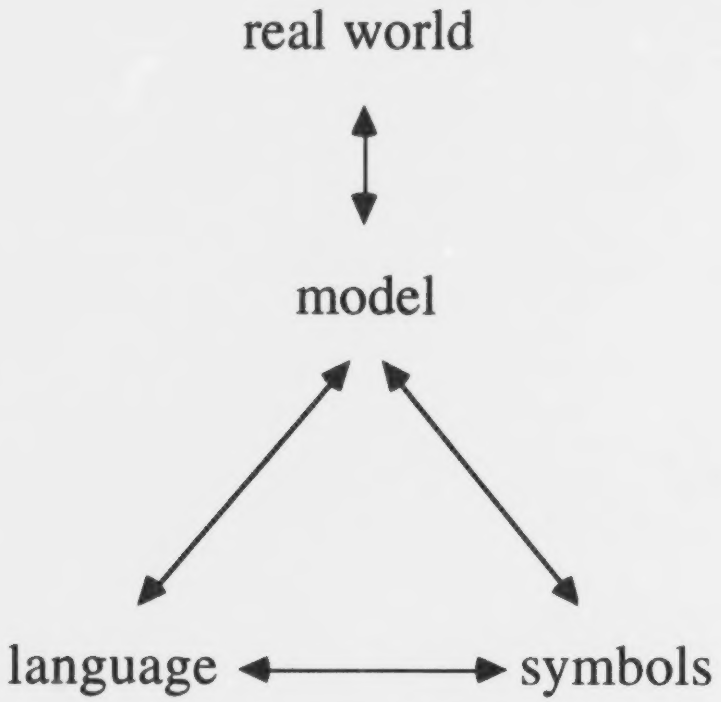
Making meaning

HO5
OHT2

...anxiety is
'the sense of disintegration which occurs
when a meaning-making organism
finds itself
unable to make meaning'.

Buerk 1985, p. 67

... creating connections ...
... constructing meaning ...



real world

model

language

symbols

Looking for patterns ... HO6

Maths is not just a matter of *number*, of comparing, counting, measuring, or even of *space*. It is also a matter of *relation*—of stating how things stand in relation to each other. These 'things' may be numbers or their representatives ($y = x^2 + 2x + 1$) and they mostly are, but they don't have to be. (Sweet peas are a subset of flowers; Town Hall is between Wynyard and Central.)

Mathematicians find pattern in the world (Pythagoras and musical octaves); they find pattern in numbers (Karl Gauss' pattern that helped him add the first hundred and the first thousand numbers); they find pattern in pattern (adding 9s, adding 8s, adding 7s). A strange compulsion. Against mess, towards tidiness? A mirror of the world, but how? The world is hardly tidy. A violent abstraction? Squeezing life into neat moulds? How come the patterns fit?

Let's look at the patterns of the 9x table:

Is this just a jumble of numbers or are there patterns?

9 left hand column: 1 2 3 4 5 6 7 8 9
18 right hand column: 9 8 7 6 5 4 3 2 1 0
27 So, as the left goes up, the right goes down.
36
45
54
63
72 what else? 18 and 81; 27 and 72; 36 and 63, and so on
81 and look at those pairs:
90 they are all combinations of numbers that add up to 9:
1 + 8, 6 + 3, 4 + 5 …

Does the pattern keep going? What about 99, 108, 117?
$1 + 0 + 8 = 9$, $1 + 1 + 7 = 9$
but $9 + 9 = 18$ … we've lost the pattern ... or have we?
After all, $9 + 9 = 18$, but $1 + 8 = 9$, so in the end we still get to 9.
Will it keep on working?
Will the digits of a multiple of 9 always add up to 9? Why ?

So if you start with 9 and keep adding 9, it seems the digits of the answers add up to 9.
What happens if you start with 8, or 5 or 4 and keep adding 9?
Is there a similar pattern?
What happens if you start with 8, say, and keep on adding 8 instead of 9?
How is the pattern different?
What if you …?

The present state of maths OHT3

Not to put too fine a point on it,
the present state of mathematics
is a threat to democracy.
Too many are denied full access to it,
too many fail it
and too many come to rely on
those few who have been initiated.
Human dignity is undermined by
the submerged guilt about inadequacy that
resides with so many of our citizens.

Boomer 1986, p. 7

Metaphors

HO7
page 1

Technique 3: Metaphors

What are they?
The Heinemann *Australian Dictionary* defines metaphor as 'a figure of speech in which one thing is identified with another'. Example: 'he was a tower of strength during the crisis'.

Teachers often unconsciously use metaphors to describe their teaching:

- *I hate having to be a police officer.*
- *Sometimes I'm more of a mother than a teacher.*

If used consciously, metaphors can illuminate teaching practice. They provide a contrast to the usual ways teachers would describe themselves. They invite the use of intuition and imagination, developing new frameworks that reflect individually.

Questions to Consider

Q1 What do you do when you teach?

For example: Teaching context—a literacy class
I listen, observe, question, negotiate, demonstrate, encourage, provide resources, mediate, organise, offer suggestions, encourage students to identify their goals, ask for feedback, help students, focus, draw out students' understanding, set up learning activities ...

Q2 What roles do you use when you teach?
What roles encompass the actions you have listed?

Continuing the above example:
organiser, mediator, evaluator, observer, negotiator, 'teacher', resource provider.

Q3 What metaphors(s) do you associate with these roles?

Continuing the above example:
I see myself as the producer of a film, with the student as director.
The student decides on the direction for the film and I provide the resources and advice necessary to achieve this.
I also see myself as a counsellor, listening, questioning, developing a learning environment in which the student constructs their own learning.

Adapted from Allan 1994, pp. 25–28.

Journal Entry

To answer Questions 4–6, reflect on your responses to Questions 1–3 in relation to your teaching.

Q4 Do these roles and metaphors represent what you do, or what you want to do, or a mix of both?

The aim of this question is to clarify what your metaphors represent. When you first develop metaphors for your teaching, it is often difficult to work out whether they represent what it is you actually do in your teaching, or your ideals or goals. It doesn't matter which of these your metaphors represent. It is important that you know what your metaphors are describing. It may take some time to work this out.

Q5 Do your roles and metaphors change when the context changes?

Depending on the teaching context, you may find one set of roles and metaphors insufficient. You may wish to develop different roles and metaphors for particular contexts, e.g. team teaching with a trade or industry class, and a numeracy class.

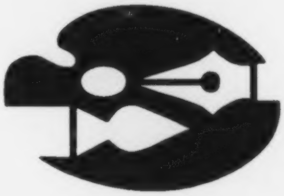
Q6 What roles and metaphors do your students see in your teaching?

Ask your students!
This can give you some fascinating feedback, and your students may wish to explore the use of metaphors in their own lives.

At home with algebra ✣ AS8

1 A person selling paintings at the Esplanade market on Sundays sells the paintings for $25 each.
She pays $75 to rent the stall.

Write a formula to express how much money she takes home, showing a connection between how much money she takes home and the number of painting she sells.



2 A TV repair person charges $75 per hour for labour and $50 attendance fee for an on-site call.

Write a formula to express how much you would pay for house calls of different lengths.

At home with algebra ✣ AS9

---

3 The number of detective novels Liz needs to last her for her holiday is one for every two days and one extra.

Write a formula for the number of books Liz needs for a holiday.

---

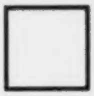
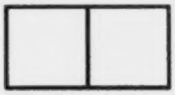
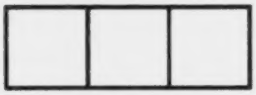
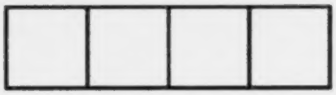
4 You have a favourite old cookbook where oven temperatures are still given in degrees Fahrenheit. A rough method to find out the number of degrees Celsius is to take off 30 degrees from the degrees Fahrenheit and then halve the answer.

Write a formula for changing degrees Fahrenheit into degrees Celsius using this rough method.

Squaring with matches

HO8

Can you work out how many matches you need to make each of the following shapes?

Shape:	Number of matches:
	
	
	
	

Without drawing it, can you predict the number of matches for *ten* squares?

What about for one hundred squares?

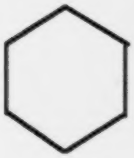
Can you find a rule (in your own words) which generalises how many matches you need?

Can you write the rule in symbols?

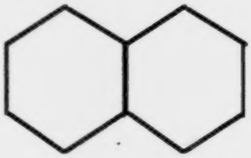
Writing your own rules ✥ AS10

WRITING YOUR OWN RULES—1

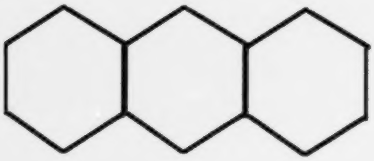
Can you find a rule (in your own words) which generalises how many matches you need for the 4th shape, 5th, … 10th, … 100th shape?



1st



2nd

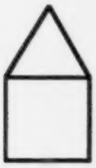


3rd

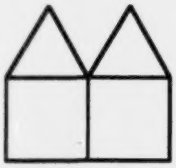
Can you translate the rule into symbols?

WRITING YOUR OWN RULES—2

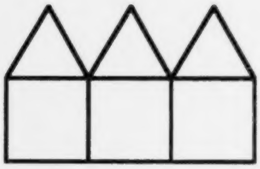
Can you find a rule (in your own words) which generalises how many matches you need for the 4th shape, 5th, … 10th, … 100th shape?



1st



2nd



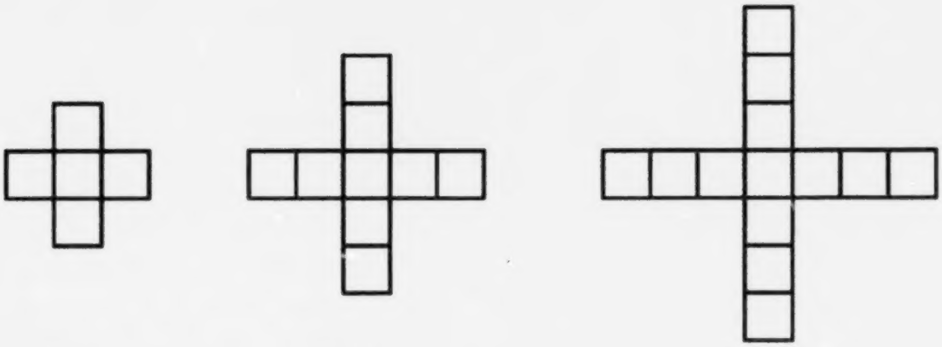
3rd

Can you translate the rule into symbols?

Writing your own rules ✣ AS11

WRITING YOUR OWN RULES—3

Can you find a rule (in your own words) which generalises how many matches you need for the 4th shape, 5th, … 10th, … 100th shape?

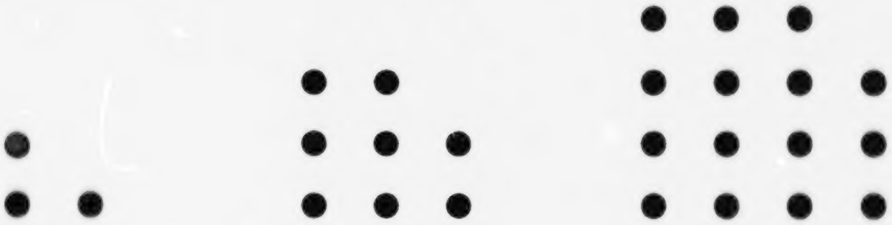


1st 2nd 3rd

Can you translate the rule into symbols?

WRITING YOUR OWN RULES—4

Can you find a rule (in your own words) which generalises how many dots you need for the 4th shape, 5th, … 10th, … 100th shape?



1st 2nd 3rd

Can you translate the rule into symbols?

Choose a number

HO9

The following number trick is illustrated with two different numbers.
Proofs are shown with boxes and dots and with algebraic symbols.
Use your cups and counters or jelly beans to follow the steps.

Step:	Examples		Proofs	
Choose a number	5	8	□	n
Multiply by three	15	24	□□□	$3n$
Add six	21	30	□□□ ●●●●●●	$3n + 6$
Divide by three	7	10	□ ●●	$n + 2$
Subtract the number you first thought of	2	2	●●	2
The answer is 2.				

For each of the following number tricks use your cups and counters or jelly beans to work out the answer. Write out the proofs as in the above example.

1 Choose a number
Add three
Multiply by two
Add four
Divide by two
Subtract the number first thought of
The result is ________

2 Choose a number
Double it
Add nine
Add the number first thought of
Divide by three
Add four
Subtract the number first thought of
The result is ________

3 Choose a number
Add the next larger number
Add seven
Halve it
Subtract the number first thought of
The result is ________

4 Choose a number
Triple it
Add the number one larger than the number first thought of
Add eleven
Divide by four
Subtract three
The result is ________

Adapted from Jacobs 1982, pp. 40–41

Filling the gaps ... HO10

wanganny
marrma
lurrkun
dambumiriw
wanganny rulu
wanganny rulu ga wanganny
wanganny rulu ga marrma
wanganny rulu ga lurrkun
wanganny rulu ga dambumiriw
marrma rulu
marrma rulu ga wanganny
marrma rulu ga marrma
....
marrma rulu ga dambumiriw
lurrkun rulu
lurrkun rulu ga wanganny
lurrkun rulu ga marrma
lurrkun rulu ga lurrkun
lurrkun rulu ga dambumiriw
....
dambumiriw rulu ga wanganny
dambumiriw rulu ga marrma
dambumiriw rulu ga lurrkun
dambumiriw rulu ga dambumiriw

The word for five ... HO11

The rich and interesting field of Australian Aboriginal and Torres Strait Islander mathematical concepts has been generally ignored by anthropologists, linguists and other researchers. Some mathematical knowledge has been lost forever, particularly where English or English-derived terms have replaced traditional terms. Much knowledge remains however, where traditional languages are still spoken, and much can still be studied by those who are prepared to question the false and misleading generalisations in the literature … Researchers, not all of them in the past, have generally taken absence of verbalisation to mean absence of counting, so we find the literature full of false statements:

> *… in various … parts of Australia, the natives show habitual uncertainty as to the number of fingers they have on a single hand.* [Smith 1923:7]

Smith obtained his information on Australia from Crawford(1863).
It is easy to see how, with poor ethnology and preconceived low expectations this misconception was reached. In many Aboriginal languages the word for five *is* "hand'. Asked how many fingers they have, apart from finding it a stupid question (who doesn't know that?), these Aboriginal people would hold up five fingers. This would not be accepted by the researcher who would try to elicit a verbal response, which could only be hand. It is the researcher who is ignorant, not the informant …

Harris, J. 1987, 'Australian Aboriginal and Islander Mathematics',
Australian Aboriginal Studies, 2

Why count...? HO12

Counting, says Denny*, serves as a way of 'apprehending objects which cannot be perceptually or conceptually identified'.

He describes a court case about land rights where an Inuit hunter was unable to say how many rivers were in the disputed area, a failure which was taken by the opposition as clear evidence that the man was unfamiliar with the region. In fact, the man probably knew the actuality of each river, of each bend of each river. There was no use in knowing the number of them. The point of the story, says Denny, is that

> *we count things when we are ignorant of their individual identity—this can arise when we don't have enough experience of the objects, when there are too many of them to know individually, or when they are all the same, none of which conditions obtain very often for a hunter ... [whereas] articles in industrial society often cannot be individualised because they are identical—all one can do is count.*

* Denny, J.P. 1986, 'Cultural ecology of mathematics: Ojibway and Inuit hunters' in M. Closs (ed.) *Native American Mathematics*, University of Texas Press, Austin.

Counting

What things do we count?

What things do we not count?

Why do we count?

Do all societies count?

Coloured counters ✣ AS12

How could you use these counters to represent 375?

(Think about how coins of 3 different sizes could be used to represent $3.75.)

Now, underneath 375, represent 847.

Can you use the counters to add 375 and 847?

Can you use the counters to multiply 3 x 24?
10 x 24?
40 x 24?
43 x 24?

Dienes blocks ... MAB blocks ✣ AS13

How could you use these blocks to represent 375?

Now, next to 375, represent 847.

Can you use the blocks to add 375 and 847?

Can you use the blocks to multiply 3 x 24?
10 x 24?
40 x 24?
43 x 24?

Can you use the blocks for this division 42 ÷ 3?
and this one ... 420 ÷ 10?

The abacus

✥ AS14

How could you use the abacus to represent 375?

Now represent 847.

Can you use the abacus to add 375 and 847?

Can you use the abacus to subtract 347 – 235?

814 – 577?

Can you use the abacus to multiply 3 x 24?

10 x 24?

40 x 24?

43 x 24?

Multiple embodiments

HO13
page 1

From Fuys & Tischler 1979, *Teaching Mathematics in the Elementary School*, p. 208,
© Little, Brown & Co., Boston..

The use of different types of materials is justified by what Zoltan Dienes calls the multiple embodiments principle. Dienes maintains that children are better able to form the abstraction behind an operation if they experience it in various models (or in multiple embodiments) and that this broad experience will help children to transfer their leaning of the operation from these classroom situations to other new situations. Most commercial textbook series appear to accept this principle, because different types of materials for number and operations on numbers are pictured. However, this principle may not apply to all children. For certain learning-disabled children a variety of embodiments may create confusion that gets in the way of learning the operations. Mathematics programs for such children may concentrate almost exclusively on one material and on one set of procedures of acting out problems involving basic facts …

You have seen that there are many available materials with which children can illustrate number. (the following are examples: chips, coins and dollar bills, abacus, pebbles in bags, bundles of sticks, wooden multi base blocks, squared material, bean sticks, beans, and the three on page 318.) …

In the activities in this chapter, you began by using coloured chips as a model for place value, and then later used structured materials (multi base blocks or squared material). However, other sequences may be better for children. They may find it easier to begin with some material that they themselves can form into groups (such as a grouping material), then go onto a material that is already formed, but that shows the relationship between pieces (such as a structured material), and finally use the more abstract place-value models, such as colour chips, an abacus, or a place-value chart.

Could you play exchange games (in base ten) with all of the materials listed above? If not, which materials would not be suitable?

Some models are much less convenient for exchange games with three places because the model for 100 is unwieldy (for example, pebbles in bags) but we certainly could play the two-place game with any of these models, provided that each child has sufficient materials. Is it desirable or even feasible to use all 13 of these models with a second grade class? Is it desirable to use one model exclusively?

Many teachers give their students experience with several different models, chosen carefully to suit the children's preferences, the class budge, and available storage space. A teacher might keep materials for exchange games in separate boxes—with directions inside each one—where each box uses different materials, perhaps chips, or coins, or play money, or cuisenaire rods, or bean sticks. Then children could choose the game they prefer or be directed to a new box if they need more experience with the corresponding model for place value. There are, however, more fundamental reasons for offering a variety of models for place value, or multiple embodiments.

HO13

page 2

1 Individual differences in learning style. One child may find a given material useful for learning a topic, while another may not. One second grader might attach manning to two-digit numerals through multi base blocks, while another might find it easier to use a grouping material such as bundles of sticks (perhaps because he has not had enough experience grouping). Thus, using several different types of materials for a topic enables the teacher to reach all of the pupils in the class more effectively.

2 Transfer of learning. Transfer means the ability to apply some knowledge of process in a new situation. The use of multiple embodiments facilitates transfer of learning. For example, experiences with coloured chips may help children to understand coinage, while multi base locks may help them to understand the relation between metric units (centimetre and decimetre).

3 Extension to other mathematical topics. Which material, multi base blocks or an abacus, is more suitable for showing the number 12,542?

It is very difficult to use the multi base blocks to show large numbers, while an abacus provides a convenient model. However, decimals such as 1.32 can be represented easily with multi base blocks, while children find an abacus a more difficult model for this concept. A fifth grade teacher who was teaching decimals and large numbers might wish that his students had used both embodiments in earlier grades, because he could then build on this prior experience.

What about little numbers? HO14

1 Without using concrete materials, arrange the following numbers in order of magnitude:

0.09 0.3 0.064 0.007 0.25 0.150 0.07

Now show how you would use MAB blocks

- to check your answer, or
- to help a student grasp the meaning of this task.

2 With the blocks, model and find the answer to

3.72 + 5.53
5.53 – 3.72
3 x 3.72 3 lots of 3.72
3 x 0.372

3 How could you use the blocks to model 0.1 x 0.3 ?
You might start by saying ...

- What about 10 x 0.3 … is that any easier to think about?
- Yes, OK, *10 lots of* 0.3 …
- Well, what is 0.1 x 0.3 then?

- A *0.1 lot of* 0.3 ... a tenth of 0.3 ?

- So make 0.3 ... 3 flats … and what is a tenth of them?
- A tenth of each flat is a long ...0.01
  So, a tenth or a *0.1 lot of* 3 flats is 3 longs ... 0.03!

4 Can you extend this to model 0.4 x 0.3?

5 How could you think about and model 1.5 divided by 0.3 ?
or divided by 0.03 ?

Murdering the innocents

HO15
page 1

The One Thing Needful

Charles Dickens, *Hard Times*, 1985 edition,
© Penguin Classics, London, pp. 47–50.

Thomas Gradgrind, sir. A man of realities. A man of facts and calculations. A man who proceeds upon the principle that two and two are four, and nothing over, and who is not to be talked into allowing for anything over. Thomas Gradgrind, sir—peremptorily Thomas—Thomas Gradgrind. With a rule and a pair of scales, and the multiplication table always in his pocket, sir, ready to weigh and measure any parcel of human nature, and tell you exactly what it comes to. It is a mere question of figures, a case of simple arithmetic. You might hope to get some other nonsensical belief into the heart of George Gradgrind, or Augustus Gradgrind, or John Gradgrind, or Joseph Gradgrind (all suppositions, non-existent persons), but into the head of Thomas Gradgrind—no, sir!

In such terms Mr Gradgrind always mentally introduced himself, whether to his private circle of acquaintance, or to the public in general. In such terms, no doubt, substituting the words 'boys and girls', for 'sir'. Thomas Gradgrind now presented Thomas Gradgrind to the little pitchers before him, who were to be filled so full of facts.

Indeed, as he eagerly sparkled at them from the cellarage before mentioned, he seemed a kind of cannon loaded to the muzzle with facts, and prepared to blow them clean out of the regions of childhood at one discharge. He seemed a galvanising apparatus, too, charged with a grim mechanical substitute for the tender young imagination's that were to be stunned away.

'Girl number twenty,' said Mr Gradgrind, squarely pointing with his square forefinger, 'I don't know that girl. Who is that girl?'

'Sissy Jupe, sir,' explained number twenty, blushing, standing up, and curtsying.

'Sissy is not a name,' said Mr Gradgrind. 'Don't call yourself Sissy. Call yourself Cecilia.'

'It's father as calls me Sissy, sir,' returned the young girl in a trembling voice, and with another curtsy.

'Then he has no business to do it,' said Mr Gradgrind. 'Tell him he mustn't. Cecilia Jupe. Let me see. What is your father?'

'He belongs to the horse-riding, if you please, sir.'

Mr Gradgrind frowned, and waved off the objectionable calling with his hand.

'We don't want to know anything about that, here. You mustn't tell us about that, here. Your father breaks horses, don't he?'

'If you please, sir, when they can get any to break, they do break horses in the ring, sir.'

'You mustn't tell us about the ring, here. Very well, then. Describe your father as a horsebreaker. He doctors sick horses, I dare say?'

'Oh yes, sir.'

'Very well then. He is a veterinary surgeon, a farrier, and horse breaker. Give me your definition of a horse.'

Sissy Jupe thrown into the greatest alarm by this demand.

'Girl number twenty unable to define a horse!' said Mr Gradgrind for the general behoof of all the pitchers. 'Girl number twenty possessed of no facts, in reference to one of the commonest of animals. Some boy's definition of a horse. Bitzer, yours.'

The square finger, moving here and there, lighted suddenly on Bitzer, perhaps because he chanced to sit in the same ray of sunlight which, darting in at one of the bare windows of the intensely whitewashed room, irradiated Sissy, being at the corner of a row on the sunny side. For the boys and girls sat on the face of narrow interval: and Sissy, being at the corner of a row on the sunny side, came in for the beginning of a sunbeam, of which Bitzer, being at the corner of a row on the other side, a few rows in advance caught the end. But, whereas the girl was so dark-eyed and dark haired, that she seemed to receive a deeper and more lustrous colour from the sun, when it shone upon her, the boy was so light-eyed and light-haired that the self-same rays appeared to draw out of him what little colour he ever possessed. His cold eyes would hardly have been immediate contrast with something paler than themselves, expressed their form. His short-cropped hair might have been a mere consternation for the sandy freckles on his forehead and face. His skin was so unwholesomely defiant in the natural tinge, that he looked as though, if he were cut he would bleed white.

'Bitzer,' said Thomas Gradgrind. 'Your definition of a horse.'

'Quadruped. Graminivorous. Forty teeth, namely twenty-four grinders, four eye-teeth and, twelve incisors. Sheds coat in the spring; in marshy countries, sheds hoofs, too. Hoofs hard, but requiring to be shod with iron. Age known by marks in mouth.' Thus and much more, Bitzer.

'Now girl number twenty,' said Mr Gradgrind. 'You know what a horse is.'

Talking about fractions

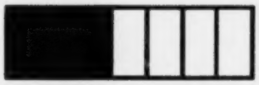
HO16

fractions!

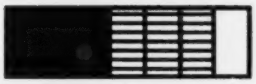
$\frac{3}{7}$ MEANS

(amongst other things)

'cut your loaf into 7 pieces and take 3' i.e.



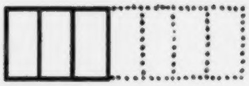
so, $2 \times \frac{3}{7}$ is two lots of $\frac{3}{7}$ i.e.



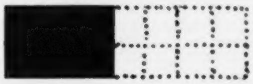
or $\frac{6}{7}$

and what about $\frac{1}{2} \times \frac{3}{7}$? i.e. a half lot of $\frac{3}{7}$?

well, here is $\frac{3}{7}$



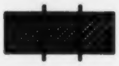
shade in $\frac{1}{2}$ of it



So, what have we done?

This is the same as cutting the loaf into 14 pieces

and taking 3 of them i.e.



or $\frac{3}{14}$

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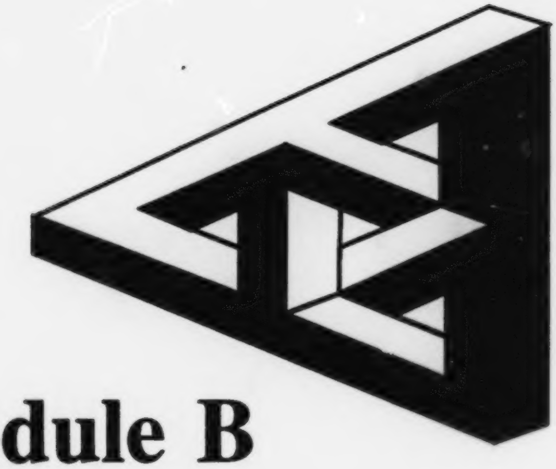


Module B
Mathematics as a human construction

Presenter's notes

Module B: Maths as a human construction

Nominal time: 24 hours

Brief description

Participants think about how people name and use shapes in their worlds, they investigate aspects of symmetry, mosaics, plane geometry, topology and scale, consider the nature of maps and models and reflect on ways of learning and teaching these topics.

Rationale/aims

During the course the participants will be engaged in two mathematical journeys, in which they will explore a number of mathematical concepts and ways of thinking. The first journey, undertaken here in Module B, consists of five sessions and focuses on ideas of space and shape, trying to link these with a variety of historical, cultural and natural contexts. The second journey, woven through the six sessions of Module C, focuses more on the nature of number and how it is used in our society. The emphasis in this journey, reflected also in the accompanying Curriculum Project 2, is on understanding and developing the mathematical concepts. The emphasis of the second journey, reflected in its accompanying Curriculum Project 3, will be on how the relevant mathematics is best learnt and taught.

Learning outcomes

Participants should be able to:

- Identify and analyse how mathematics is a human and social construction
- Analyse and apply specific areas of mathematics, and identify teaching and learning strategies and resources for these specific areas.

Assessment requirements

1 Maintain a written journal of reflections on the content of the module and any teaching implications. The journal is to comprise personal reflections on the course to be done out of workshop time, consisting of at least 2 brief informal entries for the module.

2 Complete Curriculum Project 2 (CP2) – Exploring Maths.

CP2 is preferably done as a team activity.

A contract for the content of CP2 is to be negotiated between the participants and course presenter. CP2 is a mathematical investigation of a mathematical topic of the participants' choice. It will include planning, exploring the topic, and a presentation to the Group. Each participant must write their own report (2000–2500 words) of either this curriculum project or of Curriculum Project 3.

References

A bibliographical list of references for this module is given at the end of Resources B [HO22].

Module B outline

section	*time*	*development of issues and activities*	*maths involved*
B1 – Shaping the world			
B1.1 Shaping our world	2 h	– *shaping our world* – *playing with mirrors to look at angles and regular polygons* – *different worlds*	classification of geometric shapes angles
B1.2 Filling space	1 h	– *why do bees make hexagonal cells?* – *but fitting is not enough*	angles, polygons area
B1.3 The patchwork connection	1 h	– *patchwork samples* – *excavating patchwork*	parallel lines, area, polygons, angles
B1.4 Mostly Pythagoras	2 h	– *a Pythagorean puzzle* – *cultural connections*	right-angled triangles, area, parallel lines, angles, Pythagoras' theorem
B2 – Circles and culture			
B2.1 Problem solving stations	1 h		measuring circles etc., estimation, symmetry, perimeter, area, topology
B2.2 Investigating pi	2 h	– *tins and circular objects* – *squaring the circle* – *pi in history*	length and area related to circles
B3 – Problem solving			
B3.1 Problem solving … problem posing	1 h	– *the handshake problem* – *problem solving strategies*	problem solving, algebra, relationships and pattern
B3.2 Effects of scale and shape	2 h	– *why do babies dehydrate faster than adults in summer?* – *looking at boundaries in 3-D* – *arranging space*	length, area and volume scale modelling problem solving

B4 – Maps and models			
B4.1 Maps and models	2 h	– *measuring the coastline*	length, scales, ratios etc. fractals, topology
B4.2 Mindmaps: a review	1 h		
B5 – Exploring maths			
CURRICULUM PROJECT 2 – EXPLORING MATHS	9 h	Curriculum Project 2: Exploring maths *An investigation of a mathematical topic of the participants' choice*	participants' choice of topic

B1 Shaping the world

B1.1 Shaping our world

Brief description

Participants think about how people name shapes in their worlds and investigate aspects of symmetry.

Rationale/aims

This session is the first stage of the journey looking at mathematical concepts of space and shape. It gives participants a chance to consider how concepts of shape emerge from particular cultural needs, how we categorise shapes and how we might analyse their properties by using ideas of symmetry.

Preparation

Presenter
Read
Naming shapes activity
Participants
Read
What's so great about regular shapes? [P3]

Time: 2 hours

Materials needed

Concrete materials
For *Naming shapes activity:*
- dictionary
- at least ten 5 x 8 cm pieces of coloured card or paper for each participant
- glue
- 1 large sheet of paper for each group of 4 participants
- textas—1 for each group of 4

For *Mirrors activity*
- pocket mirrors (rectangular, non-bevelled edge, about 12 cm x 8 cm, 1 to each pair)
- small bits of bright plastic or glass or paper
- sticky tape
- protractors (1 per pair)
- butcher's paper or A3 sheets

Handouts
Using symmetry [P1]
The circle and the line [P2]
Naming geometric shapes [HO1]
Names [HO2]
Experimenting with mirrors [HO3]
OHTs/paper
Shaping our worlds [OHT1]

B1.1 Shaping our world

Detailed procedure

The session consists of three activities, using a variety of Group and small group organisation

–	**Shaping our world**	(45 min)
–	**Playing with mirrors**	(45 min)
–	**Different worlds**	(30 min)

Shaping our world

1 Remind participants about plane shapes that they encountered in Module A (in *Looking at boundaries*, and possibly in one of the cooperative logic problems) and proceed with activity: *Naming geometric shapes* [HO1]

Note: *Only use Worksheet 1* [HO1] *if there is time.*

2 Some extra questions to address. Ask:

Where do the names come from? Give out *Names* [HO2]

What is regular about a regular polygon?

What would a regular polygon with a thousand sides look like?

3 And a quick diversion to the similar names and questions for polyhedra.

What are their names?

Where do the names come from?

And what are prisms and pyramids?

Some students find it useful to think of a prism as a solid shape that can be sliced into identical pieces, the shape of the slice giving its name to the prism.

What is regular about a regular polyhedron?

What would a polyhedron with a thousand faces look like?

Are prisms and pyramids regular?

Playing with mirrors

1 This activity involves using a simple kaleidoscope. Give participants a mirror each, and get them to work in pairs to hinge two mirrors together to make a kaleidoscope.

2 Get the participants to examine their own reflections in the hinged mirrors.

Are they the same as in an ordinary mirror? Why not?

3 Ask pairs of participants to carry out the activity *Experimenting with mirrors* [HO3]. During this activity individuals or the Group may need advice on how to use protractors.

4 Conclude the activity by talking briefly about line and point symmetry. Referring to *Using symmetry* [P1] make the point that it is not enough to be able to identify symmetry, we need also to understand why it is useful.

Different worlds

1 Now ask:

Where do we find examples of these shapes in the everyday world?

Elicit and list suggestions—'bricks are rectangular prisms', 'roofs are cones or triangular prisms'—and include (for later reference) 'bees' cells are hexagons'.

It is likely that the majority of examples will be human artefacts.

2 Give out *The circle and the line* [P2]. Ask half the Group to go through the Davis and Hersh quote and list the examples of a straight line given there, and the other half to list the examples of a circle given in the Black Elk quote.

Collect the lists on the board and discuss the differences between them.

The bulk of the examples in the first are of human artefacts, whereas in the second the majority are from the natural world.

3 Point out that we have been paying a lot of attention to regular shapes, or at least shapes with some degree of regularity. Ask:

Do all societies have the same interest in geometric shapes?

Address the question by discussing *What's so great about regular shapes?* [P3].

Bring out the author's points that the dependence of hunters on wild animals and plants means that:

- they subsist in a relatively unaltered environment with limited need for technology
- in such environment, shapes are irregular and variable and there is a lack of the job specialisation which is one factor generating the need for geometric (measured, regular) forms.

4 Conclude with the point Ascher makes in the last paragraph, *Shaping our world* [OHT 1]:

> '[Both] writers share their degree of conviction in the rightness of their ideas and support their view with nature, God, achievement of goals, and proper human development. Taken separately or together, the statements highlight the fact that geometric ideas are an integral part of a culture's world view.'

B1.2 Filling space

Brief description

Participants tease out some mathematical aspects of mosaics.

Rationale/aims

This session continues the journey through mathematical concepts of space and shape, engaging participants in an exploration of cultural and mathematical aspects of tiling, packing and mosaics.

Preparation

Presenter
Bring a book of Escher prints, including mosaics
Prepare sets of regular polygons [AS1]

Time: 1 hour

Materials needed

Concrete materials
set of regular polygons (1 set per pair) [AS1]
Handouts/OHTs/paper
Why do bees make hexagonal cells? [HO4]
Hexagon etc. [AS2]
Centimetre grid paper [AS3]

B1.2 Filling space

Detailed procedure

This section consists of two activities, with participants working in pairs at first and then coming together for a Group discussion and conclusion.

- **Why do bees make hexagonal cells?** (40 min)
- **But fitting is not enough** (20 min)

Why do bees make hexagonal cells?

1 Talk about the question.

Although many regular shapes would be found in the socially constructed environment—buildings, furniture, roads, clothes, packages—there are some to be found in the natural environment. We noted earlier that bees make hexagonal cells, which form a mosaic pattern. Mosaics, both culturally and naturally occurring, offer a rich field for mathematical investigations.

Talk briefly about this, about tiling, tessellation and mosaics as patterns of shapes that fill or cover 2-D space.

Collect from the Group examples of mosaics that they can see or are familiar with.

Ask if there is a 3-D equivalent, and elicit ideas about packaging and examples like books, blocks, sardines, Toblerone packages, tetrapacks…

2 And now pick up on that throw-away line about bees:

- *Well, why do bees make hexagonal cells?*
- *Would pentagons work as well?*
- *Or triangles or octagons?*

3 Before starting the next activity, suggest that it might be useful if participants could remember the sum of angles in a triangle. This would be a good time to demonstrate the relationship, by making a quick triangle, marking its corners, tearing them off and reassembling them so that they make a straight line.

4 Get participants to work in small groups on *Why do bees make hexagonal cells?* [HO4]. Ask them to draw diagrams or tessellations on the butcher's paper or A3 sheets of paper.

But fitting is not enough

1 So far we have seen that one reason that bees' cells are hexagonal rather than octagonal or circular is that hexagons fit together. So we have a partial—mathematical—explanation.

There are probably plenty of other biological understandings that might help too, but even confining ourselves to the mathematics, we can go a little further...

Talk about this, and ask—or preferably elicit the question from the participants:

OK, hexagons fit, but so do triangles and squares—and many other less regular shapes—so why not them?

2 Suggest, if necessary, that it may be something to do with the space inside—perhaps it is time for revisiting the area/perimeter question from *Looking at boundaries* (Section 1.3).

3 As a Group activity, use *Hexagon etc.* [AS2] and *Centimetre grid paper* [AS3] to get participants to work out the areas of different regular polygons (and the circle) with equal perimeter. Encourage a variety of ways of counting the squares to find the area. Ask them to put the shapes in order of size, and to describe what is happening from smallest to largest, e.g.

When the perimeters are equal then the more sides the polygon has, the larger its area (and the circle, as a polygon with infinite sides, is the biggest).

4 So, why *do* bees make hexagonal cells?

> For a given amount of work and material making wax walls, the circle gives the most room. But circles don't fit—and what's the best compromise? Hexagons! Unlike octagons and circles, they fit together, and compared with triangles and squares which also fit, they have more space inside.

5 Show some Escher mosaic designs and discuss why the mosaics fit together, and how Escher might have designed them.

6 Finish this session by asking participants to collect and bring in for the next session some examples of tiling patterns:

- brick paving
- examples in nature
- Islamic mosaics
- patchworks etc.

and in particular any information they can find out about patchwork.

B1.3 The patchwork connection

Brief description

Participants use patchwork as a starting point for excavating mathematics.

Rationale/aims

Continuing the focus on tiling and tessellations from Section B1.2, this session involves participants in developing an awareness of the possibilities of using real world resources for understanding and teaching mathematics. Using patchwork, participants are encouraged to go beyond a simple appreciation of the presence of mathematical concepts or techniques ('look, there are hexagons here') to a questioning of that presence ('why hexagons?').

Preparation

Presenter
Make 2 or 3 copies of each of the resources AS4–AS12 so that there are enough for each small group to have two or three each to use in the activity excavating patchwork.
Put each group's set (of two or three) in an envelope for ease of distribution in the session.
Participants
Bring in any information or examples they can find about patchwork.

Time: 1 hour

Materials needed

Concrete materials
strips of paper 5 cm x 30 cm, in at least three different colours or patterns (about 5 per participant)
strips of paper 2 x 30 cm, in three different colours or patterns (about 2 per participant)
graph paper
Handouts/OHTs/paper
Helpful formulas [AS4]
Transformations [AS5]
Determining yardage [AS6]
Organising design elements [AS7]
Seminole patchwork [AS8]
Dividing squares [AS9]
Side triangles [AS10]
Seam allowances [AS11]
Seminole: a patchwork technique [HO5]
Excavating patchwork [HO6]

B1.3 The patchwork connection

Detailed procedure

This section consists of a whole group activity, followed by small group excavations

–	**Patchwork samplers**	(30 min)
–	**Excavating patchwork**	(30 min)

Patchwork samplers

1 Get participants to show briefly the tiling examples they have brought in.
Discuss the aims for this session: that participants will have a chance to excavate mathematics from a particular real world source.

2 If anyone has found out information about the origins of patchwork, ask them to share what they found out. If not, leave this until it emerges in the course of the excavation.

3 Lead participants through the activity *Seminole: a patchwork technique* [HO5].

4 Brainstorm what mathematics might be involved in this activity.

Excavating patchwork

1 Handout the materials on patchwork (AS4–AS12) so that each group has at least two.
Include graph paper with *Transformations* [AS5] and *Dividing squares* [AS9].

2 Also hand out *Excavating patchwork* [HO6], and set the groups to work.
Encourage participants to share any patchwork information or materials they have brought in as well. Have graph paper available.

3 In the last ten minutes, get groups to report back matters of mathematical interest.
Conclude by listing and discussing the possibilities for mathematical exploration—learning and teaching—that participants have been able to dig out of their different materials.
For a possible list, see the table on the next page.
Encourage participants to go beyond the construction of the list, from a simple appreciation of the presence of mathematics: ('Look, there are hexagons here,' or 'There are some parallel lines.')
to a questioning of that presence: ('Why hexagons?' or 'Why parallel lines?' and 'How do you draw them?').

The maths of patchwork

doing patchwork	*relevant maths*
length (in metres or yards)	measurement estimation area and perimeter
formulas for blocks	algebra order of operations
cutting out blocks	shapes: squares, triangles etc. angles arcs grain stretchable hypotenuse
templates and blocks	symmetry: translations, rotations and reflections shapes: squares, hexagons etc. 3-D design and colour scale: enlarging and shrinking
assembly of blocks	tessellations parallel lines, angles logical analysis
borders	perimeter and area

Source: Jynell Mair and Sue Turner, Curriculum Project 2, ANT Pilot Course, 1994

B1.4 Mostly Pythagoras

Brief Description

Participants continue to look at shape and space relationships and investigate a number of proofs of Pythagoras Theorem and look at some of its uses throughout history.

Rationale/aims

The relationships between the sides of a right-angled triangle have been known and used by many nations over a long period of time, and as seen in the previous session have been utilised in art as well as in other more practical applications.

This session enables participants to investigate what has become known as Pythagoras Theorem and to look at a number of physical proofs of the theorem, which will probably be new to most participants who may have had the theorem presented to them in school in a traditional algebraic way, or who may never have seen a proof at all.

Preparation

Presenter
Make enough copies of each of the handouts described under Materials needed so that there are enough for each small group.

Time: 2 hours

Materials needed

Concrete materials
set squares and rulers
scissors
grid paper (use AS3) or graph paper

Handouts/OHTs/paper
A Pythagorean puzzle [HO7]
Making your own Pythagorean puzzle [HO8]
How far does it stretch? [HO9]
Pythagorean pieces [HO10]
Piling up of rectangles [HO11]
The kou ku theorem [HO12]

References

Bronowski, J. 1973, chapter 5, 'The music of the spheres', *The Ascent of Man*, BBC, London.
Gerdes, Paulus 1988, 'A widespread decorative motif and the Pythagorean theorem', *For the Learning of Mathematics*, Vol. 8, No. 1, pp. 35–39.
Hogben, Lancelot 1936, *Mathematics for the Million*, George Allen & Unwin, London, pp. 55–59.
Joseph, George Gheverghese 1991, *The Crest of the Peacock: non-European roots of mathematics*, I.B.Tauris, London, pp. 180–2.
Kolpas, Sidney J. 1992, *The Pythagorean Theorem: eight classic proofs*, Dale Seymour Publications

B1.4 Mostly Pythagoras

Detailed procedure

This section consists mainly of small group activities and investigations.

- **A Pythagorean puzzle** (1 hour)
- **Cultural connections** (1 hour)

A Pythagorean puzzle

1 Introduce the session by explaining that we are going to look at some characteristics of a right-angled triangle—a shape that has long been used in architecture and building and, as we have seen in the last session, in art and crafts.

You may need to describe a right-angled triangle and the naming of the hypotenuse.

2 Give each participant *A Pythagorean puzzle* [HO7]. Get them to work in small groups of 2 or 3, following the instructions to cut out the shapes in the two squares on the shorter sides of the triangle and try to make them fit onto the large square on the longest side.

Ask participants to decide what this tells them about the relationship between

- the areas of the three squares
- the three sides of the triangles.

3 Now ask them to see if they can create this puzzle for a right-angled triangle of any size. This requires an ability to be able to use a set square and ruler to draw parallel lines, so first of all ask participants to work together to teach each other how to use a set square and ruler to draw parallel lines.

When they have worked this out, give them *Making your own Pythagorean puzzle* [HO8] and ask them to follow the instructions to construct their own Pythagorean Proof.

After the activity, discuss with participants the wording for the theorem, such as:

> *'In a right-angled triangle, the sum of the squares of the legs is equal to the square of the length of the hypotenuse.'*

Cultural connections

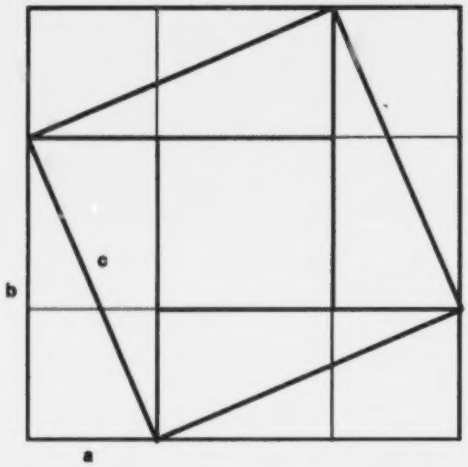
1 Hand out *How far does it stretch?* [HO9] and *Pythagorean pieces* [HO10]. Refer participants to other books and articles (see References on previous page).

Discuss them as a Group and look at the historical and cultural uses of the Pythagorean relationship, and when and where in its usage Pythagoras actually became involved.

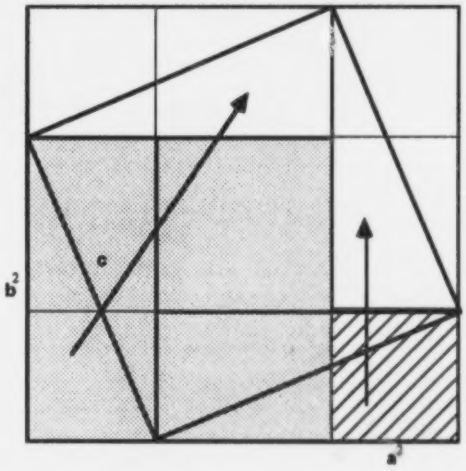
Link this to any discussion held in the previous session which looked at the Pythagorean relationship in art, and when and where it was used.

You could ask participants to use a piece of string (ONLY—no rulers etc.) to construct a right angle on their desk or table.

2 Introduce another method of proving the Pythagorean relationship, a Chinese method called 'Piling up the Rectangles'. This is explained in *Piling up the Rectangles* [HO11]. The final drawing should look like something like this:



At this point it is matter of seeing an area for a^2 and b^2 and relating this to the area for c^2. This can be seen by moving two of the right-angled triangles from the area for $a^2 + b^2$ into the area for c^2 as shown below. Note: the tilted square is c^2.



3 Give out *The kou ku theorem* [HO12] as a summing up of some of the Chinese proofs.

Finish the session by discussing the importance of the Pythagorean theorem and its place in history and culture. Relate the discussion to the traditional way that Pythagoras has been treated in the traditional maths classroom. Discuss how the material covered in this session could be used in an ALBE teaching/learning context.

B2 Circles and culture

B2.1 Problem solving stations

Brief description

Participants tackle some problems about spatial relations, focused around circles, and engage briefly with some topological puzzles.

Rationale/aims

By using a number of problem solving activities as stations, this session allows the participants to encounter a variety of situations, and to use a variety of problem solving strategies. Participants will be able to refer back to some of these strategies when they analyse problem solving in more detail in Section B4.1. The activities introduce participants to—or remind them of—a number of spatial relationships, some geometrical (around the theme of the circle), some topological.

Preparation

Presenter
Read
O'Shaughnessy (1983)
Prepare the materials for the stations.

Time: 1 hour

Materials needed

Concrete materials
As specified under instructions for each station.

Reference

O'Shaughnessy, L. 1983, 'Putting God back in math', *The Mathematical Intelligencer*, Vol. 5, No, 4, pp. 76–77.

Taylor, E. & Giffard Huckstep, S. 1992, *Taking on Technology: a manual for technology education*, Women in Engineering, University of Technology, Sydney.

B2.1 Problem solving stations

Detailed procedure

In this section participants circulate around the room, engaging in the different problems posed by the six posters which have been placed around the walls, with appropriate materials nearby. Your task is to circulate also, giving support and asking questions.

Try to ensure that most participants engage in most of the activities:

1 heads and tennis balls
2 sheep in a hole
3 handcuffs
4 a twist a side
5 symmetry
6 getting square

Station 1: Heads and tennis balls

Materials

Heads and tennis balls [AS12] poster
string
scissors
3-ball tennis can
3 tennis balls

Debriefing

A lot of people make an estimate of about 6 or 7. Why?

- We haven't much practice at estimating circular distances.

The circumference is a bit more than 3 times the diameter of a ball.

- Is this just peculiar to tennis balls? What about cans … cups?

Talk about pi.

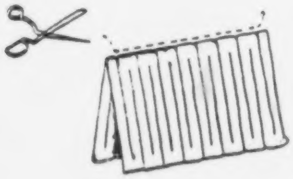
Station 2: Sheep in a hole

Materials

Sheep in a hole [AS13] poster
scissors and small pieces of paper: about half of A4 (i.e. A5)

Debriefing and development

If you're stuck try …



Go back to boundary issues:

with a given area (the paper) what is the longest perimeter to contain it?

Related and fascinating issues are *Measuring the coastline* and The *pathological snowflake* which will be investigated in Section B5.1.

Note that the perimeter can be indefinitely (infinitely) extended, but the area is limited.

Station 3: Handcuffs

Materials

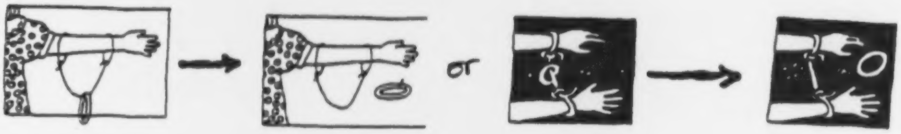
Handcuffs [AS14] poster

6 pieces of string, about 1m long, tied with loops at either end

and for a hint, a bangle

Debriefing and development

Practise this! If you are stuck, try these first:



Questions to ask as participants try it, if they are getting frustrated:

Can you try it with one loop and the bangle first?

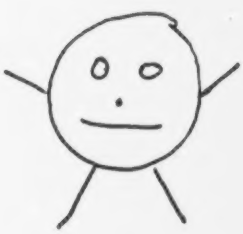
If you have two closed interlocking circles can you separate them? (no)

Do you see these as two closed interlocking circles? (probably)

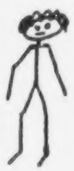
Can you see them in any other way?

What assumptions are you making? (probably that the wrist loops are tight)

This is a branch of mathematics known as topology (topo-logy = discussion of shape and form) where measurement is not important as it is in geometry (geo-metry = measurement of the earth). Also symmetry (sym-metry = same measurement), lengths, areas and the size of angles don't matter. The things that do are: open/closed curves, junctions, inside/outside, ordering. An example is children's early drawings which are more topological than geometrical in nature:



<— Legs and arms are not proportional, but *joins* are in the right place more or less, and faces are represented by *closed* curves, and eyes and mouth are *inside* that closed circle. It is only later that the more proportional drawing develops. —>



We cannot solve the handcuffs problem as long as we see the arms with the cuffs as forming two linked closed circles, since two such circles cannot be unjoined without cutting. So we then must look further, and see that they are not simply two linked circles, but loops within links...

Station 4: A twist a side

Materials

A twist a side [AS15] poster
strips of paper
pencils
scissors

Debriefing and development

Another topological puzzle, because again the properties we are interested in are not to do with measurement but to do with open/closed, inside/outside, joining.

Ask: *Is there a use for this Mobius strip?*

Some conveyor belts and some audio tapes are made as Mobius strips because the twist gives a loop with one face and one edge only and so makes possible a more even use of the whole belt or tape (Jacobs, 1982 pp. 603–608).

Ask if any participants are aware of other uses.

The Escher illustration of the ant on the Mobius strip might be interesting.

Station 5: Symmetry

Materials

Symmetry [AS16] poster
rubbish bin
hole punches
sheets of paper (about 10 cm square)

Debriefing

This activity links back to *Playing with mirrors* [Section B.1].

In a plane, shapes can have line symmetry or rotational symmetry or both. All these ones have line symmetry, obtained by folding, sometimes several times.

How many lines of symmetry does each have?

Station 6: Getting square

Materials

Getting square [AS17] poster
scissors
shapes to cut out

Debriefing and development

Use *Maths: a new beginning* [MM1–MM5] for notes about the concept and measurement of area and connect it back to work with mosaics in the previous session.

Emphasise the point that if you can link unfamiliar situations (like these shapes) to familiar ones (like the square), then you have more tools at your disposal for analysis, in this case for working out area.

Ask them to discuss, for instance, once we know how to work out the area of a rectangle, could we use this to work out the area of other shapes, e.g. triangle, parallelogram?

Use a cut out shape, or a diagram.

Start with a parallelogram and ask:

Can we cut this to make a rectangle?

So what is the area of the parallelogram?

Start with a triangle which is not right-angled and ask:

If we had two of these could we make a parallelogram?

And from that could we make a rectangle?

So, how could we work out the area of the triangle?

Start with a regular polygon. Ask:

Can we use what we know now about triangles to work out the area of the polygon?

Debriefing the stations

1 Spend a few minutes talking with participants about their strategies for working out what to do, the general problem solving strategies that they found useful, or not useful, in these activities and others like them: cooperative puzzles, the mirror investigation etc. Suggest that participants might find it helpful to collect a list of useful strategies in their journal. These might include:

- cooperate
- talk
- read carefully
- use concrete materials or diagrams
- guess, check and improve your guess
- look for patterns
- have a rest
- organise information
- model
- simplify the problem

2 Next, debrief all or a selection of the problems encountered, according to your interests and the interests of the Group.

Some suggestions for this are included with the notes on each station.

Try to include some discussion of at least the first three stations.

Here are the important ideas to get across are in this session.

- We have been taught to think about geo-metrical (measurement) properties of shapes: pi, area, angles, symmetry.
- But topological (non-metrical) properties are also interesting—order, junctions, inside–outside.

3 Finally, spend a few minutes discussing how the activities might be suitable for use with participants' own students. Ask:

What similar activities might you use with your students?

What ideas, problems and activities might engage them?

Would you modify the activities?

How?

B2.2 Investigating pi

Brief description

Participants measure pi and look at it in a historical context.

Rationale/aims

This session allows the participants to work out that there is a constant relationship between circumference and diameter of a circle, and another relationship between radius and area. They are encouraged to think about the nature of pi, and to look at how ideas about it have evolved over time.

Preparation

Presenter
Read
Putting God back in math [P4]
Prepare sets of shapes for *Squaring the circle: shapes* [AS18]—at least one set per pair.

Participants
Bring a circular object

Time: 2 hours

Materials needed

Concrete materials
Squaring the circle: shapes [AS18]: one set per pair of participants
five or six circular objects, with widely different diameters in case the objects that the participants bring do not span a wide range
books on the history of maths (including pi) including: The Old Testament, Hogben (1936), Pappas (1986; 1991), Joseph (1991)
compass and pencil
Handouts
Squaring the circle [HO13]
Putting God back in math [P4]
Pi in history [HO14]
OHTs/paper
Pi in the sky? [OHT2]

B2.2 Investigating pi

Detailed procedure

This section begins with a whole Group activity, followed by activities and discussions for small groups and a whole Group concluding report.

– **Tins and circular objects**	(30 min)
– **Squaring the circle**	(30 min)
– **Pi in history**	(1 hour)

Tins and circular objects

1 Working in pairs, using coloured paper streamers, get participants to measure around the circular objects they have brought, cutting a strip of paper to fit. Check that they know this part of the circle is called the circumference (Latin *circum-ference*: containing around).

2 Get them to measure across the object at its widest part, and cut another strip of paper to fit. Check that they know that this part of the circle is called the diameter (Greek *dia-meter*: measurement through). Ask how they would define radius (Latin *radius*: a ray).

3 Ask participants to work out how many times the small strip fits into the longer one. Get them to find an approximate answer first and then a more precise one.

4 Then get participants to glue or fix their strips on the horizontal axis of a graph you have prepared. If some strips are too long, discuss ways of coping with the information.

5 Ask participants to write their results for $\frac{\text{circumference}}{\text{diameter}}$ in a list on the board.

6 Discuss the results:
- the popularity of this number which is just a bit more than 3
- and how the tops of the strips line up (more or less) along a diagonal straight line.

If there are contrary results, try to get the participants to initiate an investigation.

Squaring the circle

1 Hand out one set of shapes accompanying *Squaring the circle* [HO13] to each pair.
Ask participants to arrange the shapes in order of area.
Discuss definitions of area and whether the order was obvious.
The triangle and/or parallelogram often seem bigger than the square, possibly because they are more spread out, have longer edges (perimeter—back to boundaries!) and the square is more compact, with shorter edges.
These pieces are a good introduction to concepts of area, but should be accompanied by discussion of the nature of length, area and volume:

- length is the answer to the question 'how far?'
- area is the answer to the question 'how much to cover?'
- volume is the answer to the question 'how much to fill?'

A connection can be made here to 'tessellating pieces' as the units of area.

2 Hand out and introduce the worksheet *Squaring the circle* [HO13].
Get participants to work in pairs while you circulate: asking questions, commenting on the 'approximately 3' that will emerge, eliciting creative ways of finding closer approximations, and encouraging participants to venture on and expand 'the detour'.

3 In Group discussion elicit the fact that the side of the small square is equal to the radius of the circle (R), so its area is RxR, or R^2. So, the area of the circle is between $2R^2$ and $4R^2$ or 'round about' $3R^2$.
Talk about this 'approximately 3' and π. Make connections with the 'just over 3' result in the previous activity.

4 The detour: With two other pieces (the larger triangles) these five pieces make the basis of the tangram, a very old Chinese puzzle. The seven pieces are put together in a large number of ways, challenging the player to unravel the designs.

Pi in history

1 Get participants to read and discuss in small groups *Putting God back in math* [P4].
In particular get them to look at the questions in *Pi in the sky?* [OHT2].

2 Discuss them in the whole Group.

3 Hand out *Pi in history* [HO14] and get the participants to choose, in groups of 2 or 3, one or two questions they would like to spend half an hour or so pursuing. Try to make sure they cover a range of questions. Make a range of resources available for immediate use.

4 In the last 20 minutes or so participants report back briefly on their investigations.
Points that may emerge are:
How could we find a more accurate measure? How, in fact, did people find pi?

- One way used by early Indian, Chinese and Greek mathematicians was to extend this method, by inscribing and circumscribing regular polygons with more and more sides, and making increasingly closer approximations.
  There is a chart of methods and estimates of pi in Joseph (1991: 190).
- The fascination with 'squaring the circle' (i.e. constructing a square whose area is exactly equal to that of a circle with a given diameter using only a straightedge and a compass) was resolved in 1882 when it was shown to be impossible.
- Beliefs about pi were different at different times, and in different contexts.

B3 Problem solving

B3.1 Problem solving ... problem posing

Brief description

Participants work together to solve a problem and analyse the tools they use to do so.

Rationale/aims

Often problem solving is seen as a gift, rather than a learnable skill. In this session participants work together to solve the well-known handshake problem, and then analyse how they have done this, in an effort to shake this belief. The analysis will provide them tools for solving problems and for convincing their students that they too can learn to solve problems.

Preparation	Materials needed
Presenter Work through and become familiar with *The handshake problem* [HO15]. **Time:** 1 hour	**Handouts** *The handshake problem* [HO15] *Problem solving: a summary* [HO16]

B3.1 Problem solving ... problem posing

Detailed procedure

The handshake problem

1 Discuss the question: *What is a problem?*

Elicit points including the idea that a situation

- is not a problem if you already know the answer or if you are not interested in finding the answer, and
- is only a problem if there is some sort of block between you and the answer, but you have some interest in solving it.

2 Pose the handshake problem and get participants in small groups to spend some time trying to solve it. Your task is to circulate amongst the groups, acknowledging strategies, and through questions and suggesting moves when the group is stuck.

Encourage simplifying the problem, modelling, acting, drawing, organising information.

3 When most groups have some answer, choose, if possible, representatives to explain the different ways they went about it. Use points from *The handshake problem* [HO15] to extend strategies that were ignored or insufficiently developed.

Not all the points need to be made, but it would be useful to include:

- the two quite different solutions, and how they generate a new result:
  'the sum of the first *n* numbers is …'
- some use of algebra.

4 Allow time for participants to tell the strategies they used at different points of the process:

- to get started, to solve it, and to conclude.

These should include:

- reading the problem carefully
- acting it out....
- drawing a diagram...
- starting with a simpler case...
- working systematically to consider all possibilities
- changing your focus; look at the problem a different way
- using a table, organise the information
- looking for patterns
- extending the example—as underlined in *The handshake problem* [HO15].

In conclusion, hand out:

The handshake problem [HO15] and *Problem solving: a summary* [HO16].

B3.2 Effects of scale and shape

Brief description

Participants use problem solving strategies to investigate effects of scale and shape.

Rationale/aims

Participants will be able to engage in a number of mathematical activities which involve modelling situations from the everyday world. In these activities relating area and/or volume to shape and scale, participants will use problem solving strategies to investigate a mathematical problem in some depth and to explore the implications of their findings for the original context.

Time: 2 hours

Materials needed

Concrete materials
centicubes
Handouts
The tale of the polyploid horse [P5]
Why do babies dehydrate faster than adults in summer? [HO17]
Why do chips fry faster than potatoes? [HO18]
Floating or drowning [HO19]

B3.2 Effects of scale and shape

Detailed procedure

This section consists of a number of activities, mostly done in pairs or small groups, linked together with Group discussion and reflection.

–	**Why do babies dehydrate faster than adults in summer?**	(1 hour)
–	**Looking at boundaries in 3-D**	(30 min)
–	**Arranging space**	(30 min)

Why do babies dehydrate faster than adults in summer?

1 Ask this question and brainstorm some answers admitting that, as it stands, it is hard to know where to begin in order to look at what is quite a complex question.

2 Suggest that the Group uses some of the problem solving strategies that emerged in the last session, namely:

model the situation, or use concrete materials, or take a simper case.

Later, point out strategies (as you come across them) like:

organising the information, or looking for patterns, or talking.

3 *Perhaps we could make a model of a baby and an adult, but people and babies are complicated shapes, so let's make a simplified version.*

Give participants some centicubes and ask them to make a baby with 3 little cubes, and then to make an adult that is double in scale.

Let them work through, *Why do babies dehydrate faster than adults in summer?* [HO17] in twos or threes. Bring them together for Group discussion from time to time.

Points that could arise

- Double in scale is not double in size, if 'size' means volume or height only.
  A scale model will be smaller or larger, but must look like the original in all other respects—it must be in proportion.
  It might be useful to have a doll or model car to support this point.
- Make sure there is plenty of talk, making the connections explicit.
  One way of doing this is to read *The tale of the polyploid horse* [P5].
- What would a 2-D version of this problem be? (sewing, house plans, maps).
  Would it be more suitable for their own students?

4 Conclude with a debriefing, naming which problem solving strategies (as listed on *Problem solving: a summary* [HO16] from Section B4.1) have been used in this activity.
- take a simpler case
- make a model
- look for patterns
- organise the information (tables etc.)
- cooperate
- talk
- read carefully
- use concrete materials

and possibly
- guess, check, improve your guess
- use diagrams
- have a rest.

Looking at boundaries in 3-D

Explain, that they have visited and revisited *Looking at boundaries* [in Section A2.1], mostly in its 1-D and 2-D form. They will visit it again, but focus more on the relationship between 2 and 3 dimensions. There will be more investigation of surfaces and volumes.

1 As a Group activity revisit Question 4 from *Looking at boundaries.*
Ask two participants to use 2 pieces of A4 paper to make two different cylinders, and ask the Group whether the cylinders are the same size. Questions about size (is it height? surface area? volume?) will arise and will need to be clarified.

2 When all participants have made a private decision about whether or not the cylinders are the same size, get two participants to test them, by filling the cylinders with a suitable material, e.g. centicubes, macaroni or beads.

3 Discuss how this is the same as the fencing question of *Looking at boundaries.*
Ask participants if they can generalise both results in a statement that covers both situations, something like:
If we are told that a shape has a given exterior measurement, it doesn't necessarily tell us what its interior measurement is.

Arranging space

Explain that the next activity, using *Why do chips fry faster than potatoes?* [HO18] gives us another way of looking at how different ways of arranging insides (here, volumes) can lead to quite different measurements for outsides (here, surfaces).

If there is time, in this activity, and the related activity, *Floating or drowning* [HO19], we will explore further the implications of the relationship between volumes and surface areas in the physical world.

1 Participants work in small groups or pairs on HO18 and, if there is time, on HO19.

2 When debriefing these activities, include this summary:
For a shape of a given volume, the more compact it is the less surface area it will have.

- Participants might like to suggest what is the most 'economical' 3-D shape, perhaps generalising from the result of the circle in 2-D.
  If so, point to generalisation as a common mathematical procedure.
- Lead some discussion of modelling as a kind of mapping (or vice versa), and point out that all models, like all maps, stress some features while ignoring others.
  This is what creates the power of the model, and also its limits.

3 Conclude with the question:
What are some of the mathematical factors that influence the shape and size of animals, plants, objects?
Allow participants to tie together the issues of scale and 'bumpiness and smoothness' with the concepts of surface area and volume.

B 4 Modelling our world

B4.1 Maps and models

Brief description

Participants meet a fractal and consider the nature of measurement, maps and models.

Rationale/aims

In this session, participants will first engage in an activity that problematises some aspects of measurement, an excursion that leads to a brief encounter with fractals. From this problem about measurement, they will proceed to an investigation of the nature of maps and models, in both the mathematical and wider use of the concepts.

Preparation

Presenter
Bring maps: a street map, a metropolitan rail map, two world maps with different projections, a globe.
Read
Peterson (1988) pp. 115–6: *The language of nature* [P6]
Why your tape measure isn't long enough for mother nature [P8]

Participants
Bring a map of Australia or their state/territory.
Read
Invariants [P9]

Time: 2 hours

Materials needed

Concrete materials
For *measuring the coastline:*
string
centicubes
maps of Australia or your state or territory— of three or four quite different scales
if undertaking the drawing of the pathological snowflake, use copies of *Isometric Grid* [AS20]
for *maps and models*
    a street map
    a metropolitan rail system map
    a globe of the world
    two world maps with different projections

Handouts
The language of nature [P6]
Rugged coastlines [P7]
Why your tape measure isn't long enough for mother nature [P8]
The pathological snowflake [HO20]
Invariants [P9]
Map of the first journey: a conducted tour [HO21]

OHTs/paper
Maps, models, theories [OHT3]

Reference

Peterson, I. 1988, *The Mathematical Tourist: snapshots of modern mathematics,* W. H. Freeman & Co, New York, pp. 115–6.

B4.1 Maps and models

Detailed procedure

This section consists of two main activities

- **Measuring the coastline** (1 h 30 min)
- **Maps and models** (30 min)

Measuring the coastline

1 Start with a story which has links with *The sheep in a hole* [station 2 in Section B3.1].

There is an old story about a woman, perhaps Queen Dido, who claimed the right to some land. She was given an ox hide and told that she could have as much land as she could enclose with the hide. Being a person of some resources, she cut the hide into many very thin strips and using the coastline on the beach as a diameter, she laid the strips end to end in a semi-circle and claimed the land inside the circumference.

2 Make links with *The sheep in a hole* [Section B3.1]:

How was what the woman did the same as what we did?

In both situations a fixed amount of exterior material (paper, ox hide) was cut thinly.

In both situations, the more thinly it was cut the longer the exterior could be.

Is there a limit in theory? (No) In practice? (Yes)

3 Make links also with *Filling space* [Section B1.2]:

The circle is the shape with most interior for a given exterior.

4 Was the woman cheating by not including the length of the beach?

This takes us to a related question. Ask:

Can we measure the coastline of Australia?

Discuss what estimate would seem sensible.

5 Talk about how the scale of a map is written and what it means e.g. 1: 24 000 000 can be more concretely understood as '1 cm represents 240 kilometres'.

You could discuss why measurement is never exact, but only precise within certain parameters—usually of a practical nature.

6 Divide the participants into small groups, each with one or two different sized maps of Australia, or their state/territory, and get them to make a more precise measuring of the length of the coastline for their map. Make sure that there is a wide range of scales.

One way of measuring might be to mark off the coastline in 1 cm steps, and then count the number of steps—or use string.

7 On the board draw up a table something like the one below:

scale of map (1 cm =.....)	**length of coastline (km)**
1100 km	
480 km	
240 km	
100 km	
25 km	
etc	

Ask each group to report back on their findings and put the figures in the table in order of the scales of their particular maps.
The aim is to see that the finer the scale then the longer the length of the coastline.
However this will obviously depend on how each group has measured the distance.

8 Gather the responses in the large Group. There will be a wide range of answers.
Is this just because some people have been more careful than others? Or is there a pattern?
By discussion elicit the conclusion that on the whole, finer scales show longer coastlines.

9 Ask:
Why do finer scales result in longer measurements?
Read *The language of nature* [P6] and *Rugged coastlines* [P7].
This is a surprising result: the coastline is in some sense infinite.
What about the area? Is it also infinite? No, it may be hard to be exact, but it has an upper limit. Draw an oval around the map.

10 Talk about fractals and the Koch curve in *Why your tape measure isn't long enough for mother nature* [P8] and *The pathological snowflake* [HO20] and if there is time use the isometric grid [AS20] to ask participants to draw the snowflake.

Maps and models

1 Ask each participant to draw a quick map of the room for use later in the session.
Give as little help as possible—it will aid the discussion if the maps are not all the same.

2 Now use two maps of a familiar city: one a page from a street directory, one a metropolitan rail map. Ask:

What is similar about these maps? What is different?

What sort of information do they give or not give?

Why do you think this is so?

Elicit in discussion the following points:

	street map	**rail map**
differences	- distances are to scale - angles are same as original - parallel and perpendicular lines are kept	- distances not to scale - angles are largely ignored - parallel and perpendicular lines are ignored
similarities	- order of streets kept - junctions/terminals kept - closed loops kept	- order of stations kept - junctions/terminals kept - closed loops kept

3 Ask: *What is it that we need to know from a rail map? From a street map?*

- Basically on a train, we need to know where to get off, get on, change trains; and the information on the map tells us exactly this: order and junctions and loops are particularly important, distance is not.
- When walking or driving around a city, we need to recognise corners, estimate distances, prepare for sharp turns, as well as know about order and junction and closed or open circuits. Measurement is crucial.
- Thus we have two different maps… one geometrical, the other topological.

Refer participants back to the *Handcuffs* [station 3, Section B3.1] discussion.

4 Ask: *Can you think of other topological maps that we use?*

Electrical wiring diagrams is one example. A possible avenue to explore would be whether some Aboriginal paintings can be seen as topological maps.

5 Point out that all maps are approximations, and particular maps stress certain features, and ignore or distort others.

Ask: *If topological maps ignore or distort length, what maps distort area?*

Topological maps distort areas, but so also do flat world maps, especially, in most, towards the poles so that Greenland for instance, often looks disproportionately large.

Ask: *What do street maps ignore or distort?*

Trees, buildings (often), but in particular, hills and valleys—contours.

6 Look back at the room maps that participants drew at the beginning of this section. Are they all the same? How do they differ? What was included? distorted? ignored?

7 Read *Invariants* [P9].

8 Conclude by discussing *Maps, models, theories* [OHT3].

B4.2 Mind maps: a review

Brief description

Participants review the journey they have undertaken in Module B.

Rationale/aims

Participants have covered a lot of ground in the journey through Module B and this session gives them a chance to review both the mathematical and pedagogical content by mapping the journey, as it has seemed to them.

Time 1 hour	**Materials needed** **Handouts** *Curricular issues* [P10] *Map of the first journey; a conducted tour* [HO21] *References for Module B* [HO22]

B4.2 Mind maps: a review

Detailed procedure

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1 Review with the Group, in some detail, what they have encountered and learnt in the five 3-hour sessions of this whole mathematical journey [Module B].
Include in the review both mathematical material and material about teaching and learning.

2 Get participants to work in twos or threes with coloured pens and butcher's paper to draw a map of the journey, combining their experiences where possible, and indicating detours for further exploration. Ask them to indicate roads that lead directly into their practice as teachers, others that lead to their own enrichment, and others that are dead ends.

3 Get the different groups to share their mind maps with the rest of the Group.

4 Ask them to reflect on what their maps (like all maps) are stressing, and what they are ignoring or distorting. Add this information to the maps if possible.

5 Ask participants to revisit the metaphors they developed for their teaching in Section A3.1 and Section A6.3. In view of what they have learnt in Module B, can they further elaborate those metaphors, or change them? Encourage participants to be clear about whether they are talking about how they see themselves now, or how they would like to see themselves. What do these metaphors stress or ignore?

6 Hand out *Map of the first journey: a conducted tour* [HO21], as the map that the guides were using. Discuss what this map stresses, ignores or distorts.
Refer participants to *References for Module B* [HO 22].

7 Hand out *Curricular issues* [P10]. Which of the topics have been visited?

Conclude by discussing the framework suggested and how it might be used with the participants' students.

B5 Curriculum project 2: Exploring mathematics

Time: 9 hours over several weeks

Brief description

Participants investigate a mathematics topic they are interested in, following a detour from the first Mathematical Journey just concluded in Module B.

Rationale/aims

'Exploring mathematics' is the second of the four Curriculum Projects in the course. It is a more substantial project than the first one, giving participants an opportunity to experience what it means to be involved in their own mathematical quest. Preferably in groups, they will be expected to investigate in some mathematical depth a topic of interest to themselves that has arisen out of the activities of the first Mathematical Journey. This kind of experience is necessary if they are to encourage similar curiosity and investigation in their own students. The Curriculum Project will also involve participants in developing, from their investigation, one or two activities suitable for use with their own students.

Preparation

Presenter
Read
Boomer (1992), Brown (1984) and Lerman (1989)
Bring a large number of suggested topics and reference books—see *Map of the first journey: a conducted tour.*

Participants
Spend some time reading the following and thinking about possible detours...
Curriculum project 2: Exploring mathematics [CP2–1] [CP2–2] [CP2–3] and [CP2–4]

Materials needed

Handouts
Curriculum project 2: Exploring mathematics [CP2–1]
Notes on negotiation [CP2–2]
A negotiated contract [CP2–3]
Contract form [CP2–4] all from the **Curriculum Projects and Numeracy Journal** section.

References

Boomer, G. 1992, 'Negotiating the curriculum reformulated', in G. Boomer, N. Lester, C. Onore & J. Cook (eds), *Negotiating the curriculum: educating for the 21st century*, Falmer Press, London.

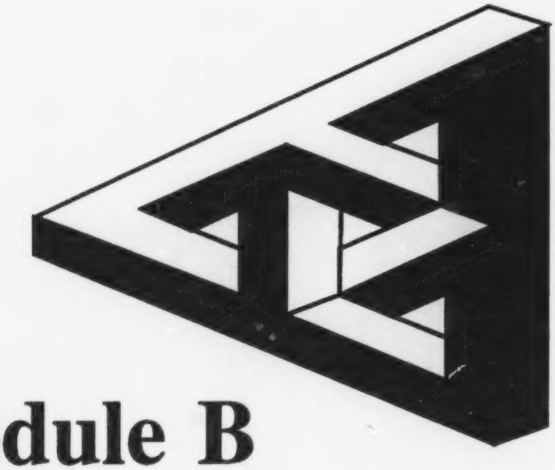
Brown, S. I . 1984, 'The logic of problem generation: from morality and solving to deposing and rebellion', *For the Learning of Mathematics*, Vol. 4, No.1, pp. 9–20.

Lerman, S. 1989, 'Investigations: where to now?' in P. Ernest, (ed.) *Mathematics Teaching: the state of the art*, The Falmer Press, East Sussex.

Procedure

This project should be timed to start towards the end of the first Mathematical Journey (Module B), and should extend over at least three weeks. The first quite difficult task is to negotiate contracts with the participants.

Detailed instructions for this Curriculum Project will be found in the **Curriculum Projects and Numeracy Journal** section.



Module B

Mathematics as a human construction

Resources

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AS = Activity sheets to be used and returned to presenter ✥
HO = Materials for participants to use and retain ✋
OHT = Overhead projector transparencies ▤

List of readings

Make copies of the following readings and distribute them before the course begins, at the beginning of Module B or when indicated in Module B Presenter's Notes.

P1 Using symmetry 153

P2 The circle and the line 155

P3 What's so great about regular shapes? 157

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All these readings are supplied in Resources B, on the pages indicated.

Naming Geometric Shapes

HO1
page 1

Adapted from unpublished work produced by Beth Marr and Sue Helme for the Teaching Mathematics to Women Project, Northern Metropolitan College of TAFE, Victoria.

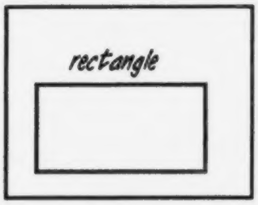
Skills developed
- identification of 2D geometric shapes and
- classification of 2D geometric shapes

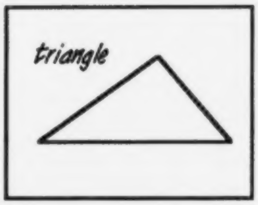
Materials
- dictionary
- glue
- textas: 1 for each group of 4 students
- 1 large sheet of paper for each group
- Worksheet 1
- at least ten 5 cm x 8 cm pieces of coloured card or paper for each person

Aims
This session enables students to tell you and each other what they know about the names of geometric figures. It also encourages them to compare and discuss the geometric properties of these shapes by making a classifying chart.

Session outline
Give out the 10 pieces of paper or card (5 cm x 8 cm) to each student.

Get each student to write the names of all plane (i.e. flat or 2-D) geometric shapes known (one on each card) and to make a rough sketch if possible, e.g.





Divide the students into groups of 3 or 4 and ask members to pool their cards.

Give each group a large sheet of paper, glue and a texta and ask them to make some sort of sensible chart of their shapes, discarding duplicates.

Circulate amongst the groups offering suggestions or answering questions. Answer specific questions such as:
- *What is a pentagon?*
- *What is this kind of triangle called?*

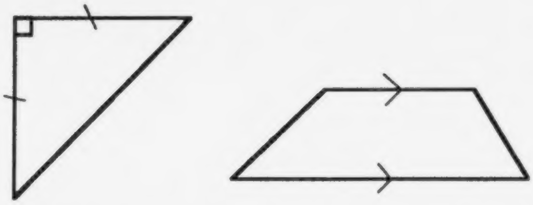
By the end of 15 minutes you will probably have answered many questions and listened to many lively arguments, and the students will have clarified the meanings of words like polygon, quadrilateral, kite and trapezium.

All that remains is for you to bring all students together to compare charts and look at the differences.

If general mathematical terms haven't come up then ask specific questions, e.g.
What do we call all straight-sided figures? (polygons)

Discuss with students the naming of commonly known polygons such as pentagon, hexagon and octagon. If you wish, extend this to less well known polygons: heptagon—7 sides, nonagon—9, decagon—10, dodecagon—12.
What about all 4-sided figures?
Does anyone know a 'family name' for them? (quadrilaterals)

Also discuss geometric conventions for showing right angles, equal lengths, equal angles and parallel lines on diagrams, e.g.

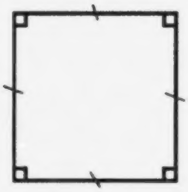


Finally put all the charts on the wall for the next part of the exercise.

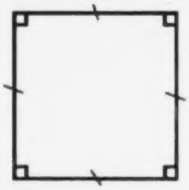
You will probably need to point out that some figures are just special cases of others, e.g.

- a rectangle is a special parallelogram
- a square is a special rectangle
- a square is also a parallelogram.

One way to do this: hold up (or draw) specific shapes, giving all necessary facts using standard geometric conventions, e.g.



Ask students to write down as many names for the figures as they can, e.g.



This is a square
but it is also a
- polygon
- rectangle
- parallelogram
- rhombus
- quadrilateral.

After each one, list on the board the names given and discuss whether they are true or not.

After a few of these they will have a good idea what you mean and will have gained practice at using the words confidently.

If you want to explain why it is important to classify shapes, then discuss their geometric properties, e.g.

> *if we know, say, that for all parallelograms the diagonals bisect each other then we will know this is true for a rectangle, a square and a rhombus, without having to find out all over again for each case.*

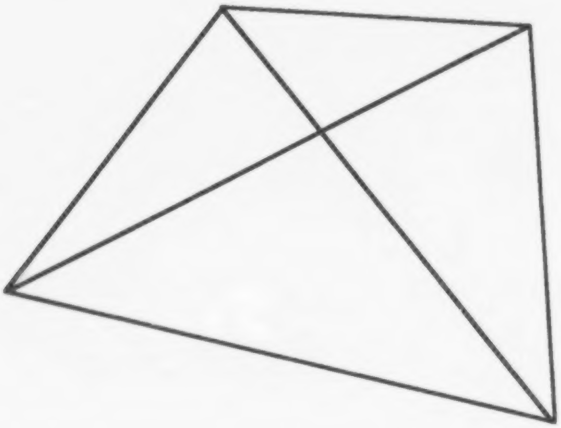
If the students have done some work on Venn diagrams see if they can make a diagrammatic representation of the classification of the polygons which you have discussed.

Worksheet 1 asks students to explore the relationship between the number of sides and the number of diagonals in polygons.

Questions at the end of Worksheet 1 should be considered as research questions.

Worksheet 1

Look at the quadrilateral below. All of its DIAGONALS have been drawn for you. As you can see, there are only TWO diagonals for a quadrilateral.



Draw your own pentagon, hexagon, heptagon etc., and draw in their diagonals.

By counting the diagonals for each figure, fill in the table below:

number of sides	*number of diagonals*
3	
4	2
5	
6	
7	
8	20
9	
10	

Did you see a pattern?

Can you predict how many diagonals there are for a 15-sided figure?

… or how many diagonals there would be for an 18-sided figure?

Names

HO2

poly–gon = many angles
poly–hedron = many faces

from Greek roots (except for *nona* which is Latin based)

tri	tetra	penta	hexa	hepta	octa	nona	deca	dodeca	icosa
3	4	5	6	7	8	9	10	12	20

- So why isn't a triangle called a trigon?
- Or a quadrilateral (quadri = 4, lateral = sides) called a tetragon?
- Somehow at this point the Latin influence won out.
  After this rather muddled beginning, mathematicians must have got into the act, and things became more ordered.
- And why isn't a cube called a hexahedron?

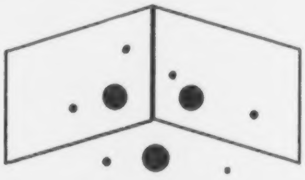
  It can be, but its part in ordinary life ensures that it has also a common name.

  Does it have any other names?

Experimenting with mirrors

HO3
page 1

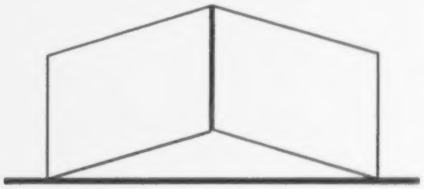
To explore the operation of a simple kaleidoscope you just need 2 rectangular pocket mirrors hinged with adhesive tape and some tiny coloured objects—bits of plastic or glass will do very well. Place the objects on the table between the standing mirrors. This creates a simple kaleidoscope.



If you look in the mirrors you will see the objects repeated in a delightful pattern. A little experimentation will show that some angles produce lovelier patterns than others. Only certain angles produce, in the words of the inventor of kaleidoscopes, Sir David Brewster, 'a perfect whole—a finite number of identical regions arranged in a circular pattern.' (Senechal 1990, p. 151)

How many different special positions can you find that give this perfect whole?

Now draw a dark line on a clean sheet of paper and place the tips of the joined mirrors on the line.



- Keeping the edges of the kaleidoscope on the line, can you open or close the mirrors so that the reflection looks like a square?
- Can you make the reflection into a triangle? pentagon? hexagon? octagon?
- To get more sides for your polygon what are you doing to the angle at the centre? what is happening to the angle at the edge?
- What is the angle at the centre when the reflection is a square? Use a protractor.

HO3
page 2

Fill in the results in the chart below, *except for the last line.*

regular polygon	*no of sides*	*centre angle*
triangle		
square		
pentagon		
hexagon		
octagon		
decagon		

- Can you find a pattern connecting the number of sides and the size of the centre angle of the polygon?
- Using your pattern, predict what the centre angle will be for the decagon.
- Checking it with the mirrors, does your pattern work?
- Can you predict what it will have to be in order to make a dodecagon (12 sides)?

Write your prediction in the last row now.

Using symmetry

P 1
page 1

From Senechal 1990, pp. 153–155.

If all we learn about symmetry is to identify it, we miss the whole point. Symmetry is an effect, not a cause. Why are so many natural structures symmetrical? For example, what atomic forces ensure that the arrangements in crystals will be orderly? Although these are profound and largely unsolved problems, a good working answer was given over thirty years ago by James Watson and Francis Crick in describing their discovery of the structure of DNA:

> Wherever, on the molecular level, a structure of a definite size and shape has to be built up from smaller units ... the packing arrangements are likely to be repeated again and again and hence sub-units are likely to be related by symmetry elements.

In other words, nature builds modular structures that organise themselves according to certain rules. Repetition of the rules tends to lead to arrangements of modules that we call symmetrical. Polyhedra provide a wealth of excellent examples of arrangements that are repeated again and again. When you build a cube with cardboard squares by attaching three squares to each corner, you are constructing a shape that satisfies a certain packing arrangement: it must be made of congruent regular polygons, and it must have the same number at each corner. By generalising this construction to other polygons, we obtain the five regular polyhedra. The arrangement can be further generalised to include the semi-regular polyhedra (Figure 15), in which more than one kind of regular polygon can be used, and the convex deltahedra (Figure 16), all of whose faces are equilateral triangles but whose vertex arrangements need not all be the same.

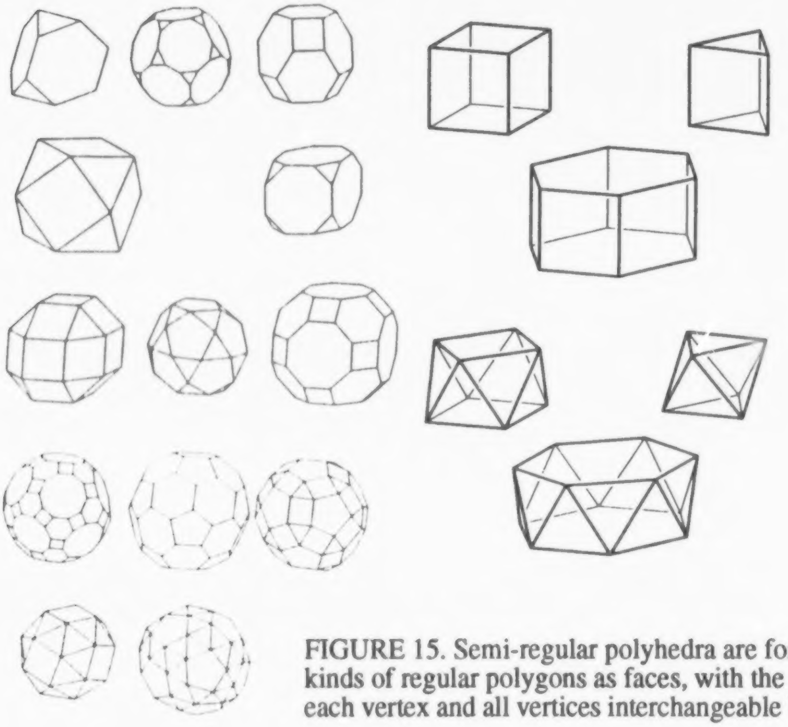


FIGURE 15. Semi-regular polyhedra are formed by using several kinds of regular polygons as faces, with the same arrangement at each vertex and all vertices interchangeable by symmetry operations.

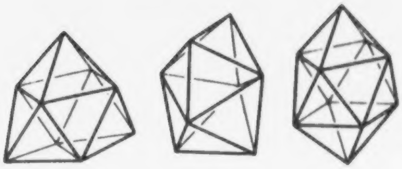


FIGURE 16. Convex deltahedra are formed from equilateral triangles arranged with differing types of vertex arrangements: three, four, or five triangles may be joined at a vertex.

The cover design for the biological journal *Virology* contains an icosahedron. The story of the discovery of icosahedral symmetry in viruses and the ongoing efforts of scientists to link that symmetry to their sub-unit structures is very instructive. Viruses are tiny capsules that contain an infective agent. The capsule is composed of protein sub-units that group together to form a closed shell. Watson and Crick realised, in the course of early X-ray investigations into virus structure, that the shells of many viruses had polyhedral or helical forms. Subsequent studies showed that the polyhedra were often icosahedra, and this suggested more recently these models have been found to be incorrect. The connection between packing arrangements and overall symmetry in viruses remains an unsolved problem. Problems such as these lead also to new developments in mathematics: they force mathematicians to rethink their definitions and the broaden the scope of their investigations.

The circle and the line

P2
page 1

From Marcia Ascher 1991, *Ethnomathematics: a multicultural view of mathematical ideas*, © Brooks/Cole Publishing Co., California, p. 123-125.

If you look around the room in which you are sitting, you will probably see many straight lines, flat surfaces, and right angles. Look, for example, at a corner where the floor and walls meet, at a door frame, at a window and its frame, or at your desk. If there are any wall decorations, they too probably contain straight lines and are in rectangular frames with the bottom edges set parallel to the floor. We define the space around us by both physically and mentally imposing order upon it. More and more in our Euro-American culture, as exemplified by the enlarging and changing cityscape in addition to interior design, that order is made up of lines, rectangles, planes, and rectangular solids. To most of us these forms are necessary, sensible, and, above all, proper.

For us, what could be more natural than looking up at the sky, spotting particular stars, mentally connecting the star-points with straight line segments, and creating constellations that are seen by generation after generation? It could, of course, be otherwise. In keeping with their spatial ideas, native Andean peoples see other constellations far more irregularly shaped, made up of darker and lighter blotches (clouds of interstellar dust) in the sky. Or contrast the two [following] statements; one is a statement about the line made by two American professors of mathematics, and the other is a statement about the circle made by Black Elk, an Oglala Sioux.

While they differ on the geometric form, the writers share their degree of conviction in the rightness of their ideas and support their view with nature, God, achievement of goals, and proper human development. Taken separately or together, the statements highlight the fact that geometric ideas are an integral part of culture's world view. Western cosmological ideas have, of necessity, influenced the course of Western mathematics, but mathematics, in turn, reinforced those ideas through art, architecture, measuring and mapping schemes, ways of seeing and describing, and even our aesthetic sense.

The Line

Reprinted from *The Mathematical Experience*, by Philip J. Davis & Reuben Hersh, by permission of Birkhäuser Boston Inc. Copyright 1981.

… in every human culture that we will ever discover, it is important to go from one place to another, to fetch water or dig roots. Thus human beings were forced to discover—not once, but over and over again, in each new human life—the concept of the straight line, the shortest path from here to there, the activity of going directly towards something.

In raw nature, untouched by human activity, one sees straight lines in primitive form. The blades of grass or stalks of corn stand erect, the rock falls down straight, objects along a common line of sight are located rectilinearly. But nearly all the straight lines we see around us are human artefacts put there by human labor. The ceiling meets the wall in a straight line, the doors and windowpanes and tabletops are all bounded by straight lines. Out the window one sees rooftops whose gables and corners meet in straight lines, whose shingles are layered in rows and rows, all straight.

continued over the page

The world, so it would seem, has compelled us to create the straight lines so as to optimize our activity, not only by the problem of getting from here to there as quickly and easily as possible but by other problems as well. For example, when one goes to build a house of adobe blocks, one finds quickly enough that if they are to fit together nicely, their sides must be straight. Thus the idea of a straight line is intuitively rooted in the kinesthetic and the visual imaginations. We feel in our muscles what it is to go straight toward our goal, we can see without eyes whether someone else is going straight. The interplay of these two sense intuitions gives the notion of straight line a solidity that enables us to handle it mentally as if it were a real physical object that we handle by hand.

By the time a child has grown up to become a philosopher, the concept of a straight line has become so intrinsic and fundamental a part of his thinking that he may imagine it as an Eternal Form, part of the Heavenly Host of Ideas which he recalls from before birth. Or, if his name be not Plato but Aristotle, he imagines that the straight line is an aspect of Nature, an abstraction of a common quality he has observed in the world of physical objects.

The Circle

Reprinted from *Black Elk Speaks* by John G. Neihardt, by permission of University of Nebraska Press.
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… I am now between Wounded Knee Creek and Grass Creek. Others came too, and we made these little gray houses of logs that you see, and they are square. It is a bad way to live, for there can be no power in a square.

You have noticed that everything an Indian does is in a circle, and that is because the Power of the World always works in circles, and everything tries to be round. In the old days when we were a strong and happy people, all our power came to us from the sacred hoop of the nation, and so long as the hoop was unbroken, the people flourished. The flowering tree was the living center of the hoop, and the circle of the four quarters nourished it. The east gave peace and light, the south gave warmth, the west gave rain, and the north with its cold and mighty wind gave strength and endurance. This knowledge came to us from the outer world with our religion. Everything the Power of the World does is done in a circle. The sky is round, and I have heard that the earth is round like a ball, and so are all the stars. The wind, in its greatest power, whirls. Birds make their nests in circles, for theirs is the same religion as ours. The sun comes forth and goes down again in a circle. The moon does the same, and both are round. Even the seasons form a great circle in their changing, and always come back again to where they were. The life of a man is a circle from childhood to childhood, and so it is in everything where power moves. Our tepees were round like the nests of birds, and these were always set in a circle, the nation's hoop a nest of many nests, where the Great Spirit meant for us to hatch our children.

But the Waischus (whitemen) have put us in these square boxes. Our power is gone and we are dying, for the power is not in us any more. You can look at our boys and see how it is with us. When we were living by the power of the circle in the way we should, boys were men at twelve or thirteen years of age. But now it takes them very much longer to mature.

Well, it is as it is. We are prisoners of war while we are waiting here. But there is another world.

What's so great about regular shapes?

From Denny 1986, pp. 129–165 (extracts), © University of Texas Press.

The distinctive thing about hunting economies is that one gains a living from wild plants and wild animals, in contrast to the domesticated plants and animals of agricultural economies. Furthermore, only human energy is used, not that of large domesticated animals or man-made engines as in agricultural and industrial societies. The dependence of the hunter upon wild plants and animals leads to two crucial features in his pattern of living. First of all, he only alters the environment to a small degree and must for the most part adapt to its natural conditions, in contrast to this, agricultural and industrial societies alter the environment to increasing degrees and strive hard to make the environment fit their needs. The second feature arises as a consequence of the first. Since the technology needed for a small degree of alteration of the environment is itself restricted, any adult knows the whole repertoire. Consequently, there need be no specialisation of occupation—anyone can kill an animal, butcher it, and cook it; anyone can cut wood and bark from trees, shape them into canoe, and paddle it. Because tasks are not shared among specialists any can support himself by his own efforts without reliance on anyone else, although co-operation with others will normally increase success. In contrast, as the degree of alteration of the environment grows in agricultural societies, the range of skills multiplies, and tasks must be divided among specialists with consequent dependence on others in gaining one's living. We will see many points at which the hunter's mathematical concepts are affected by these two features: first, adapting to a little-altered natural environment, and, second, performing all tasks oneself independently of other people. In general, we will note that because of these features the hunter needs only a small amount of mathematics, but that as the features change to their opposites in agricultural and industrial societies the need for mathematics grows. These opposites, a high degree of alteration of the environment and division of work among specialists, require mathematical thinking. In this vein, I will try to show that mathematical thought is not inevitable or innate in human beings, but arises from specific conditions in recent human history …

SHAPE AS THE PRECURSOR OF GEOMETRY

We are often told that geometry, as its name suggests, emerged from the need to measure plots of ground. Certainly such needs are not found among hunters, since ownership of such needs are not found among hunting territories is shared with other members of the band, without imposing man-made boundaries. However, there are other impulses toward recognising fundamental geometric forms such as circle and triangle. An important one is having a sufficient degree of job specialisation so that one person may design an object and another one build it—it is much easier for the designer to control the builder's actions if the design involves regular geometrical properties. This situation does not arise among hunters who always design and build their own artefacts. Consequently, complex and irregular shapes can be accommodated since there is never a need to communicate them from one person to another. The shapes of a canoe or an igloo are extremely sophisticated but do not have to be analysed in terms of geometrical properties—perceptual judgements of length and degree of curvature allow the designer-builder to control his own work. Another aspect of occupational specialisation in complex societies which calls for regular geometrical shapes is the making of parts by one worker for assembly by another. A circular hole and a cylindrical rod will fit together without difficulty even if the worker has never seen the maker of either hole or rod. However, for a hunter doing his own building and assembly an irregular-shaped rod can be fitted just as easily to an irregular-shaped hole.

If hunters do not need geometrical concepts what sort of shape concepts do they have? The answer is shape categories, exactly analogous with other basic descriptive categories such as red, hard, sour, and smooth. Psychological study of categories has shown that each one covers a range of variation, that understanding of the category frequently involves knowing its central or proto-typical members, and that the boundaries between categories are vague and require judgment in individual cases. This means that a shape category such as 'round' covers varying kinds of roundness from the circular penny to the slightly flattened tomato, the vertically elongated apple, and the quite irregular potato—all fall into the category 'round'. The circle is one possible proto-typical or central member of the category 'round'; also, the boundary line between 'round' and an adjacent category such as 'angled' cannot be fully specified but has to be judged in particular situations ...

So far, we have emphasised the role of the second ecological factor discussed in this paper, lack of occupational specialisation since each producer is both designer and builder in a hunting economy, in the use of shape categories rather than geometrical figures. However, the first factor, subsistence, derived from a relatively unaltered environment, also encourages shape categories. The shapes of the natural world are both irregular and highly variable, so that they cannot be efficiently grasped in geometrical terms. However, shape categories such as 'round and elongated' expressed by Ojibway noonim- cover a wide range of natural objects such as leaves, lakes, and fish, as well as man-made objects such as canoes and wigwams. It is only the man-made world of complex societies in which geometrical shapes are common—and they are there so that designers can accurately control the work of builders, and so that parts made by one specialist can be used by another.

The room in which I write contains almost entirely rectangular shapes so that the notion of a 90° angle captures much of what I see. To fully understand how shape categories, in their turn, capture the range of variation in shapes of natural objects, it is important to emphasise their, abstractness. Rounded figures are divided, as we have seen into more and less elongated ones. The more elongated ones designated by noonim- include all degrees of elongation beyond the vague boundary line with the less elongated ones, including infinite elongation. the infinitely elongated round figures are a sub-category which might be called categorically (not geometrically) cylindrical figures. To see this we can imagine a finitely elongated round object such a s an egg, and recognise that if it were infinitely elongated the result would be the curved body and straight sides of a cylindrical object such as a stick; in fact, the curvature of the body of a stick or log is described in Ojibway with noonim-. Although the curve of the body of a stick is infinitely elongated, the stick is of course finite, having two ends. the shape of the ends will usually not be elongated, i.e. they will be roughly circular, and will be described using waawiye-. These results show not only the abstractness of the shape categories but their intelligent use in analysing compound shapes.

Having introduced ourselves to shape categories in contrast to geometrical figures, let us now look at the whole system of Ojibway categories. First of all, curved shapes are emphasised at the expense of angular ones, since curves are what abound in the natural world of rivers, stones, sticks, fruits, and hills.

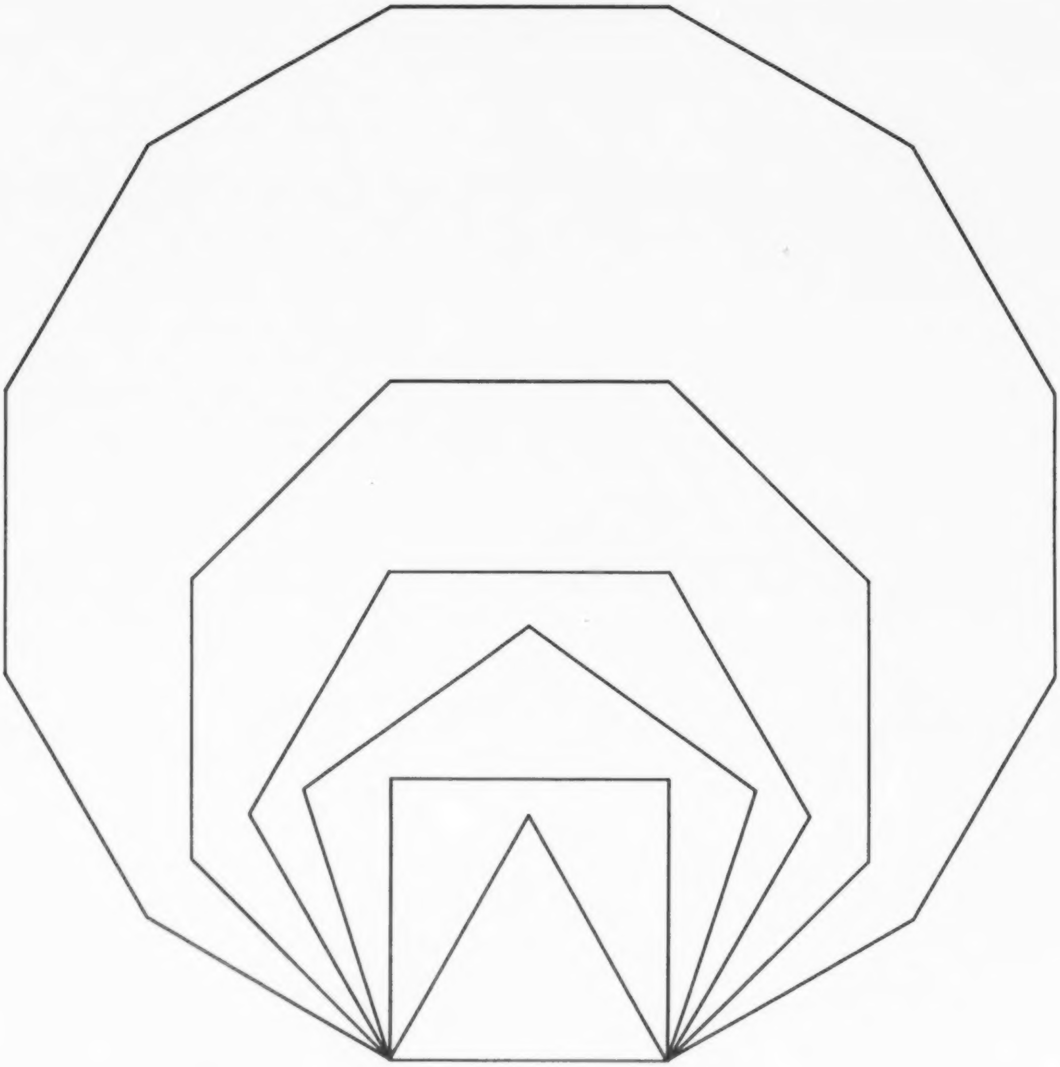
Shaping our worlds

OHT1

[both] writers share their degree of
conviction in the rightness of their ideas
and support their view
with nature, God, achievement of goals,
and proper human development.
Taken separately or together,
the statements highlight the fact that
geometric ideas are an integral part of
a culture's world view.

Ascher 1991, p. 125

Regular polygons ✣ AS1



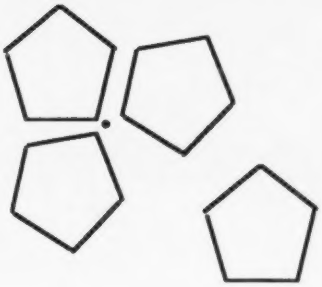
Photocopy onto coloured card if possible.
Cut out sets of each of the six polygons

Why do bees make hexagonal cells?

HO4

page 1

Using your set of regular polygons, work out which ones will surround a point exactly. To do this, draw a point in the middle of a piece of paper. For instance, take three pentagons, and place them around the point ... do they fit exactly? are there gaps or overlaps? will 2 fit? or 4?



Will an exact number of triangles fit around the point?
Try all the regular polygons in your set.
Record your results below.

which regular shape?	***do they fit around a point?***	***if so, how many?***	
triangle			
square			
pentagon			
hexagon			
octagon			
dodecagon			

So, ... , ... and ....

fit together or tessellate,

but .... , ... and .... don't.

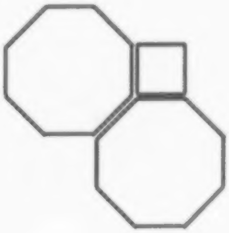
But can we be sure that no other polygons will fit together?

One way of checking is to look at what is happening to the angle of the polygon as the number of sides increases. When you were *Experimenting with mirrors* you found that the more sides there were, the bigger the angle at the edge.
How big?

✋ HO4
page 2

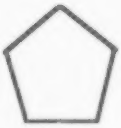
Let's go back to the earlier table, and write the word 'angle' at the top of the fourth column. For the shapes that tessellate, this column is easy to fill in, if you remember that the angles surrounding a point must add up to 360° degrees. Fill in the table for those ones. Then we'll look at the others.

Now surround a point with two octagons and a square. You know the angle for a square. Use arithmetic to find out what it will be for an octagon.



Can you find out the angle of the dodecagon in a similar way?

So, all we have left is the pentagon.
Will it fit with anything? Can you make an estimate?



If you want to check, you could think of the pentagon as being made up of three triangles ...

- *what is the sum of angles in one triangle?*
- *what is the sum of the angles in all three triangles put together?*
- *there are five equal angles here, so each one must be ...?*

Could you use a similar method to check the octagon angle, or to find out the angle of a decagon?

Look at the table. Have we confirmed 'the more the sides, the bigger the angle'?

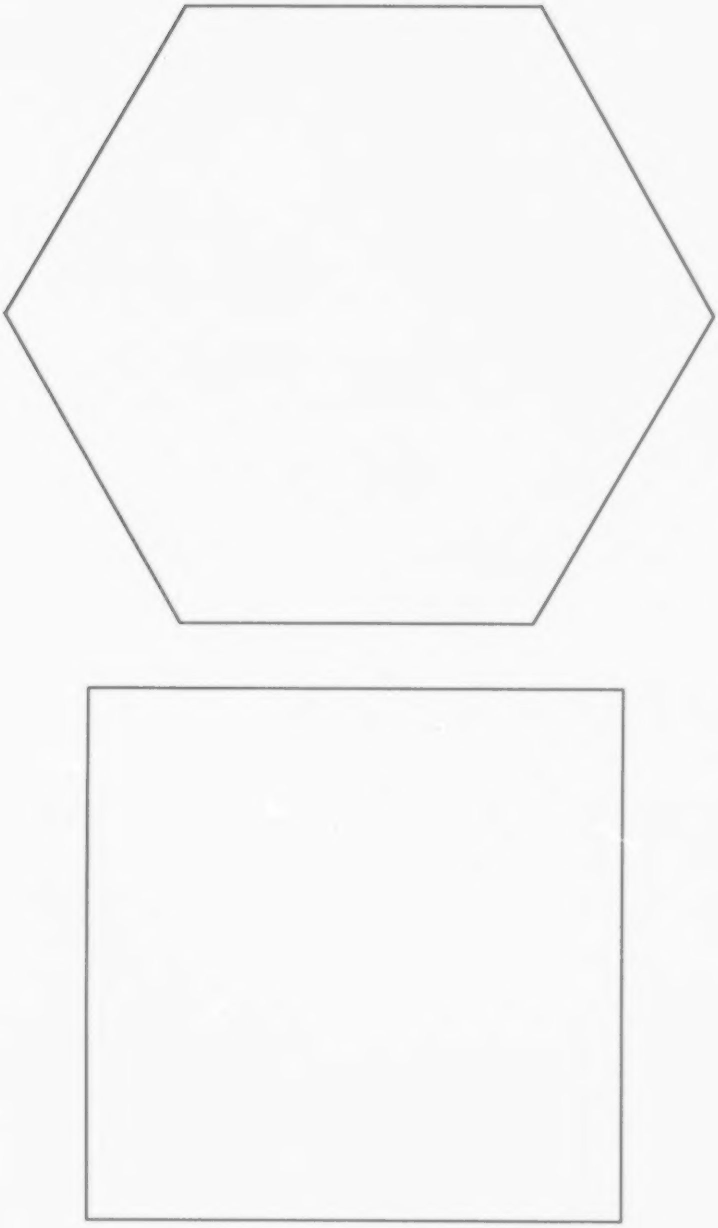
So, to surround a point with regular polygons, you need:
six triangles with angles of 60°
or *five* polygons with angles of 72° (but there aren't any such things)
or *four* squares with angles of 90°
or *three* hexagons with angles of 120°
or *two* polygons with angles of 180°

Why is this last line impossible?

So, there are only three regular polygons that will tessellate. So, no, bees won't use pentagons or octagons—or circles either—because they won't tessellate.
But why don't they use triangles or squares? Any ideas?

Hexagon etc.

✣ AS2
page 1



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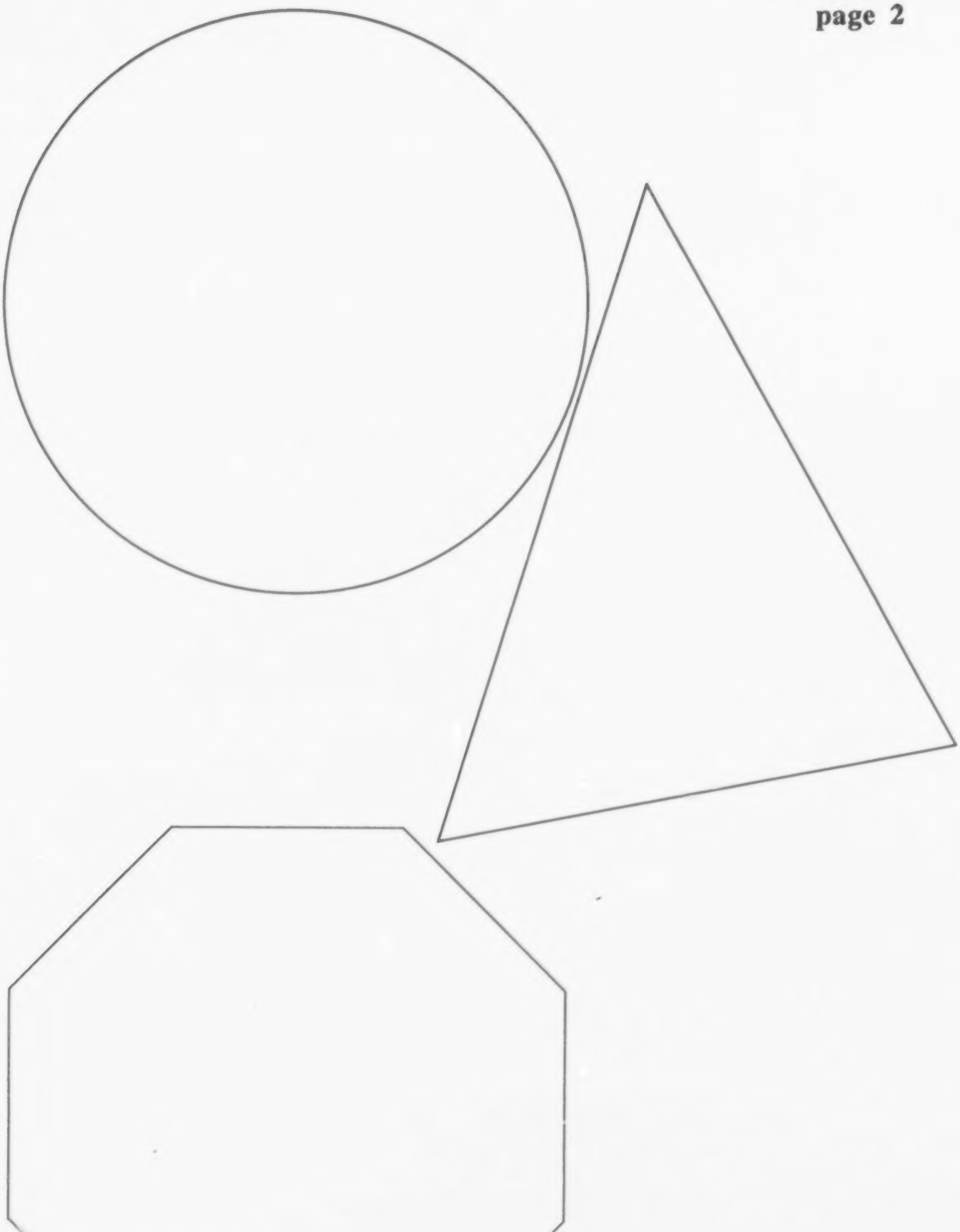
page 2



Centimetre grid paper ✣ AS3

Seminole: a patchwork technique

HO5
page 1

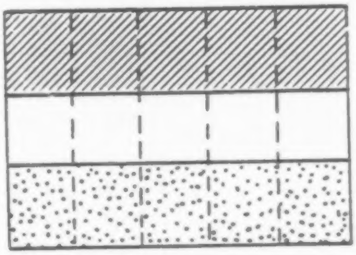
The Seminole Indians(USA) are credited with this form of patchwork although many other cultures have used it for hundreds of years. It involves joining strips of different material together, cutting it at precise intervals then rejoining it in a staggered pattern. Many different effects can be achieved in seminole work.

Today we will make three samples of seminole, time allowing.

Sampler 1.

1. Take 3 different strips of 5 X 30 cm paper. (In sewing you would have to allow 1/4" or 6mm for the seam on each long edge.)
2. Join the strips as shown in Fig. 1.

Fig 1



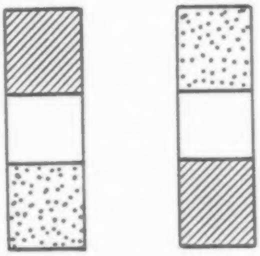


Fig 2

3. Cut the strips vertically every 5 cm as shown in Fig. 1 (In sewing you would have to allow a 1/4" or 6mm seam.)
4. Place the strips as shown in Fig. 2 + 3

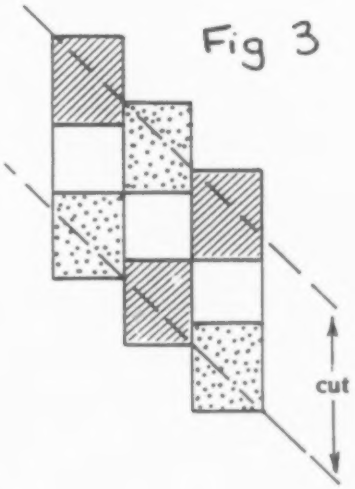
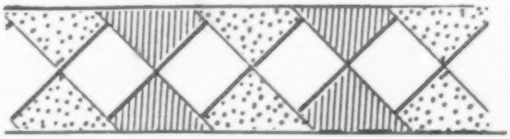


Fig 3

5. Use sticky tape to join them together.
6. Draw a line as shown in Fig. 3
7. Cut along this line.
8. The result is Fig. 4
9. Congratulations!

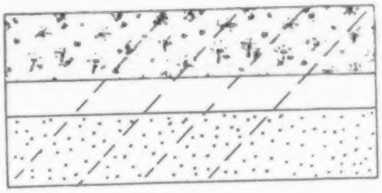
Fig 4



Sampler 2.

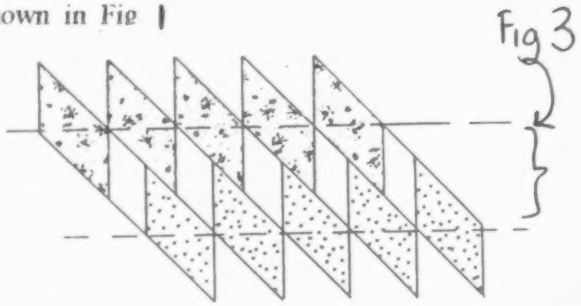
1. Take 2 different strips of paper 5cm X 30cm and one strip of paper 2cm X 30cm.
2. Join the strips as shown in Fig. 1.

Fig 1



3. Cut the strips vertically every 5cm as shown in Fig 1
4. Place the strips as shown in Fig. 2

Fig 2



5. Use sticky tape to join together.
6. Draw a line as shown in Fig. 2
7. Cut along the line.
8. The result is Fig. 3 It is quite different to the effect in Sampler 1.

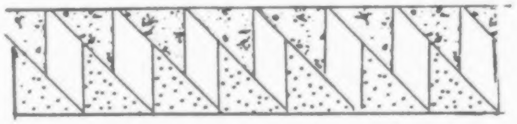


Fig 3

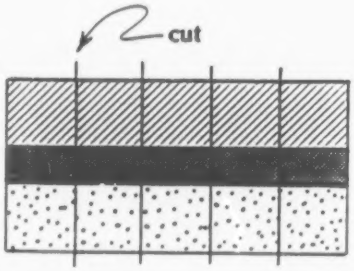
HO5
page 3

Sampler 3.

1. Take 2 different strips of paper 5cm X 30cm and another different strip 2cm X 30cm.

2. Join the strips as shown in Fig. 1.

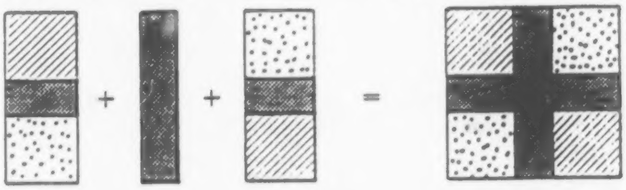
Fig. 1



3. Cut the strips vertically every 5cm as shown in Fig.

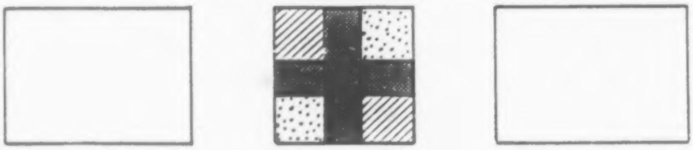
4. Now take another 2cm X 30cm strip of the centre paper and join as shown in Fig. 2

Fig. 2



5. Join a rectangle 12cm X 12cm to between each seminole patch as shown in Fig. 3.

Fig. 3



6. Draw lines as shown in Fig. 4.

7. Join together as shown in Fig. 5.

8. You could now get a contrasting piece of paper and put a border along each side. (Fig. 5)

Fig 4

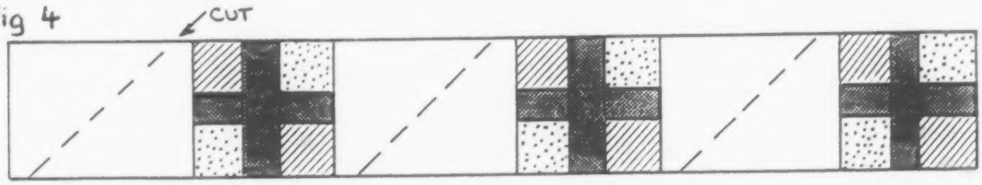
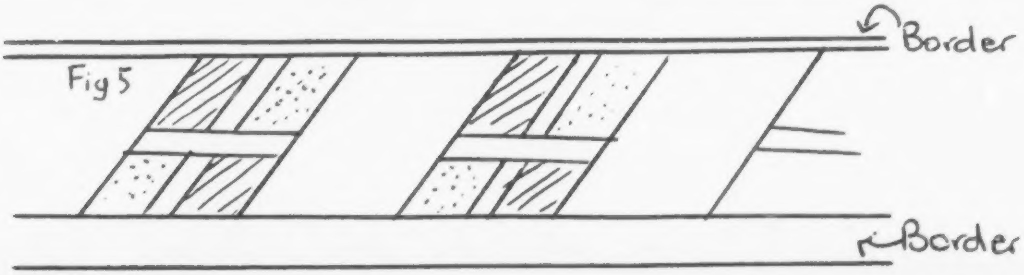


Fig 5



Excavating patchwork

✋ HO6

What is patchwork?

> It is a work made up of pieces of cloth or leather of various colours or shapes sewn together. The resulting articles can be used as a clothing, quilts, covers for cushions and for decoration.
>
> Patchwork probably dates back as far as clothing itself. The first patches were no doubt applied as a way to extend the life of a worn garment. Unfortunately, little evidence of the origins of patchwork and quilting (top stitching used in conjunction with patchwork) has survived because the textiles themselves were perishable.
>
> Few quilted and applique designs date further beyond the 17th and 18th Centuries when pieced work seemed to have become widespread. This was when cotton printed fabrics from India became available. For the early American settlers in particular fabric was extremely precious because English laws prevented the colonists from manufacturing their own or buying any cloth from anywhere but England. Because the fabric was so precious, early designs were based on simple shapes which wasted little fabric. Few of these early quilts have survived. They were made for use and were used until they fell apart; then they became filling for new quilts.
>
> Patchwork quilting flourished between 1778 and 1830. Then it became a poor relation to other forms of needlework. Then from 1890 to 1940 there was an upsurge of interest. The decline of the 50's and 60's was followed with increased interest in the 70's as a leisure activity which survives to today.
>
> Certain cultures have very distinctive styles such as the Japanese with less symmetry and the Amish with their natural dyes and extraordinary awareness of wastage a prime consideration.

A patchwork excavation

In this activity we are using patchwork as an example of a real world resource for understanding and teaching mathematics. Most of the materials involved were collected in work done by two participants, Jynell Mair and Sue Turner, in the ANT pilot course held in Sydney in 1994. AS8 and AS9 are based on work done by Trish Dunn in a student assignment at the University of Technology, Sydney. We are grateful to Jynell, Sue and Trish for their permission to reproduce these materials.

Teasing out what maths could be involved is an important first step. An even more important step is to go beyond a simple recognition of the presence of mathematical concepts or techniques ('look, there are parallel lines here') to a questioning of that presence ('why parallel lines?' and 'how were they drawn?') i.e. to go from 'what maths?' to 'why maths?'

1. Share out the resources in the envelope, and any other information or materials about patchwork you may have brought.
2. Read and discuss the information.
3. What mathematical concepts, understanding or techniques are involved in this information? List them.
4. Focus on one or two areas, and think about such questions as:
   - What maths is involved explicitly or implicitly?
   - How is the maths involved?
   - How could you explore those mathematical concepts, understandings or techniques more fully?
   - Is there other maths that might be relevant?
   - Do the patchwork makers need to know and/or understand this maths?
   - Would it help them to know and understand it? Why?

Helpful formulas ✥ AS4

Helpful formulas for determining the numbers of blocks, sashes, setting squares and alternative blocks required

Number of blocks needed

Straight set, side by side or with sashes
$L \times W$ = no. of blocks

Straight set, alternate blocks
$(L \times W) \div 2$ (rounded up to nearest whole number) = no. of blocks

Diagonal set, side by side or with sashes
$(L \times W) + [(L - 1) \times (W - 1)]$ = no. of blocks

Diagonal set, alternate blocks
$L \times W$ = no. of blocks

Number of alternate plain blocks needed

Straight set
$(L \times W) \div 2$ (rounded down to nearest whole number) = no. of alternate plain blocks

Diagonal set
$(L - 1) \times (W - 1)$ = no. of alternate plain blocks

Number of setting squares needed

Straight set
$(L + 1) \times (W + 1)$ = no. of setting squares

Diagonal set
$[L \times (W + 1)] + [W \times (L + 1)]$ = no. of setting squares

Number of sashes needed

All sets
(no. of blocks + no. of setting squares) – 1= no. of sashes

For all these formulas:
- L equals the number of blocks per row down the length of the quilt.
- W equals the number of blocks per row across the width of the quilt.
- To count blocks per row for alternate block quilts, count both pieced blocks and plain blocks.
- To count blocks per row for sashed quilts, count blocks only, not sashes.
- To count blocks per row for diagonal sets, don't count blocks in diagonal rows as you would sew the quilt. Instead, count the number of blocks whose points touch the top edge of the quilt (W) or the number of blocks whose points touch one side of the quilt (L).

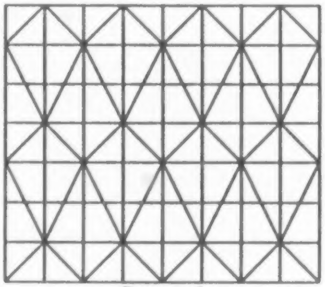
Transformations

✣ AS5

A shape can be moved to a new position by sliding it, spinning it or flipping it over. In maths these are called translations, rotations and reflections.

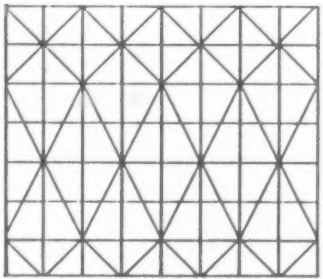
Patterns can be made by taking a shape and sliding it, spinning or turning it over. Sometimes combinations of these three movements can be used.

Pattern I has been produced by sliding the kite vertically or horizontally into a new position.

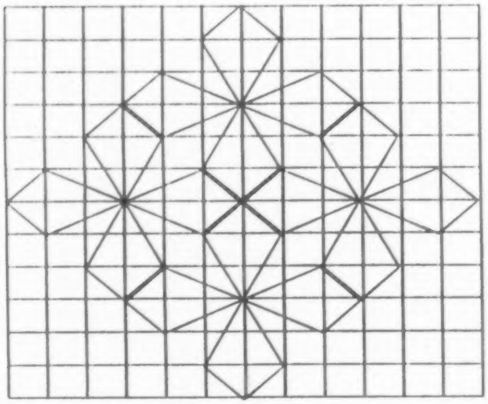


Pattern I

Pattern II has been produced by flipping the kite over or reflecting it. It could also have been produced by a combination of translation and reflection.



Pattern II



Pattern III

Pattern III has been produced by spinning the kite about different points. It could also have been made by a combination of rotation and reflection or of rotation and translation.

Determining yardage

✣ AS6

Determining Yardage Requirements

Knowing how to determine yardage requirements allows you more independence in selecting quilt designs. You won't need to rely on patterns with the yardage requirements already given nor a sales clerk's quick estimate of needed fabric amounts.

A simple sketch of the quilt showing the number of quilt blocks, lattice strips, and borders will help you visualize the amount of each fabric needed. The following formulas may look confusing, but once numbers are substituted for the letters used to represent them, it all begins to "fall into place."

1. Count the number of squares in a quilt block; call this S (for "squares"). Multiply S by the number of blocks (B) in the quilt. This will produce the total number of squares needed. Call this T (for "total"). S X B = T.
2. Divide the fabric width (W) by the width of an individual square (w) to find the number of squares (N, for "number") that can be cut per fabric width. W divided by w = N. Count only the whole number of pieces cut. Fractions of squares cannot be used.
3. Divide the total number of squares needed (T) by the number of squares cut per fabric width (N) to give the total number of rows (R) needed. T divided by N = R.
4. Multiply the rows needed (R) by the height of the squares (H) to find the total inches needed per fabric (I, for "inches"). R X H = I.
5. Divide the total number of inches needed (I) by 36 to convert to yards.

The following list may be used for reference when replacing the numbers for letters:

S = squares per block
B = number of quilt blocks per project
T = total number of squares per project
W = fabric width (most 100% cotton is 44" wide)
w = width of individual square
N = number of squares cut per fabric with
R = rows of squares needed
H = height of squares
I = inches needed per fabric

When using the grid for quick piecing triangle-squares, remember that for each set of 2 triangle-squares, the grid size needs to be the size of a finished square in the block + seam allowances.

Determine the total amount of fabric needed for each fabric required in the quilt using the above method. Then calculate the fabric needed for the lattice strips and the borders. If you will be using one of the fabrics in the quilt for the binding, this calculation must also be made. Cut the borders in long continuous strips whenever possible, then cut the rest of the pieces from the fabric.

Some shrinkage may occur when pre-washing the fabric, so be somewhat generous in purchasing the needed amounts of each fabric.

Organising design elements ✣ AS7

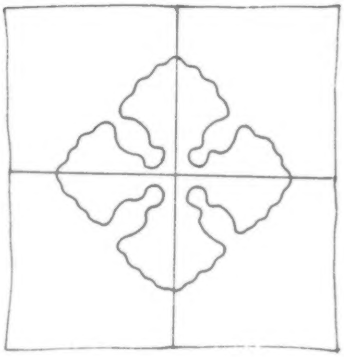
As quiltmakers, we strive to produce new quilts and designs as expressions of our lives. We have made the unique decision to quilt, not out of necessity, but as a response to the joy of the quilting process and its unlimited choices. Moved by a spirit from within and a sensitivity to the beauty of everyday life, we can create designs meaningful to our era. It should never be said that women of the twentieth century copied patterns.

The ability to create new and meaningful designs is within each of us. How we let ourselves solve the decision-making and problem-solving is half the fun. Whether a contemporary quilt is planned or develops spontaneously, color and design principles help cultivate our creativity, allowing us to say something about ourselves. An awareness of how we use color and design makes our work stronger and visually more exciting.

Images are language, a means of communication. We can capture moods, spirits and thoughts in quilts. Design inspiration is everywhere in both the natural and man-made worlds. By understanding design, we help relate our varied—and possibly disconnected—experiences.

Design, or the arrangement of lines, masses and color in a harmonious composition, is much like ballet, music or painting: the final product should be a well-organized, coordinated whole. Design principles are flexible generalizations, or tools. By gaining an understanding of the elements and principles of design, we add impact and excitement to our work.

Point, line, plane and volume are the building blocks. They orchestrate our compositions.



The point is the smallest unit. It indicates position. Place a point above the center of a composition to give a lighter, flamboyant feeling. Place it below center for greater weight.

Point size is also a factor. An increase in the size of a point makes it come forward. Multiple points in a design are effective; for example, two points cause a pull or attraction in space. We tend to see points as joining elements. Odd numbers often have more visual interest than even numbers.

A line is the path of a moving point in any number of directions. Defining a line as a moving dot is useful because it recognizes the inherent dynamic quality of line. A line is created by movement. Vertical lines suggest the feelings of growth and dignity. Horizontal lines connote rest and quiet. Both offer structure. Diagonal lines are action-packed. The angle determines the quality of action. Curved lines (or arcs) give the sensation of movement, while zigzags or erratic lines leave us feeling surprised. An X has a feeling of stability, while a spiral unfolds and grows.

The path of a moving line becomes **a plane,** area or surface; it defines the external limits of volume, which are illusory in two dimensions. The expression of shape is the composite feeling of its lines. A bean is a graceful, feminine freeform. A rectangle is a quieter form. A curved plane may imply feelings of consistency. Parallelograms, triangles, diamonds and trapezoids all pierce the picture plane. Triangular and diagonal forms tend to be more dynamic than rectangular ones. Pyramids, when low and broad, suggest stability and permanence; tall, slender pyramids lead the eye and spirit rapidly upward. Freeform and organic curves, which abound in nature, relate to life and growth.

Emphasis, something in the work, must dominate, through form, color, texture, position or size. Balance brings all these parts of the composition into a well-ordered whole.

The simplest type of balance—the simplest to create and to recognize—is called symmetrical balance. Symmetry occurs when one half of an object is the mirror image of the other. It connotes feelings of dignity, formality and passiveness.

The second type of balance is called asymmetrical balance. Asymmetry occurs when objects on each side of a composition are equal in weight but are not identical. It offers more opportunity for freedom, often stirring our curiosity.

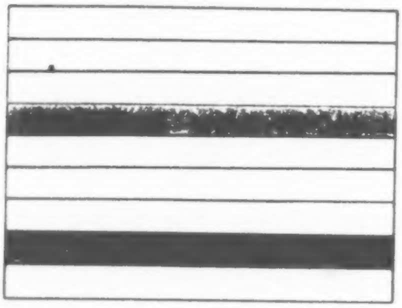
Informal or active balance suggests movement, spontaneity, casualness. A third variety of balance is called radial balance. Major parts radiate from a center, like spokes of a wheel. Radial balance gives a feeling of circular motion. The center is the potential focal point.

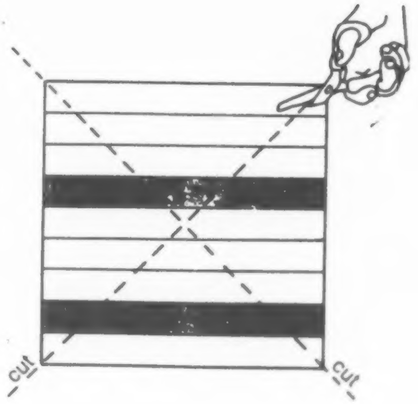
2

Seminole patchwork

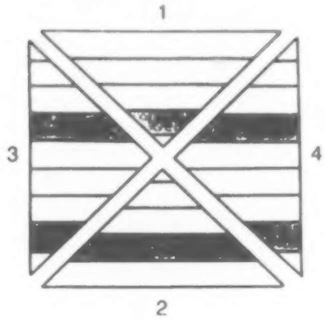
AS8
page 1

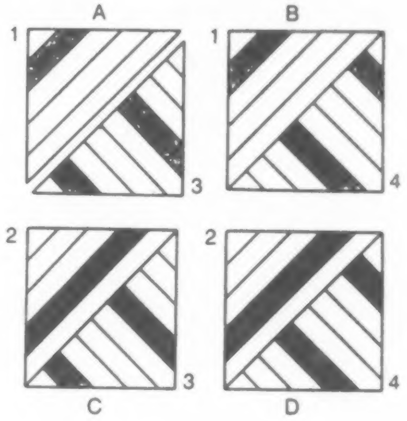
Sew together 9 strips of fabric + cut into block squares.





Divide into 4 triangles
Number 1 – 4

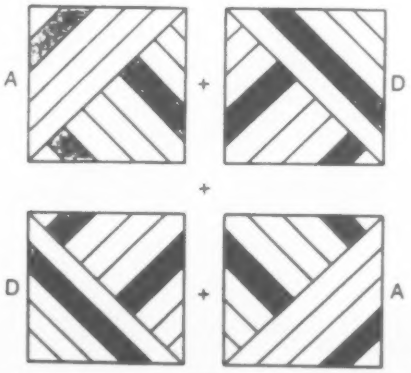


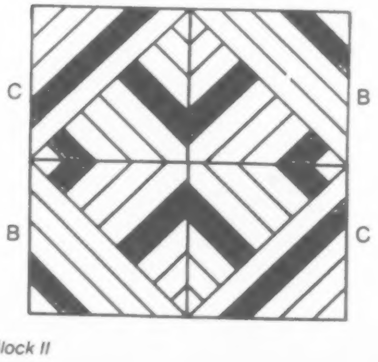

A
B
1
1
3
4
2
2
3
4
C
D

Join the triangles to form new squares

Arrange the triangles as shown.
Rejoin the squares as shown

Block I

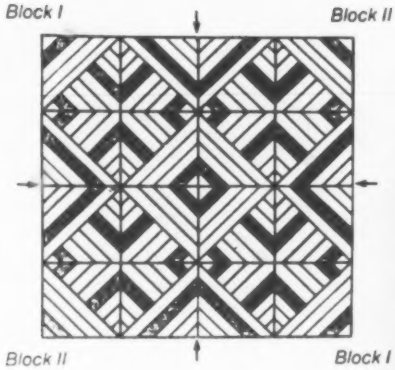


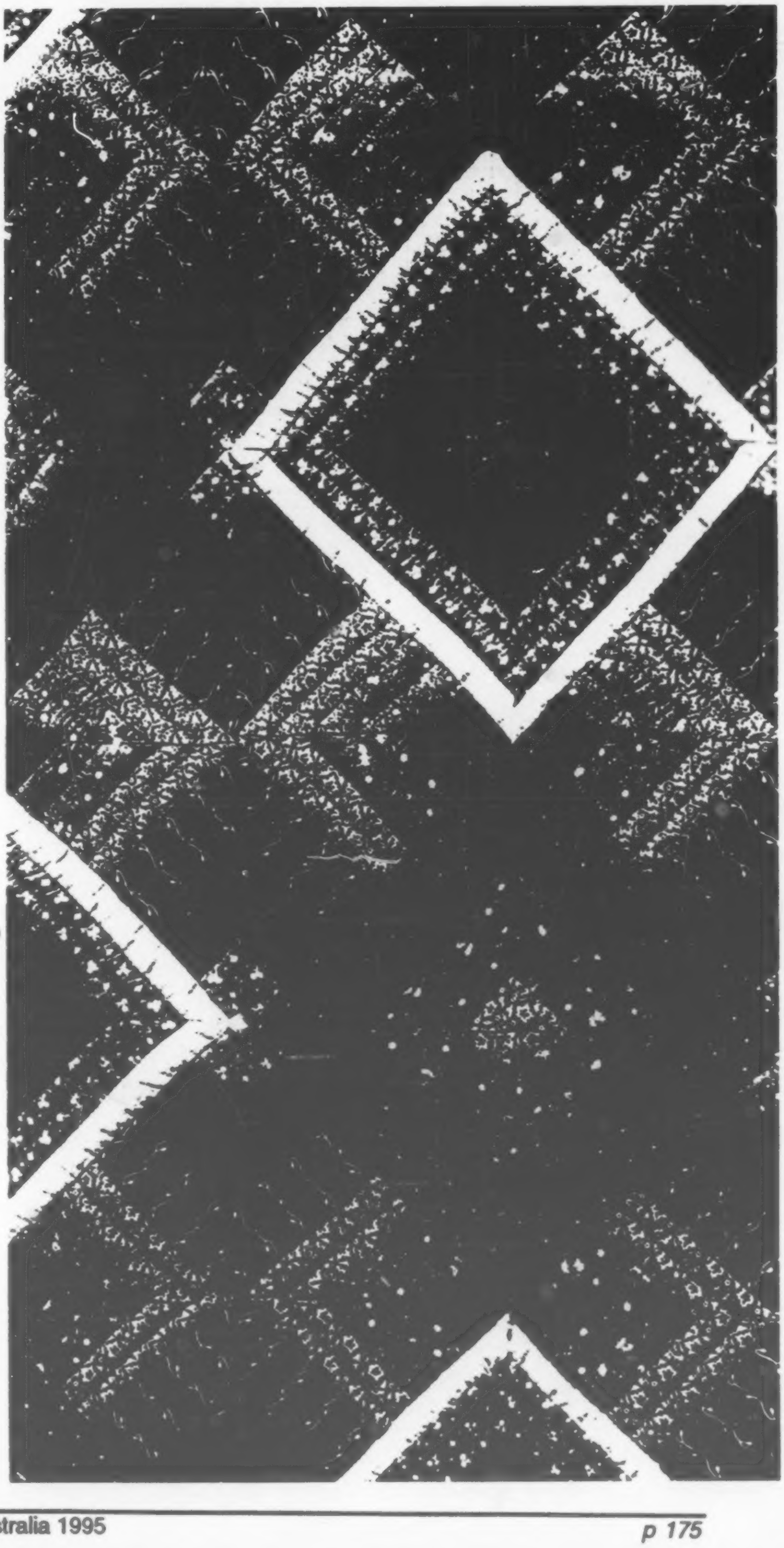


Block II

✣ AS8

page 2





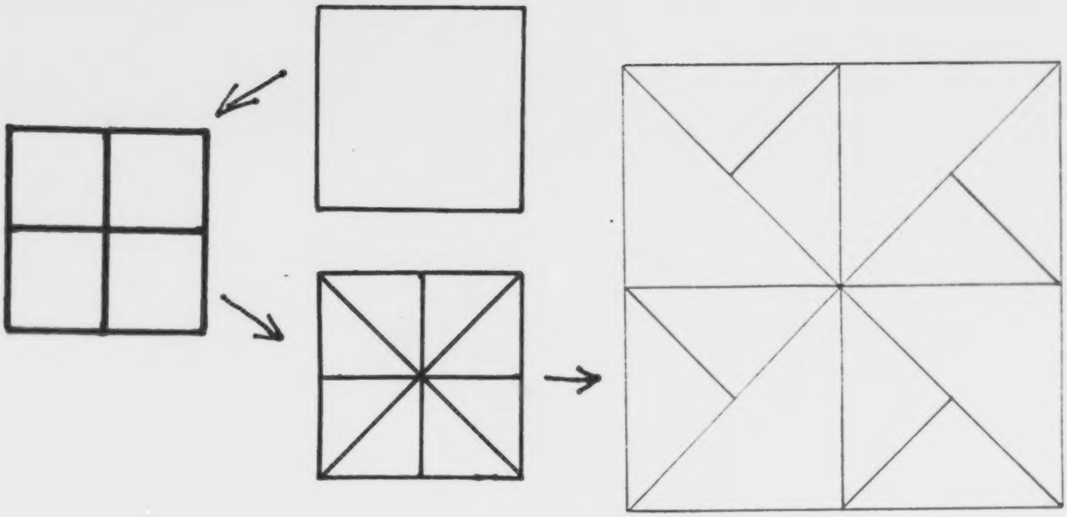
By turning these blocks around + rejoining together as shown up above the quilt on the right can be achieved.

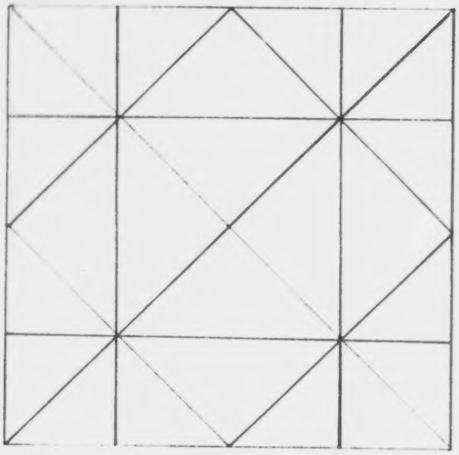
This quilt then creates many new shapes. The lines create angles + when pieced together like this they give balance of horizontals versus verticals.

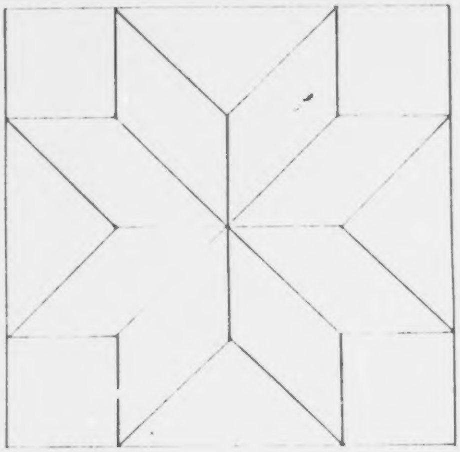
Quilt artist:
Margee Gough
"Hidden Wells"

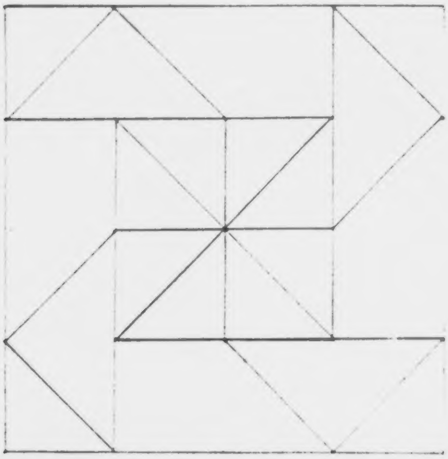
AS8 & AS9 are based on a student assignment by Trish Dunn, University of Technology, Sydney.

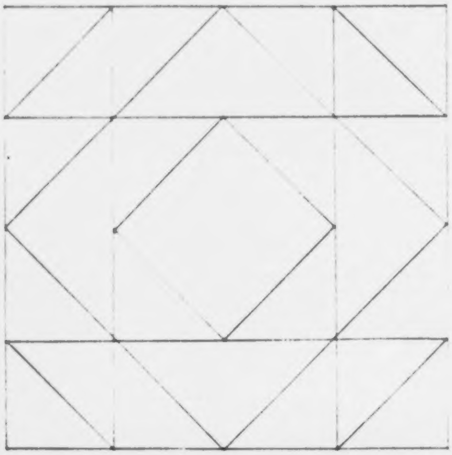
Dividing squares ✣ AS9











Side triangles

✣ AS10
page 1

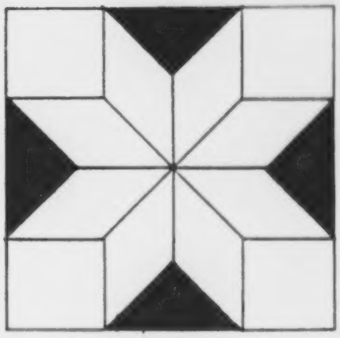


Figure 3.20

ii) The side triangles

These side triangles will have two sides adjoining the right angle which will measure 1 cm (½ inch) more than the sides of the background squares you have just cut. In the example, the squares were 44 cm (17½ inches) so the triangles will have to measure 45 cm (18 inches) on the two short sides. (Figure 3.21)

The four triangles can be cut out of one square of fabric, marked and cut diagonally, as shown below. (Figure 3.22)

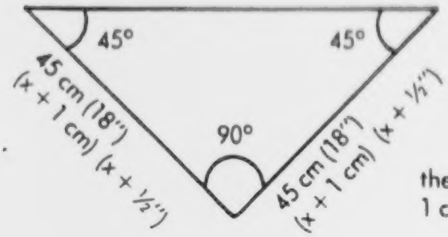
To estimate the size of the square which will yield these four triangles, you will need a calculator with a $\sqrt{\quad}$ button. The size of the square is $\sqrt{2\,(x+1)^2}$ (remember Pythagoras?). In our example, where the star points (x) measured 44 cm (17½ inches), the square for the four triangles will measure

$\sqrt{2(45)^2} = \sqrt{4050}$ $= 63.6$ cm

$\sqrt{2(18)^2} = \sqrt{648}$ $= 25½$ inches

But **don't panic!** If geometry and algebra is not your strong point, the chart on page 55 will help your calculations.

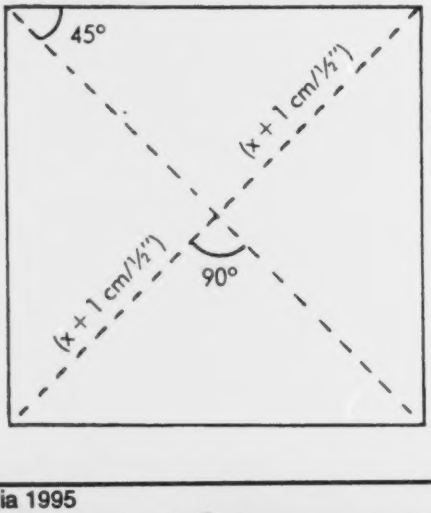
Figure 3.21



x is the size of the square

the triangle side is 1 cm (½") longer than x

Figure 3.22



✣ AS10
page 2

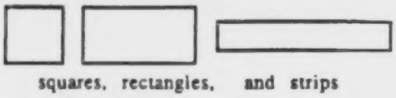
SOME LIKELY MEASUREMENTS FOR YOUR SIDE TRIANGLES (TO SAVE YOU CALCULATING)		
sides of star points measurement (average)	background squares (cut four)	squares from which to cut four triangles
42 cm	42 cm	60.8 cm
42.5 cm	42.5 cm	61.5 cm
43 cm	43 cm	62.2 cm
43.5 cm	43.5 cm	62.9 cm
44 cm	44 cm	63.6 cm
44.5 cm	44.5 cm	64.3 cm
45 cm	45 cm	65 cm
45.5 cm	45.5 cm	65.7 cm
46 cm	46 cm	66.4 cm
16½ inches	16½ inches	23⅜ inches
16¾ inches	16¾ inches	23¾ inches
17 inches	17 inches	24 inches
17¼ inches	17¼ inches	24⅜ inches
17½ inches	17½ inches	24¾ inches
17¾ inches	17¾ inches	25 inches
18 inches	18 inches	25½ inches
18¼ inches	18¼ inches	25¾ inches

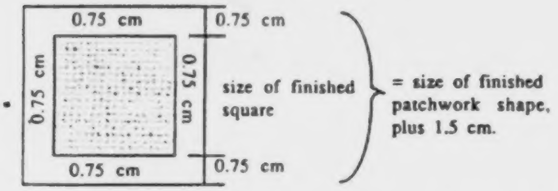
Seam allowances

✣ AS11

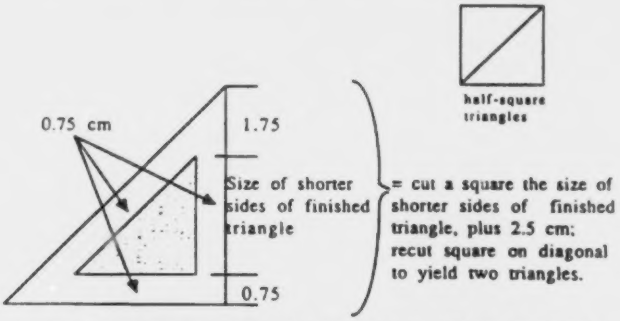
IF YOU MAKE THE SEAM ALLOWANCE 0.75 CM, THEN ALL OF THE QUICK ROTARY CUTTING TECHNIQUES BECOME POSSIBLE. This is how it works:

1. For squares, strips, and rectangles, cut all shapes plus 1.5cm, that is the size of the finished shape, plus 0.75 cm all around. For example, to cut a square which is 5 cm finished size, you would cut a square 6.5 cm x 6.5 cm. To make a rectangle which is 6 cm x 12 cm finished size, you would cut a shape 7.5 cm x 13.5cm.

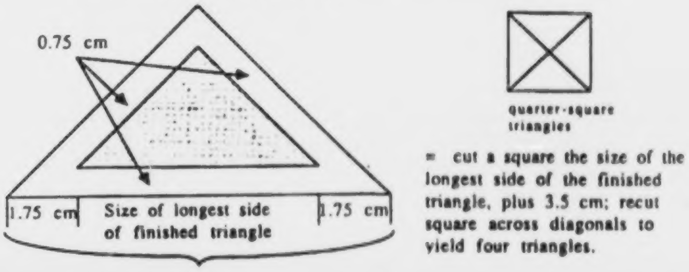

squares, rectangles, and strips


0.75 cm
0.75 cm
0.75 cm
0.75 cm
0.75 cm
0.75 cm
0.75 cm
size of finished square
= size of finished patchwork shape, plus 1.5 cm.

2. For half-square triangles, cut a square which is the size of two finished triangles put together, plus 2.5 cm. When the square is cut across the diagonal, you will have two triangles, each the correct finished size, plus 0.75 cm all around for the seam allowances.* For example, if you wanted to have a half-square triangle which was half of a 10 cm square, then you would cut a square 12.5 cm, then recut this square on the diagonal, and you have two triangles the size you require, with the 0.75 cm seam allowance all around each one.


half-square triangles
0.75 cm
1.75
Size of shorter sides of finished triangle
0.75
= cut a square the size of shorter sides of finished triangle, plus 2.5 cm; recut square on diagonal to yield two triangles.

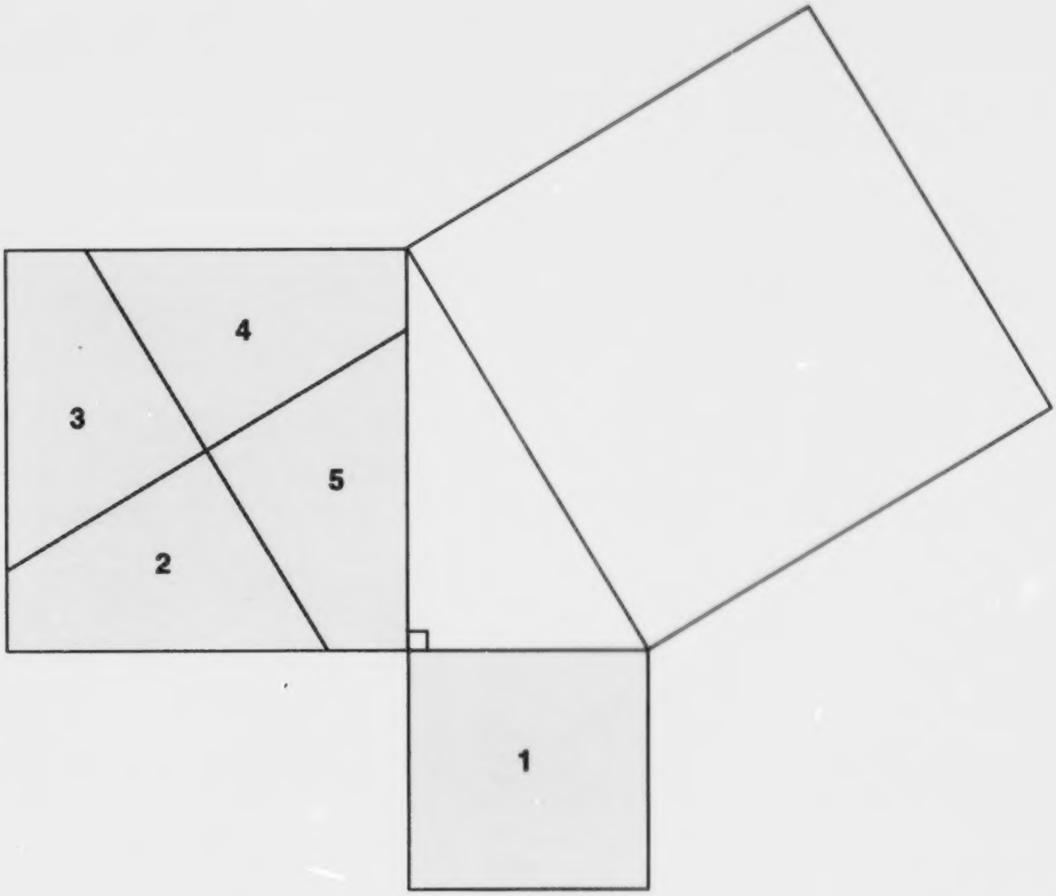
3. For quarter-square triangles, cut a square which is the size of four finished triangles put together, plus 3.5 cm. When the square is cut across both diagonals, you will have four triangles, each the correct finished size, plus 0.75 cm all around for the seam allowances. * For example, if you wanted to have a quarter square triangle which is quarter of a 12 cm square, then you would cut a square 15.5 cm, then recut this square across both diagonals, and you have four triangles the size you require, with the 0.75 cm seam allowance all around each one.


0.75 cm
quarter-square triangles
1.75 cm
Size of longest side of finished triangle
1.75 cm
= cut a square the size of the longest side of the finished triangle, plus 3.5 cm; recut square across diagonals to yield four triangles.

A Pythagorean puzzle

HO7

This is a right-angled triangle.
Cut out the regions 1 to 5 in the shaded squares and see if you can fit them into the largest square.



What does this tell you about:
- the relationship between the areas of the three squares?
- the relationship between the lengths of the three sides of the triangle?

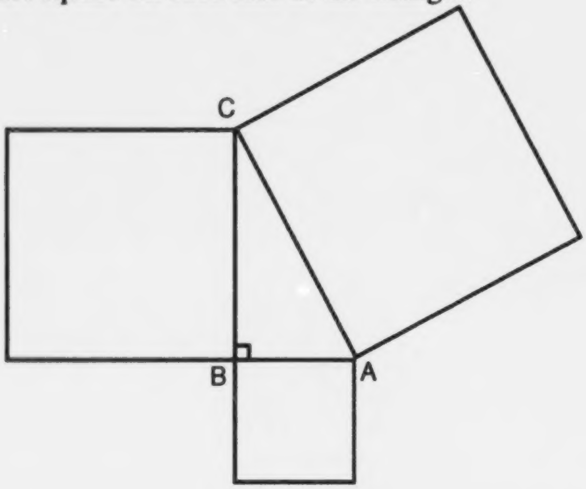
Making your own Pythagorean puzzle

HO8

Here is a way to construct your own proof of the Pythagorean Theorem.

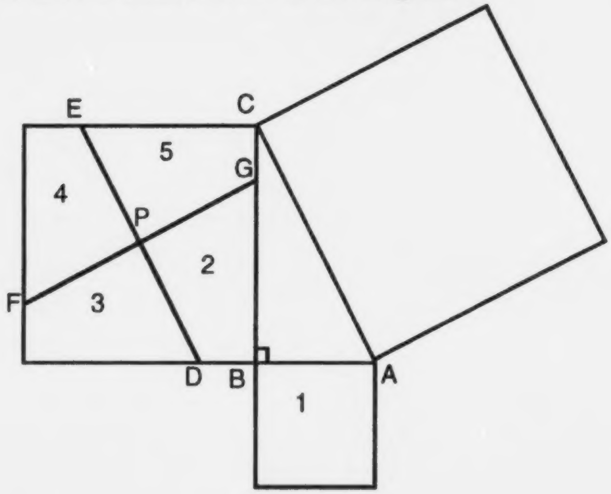
1 Carefully draw a right-angled triangle, it can be any size as long as you leave plenty of room around the outside of it (see the diagram). If you are working in a group try to draw different sized triangles.

2 Draw an exact square on each side of the triangle.



3 In the second largest square find the centre point, labelled P in the diagram below, by drawing two intersecting diagonals.

4 Draw lines through the point P which are parallel to the sides of the largest square. (The lines are DE and FG in the diagram.)



5 Cut out the regions 1 to 5. See if you can fit them into the largest square.

How far does it stretch?

HO9
page 1

From Hogben 1936, *Mathematics for the Millions*, pp. 55–59
© George Allen & Unwin, London.

Men built hotels for their celestial visitors and earthly representatives long before they had the sagacity to think about making houses fit for themselves to dwell in. The construction of calendar monuments for sighting the direction of heavenly bodies and of burial edifices for the embalmed remains of the sky-born Pharaoh entailed accurate measurements of distance. The two pyramids of Cheops and Sneferu at Gizeh are constructed on the same geometrical plan … The accuracy of construction which is characteristic of the temple architecture is bound up with their essential social function. Being built to receive their heavenly guests, these ancient monuments had to have a very precise orientation, which has been illustrated by the arrangement of the ventilating shafts in the Great Pyramid at Gizeh. For many millennia men were content to use crude anatomical units of length for most practical purposes. The Semitic peoples used the cubit or distance from the tip of the middle finger to the elbow, as farmers still use their legs to pace out a field in 'feet' or yards. For ordinary purposes they were content with a unit of length which varied from one individual to another. Temple architecture demanded a far higher standard of precision, and was based on the long-lost art of shadow reckoning in the sunnier climates where civilisation began. Heights were reckoned by the length of the shadow and the angle of the sun above the horizon, and reckoning heights in this way depends on certain simple truths about the relation between the lengths of the sides of a triangle.

The earliest mathematical discoveries belong to this class of problems. At a very early date the Babylonians knew how to make an angle of 60° by inscribing a figure of six equal sides (hexagon) in a circle (Fig. 14). All over the ancient world we find evidence of a very simple recipe for making an angle of 90°, which depends on the fact that a triangle with sides 3, 4, and 5 units of length is right-angled. According to one legend the priestly architects of Egypt laid out a right angle by knotting together three pieces of rope as in Fig. 15, the lengths of the pieces being in the ratio 3: 4: 5. If this is pegged down at the knots, a perfect set square is obtained.

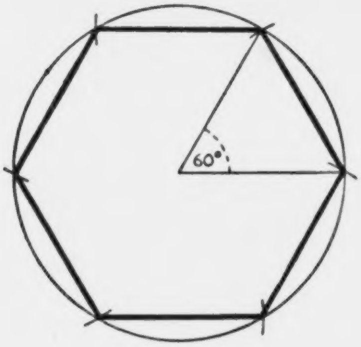
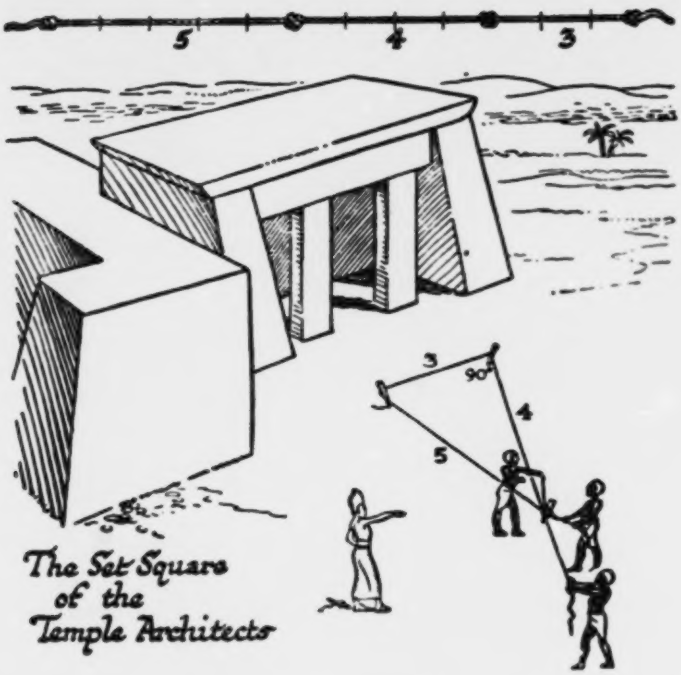


Fig. 14 A regular hexagon (figure with six sides of equal length) inscribed in a circle by marking off, along the boundary, intersecting arcs with the same radius.


5
4
3
3
90°
4
5
The Set Square of the Temple Architects

Five or six thousand years ago the Egyptians and Babylonians had discovered at least one case of a general rule about the sides of a right-angled triangle (see Fig. 16). Geometry books state it in these words: 'the square on the longest side (hypotenuse) or right-angled triangle is equal to the sum of the squares on the other two sides.' Thus a right-angled triangle is also formed if we knot together three pieces of rope, 5 yards, 12 yards, and 13 yards long, stretch it out, and peg it down at the knots as in Fig. 15. You can see at once that ... $25 + 144 = 169$

i.e. $5^2 + 12^2 = 13^2$

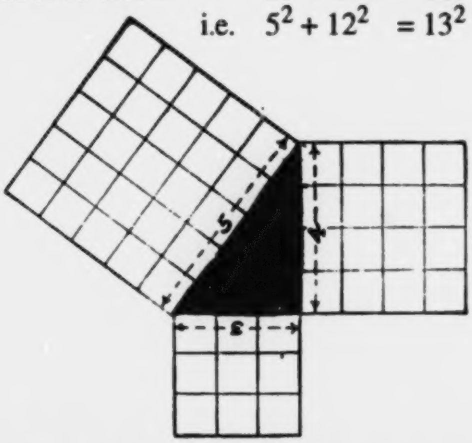


Fig. 16 THE RIGHT-ANGLED TRIANGLE OF THE TEMPLE BUILDERS

Long side: 5 feet .. .. ($5^2 = 25$ square feet)
Short sides: 4 feet .. .. ($4^2 = 16$ square feet)
3 feet .. .. ($3^2 = 9$ square feet)

$$25 = 16 + 9$$

or $$5^2 = 4^2 + 3^2$$

Pythagorean pieces

HO10

Extracts from Kolpas 1992, © Dale Seymour Publications, PO Box 10888, Palo Alto, CA.

The Pythagorean Theorem states one of the most important ideas in all of mathematics, forming part of the conceptual basis of trigonometry, analytic geometry, vector algebra, and calculus. Much of higher mathematics would be impossible without it. The theorem states quite simply that 'in a right triangle, the sum of the squares of lengths of the legs is equal to the square of the length of the hypotenuse.' The word *hypotenuse* literally means 'stretched under' and was chosen because the side so named is stretched under (opposite) the right angle. The legs are the sides that form the right angle.

Although the name of Pythagoras (c.580-501 B.C.) is inexorably linked with this theorem, there is no doubt the concept was known prior to Pythagoras. While tradition credits Pythagoras with the independent discovery of the theorem that bears his name, the theorem was known to the Babylonians more than one thousand years earlier. However, most scholars believe the first general proof of the theorem was given by the Pythagoreans.

The Pythagorean Relationship in Ancient Times

In ancient Egypt, the Nile River annually overflowed, flooding the land for miles. While this flooding watered the crops, it also destroyed property boundaries. For measuring and remeasuring their lands, the Egyptians needed to use the right angle. To construct a right angle, men called rope stretchers tied thirteen knots at equal intervals along a length of rope. They anchored the rope in the ground with two stakes at the fourth and eighth knots. Where the thirteenth knot met the first knot, they put a third stake. This of course formed a 3–4–5 right triangle. Records of the use of the 3–4–5 right triangle in Egypt come from papyrus fragments dated approximately 2000 B.C.

Around the time the Egyptians were using the Pythagorean relationship, the Hindus in India also made use of right angles. They discovered not only that the 3–4–5 triangle produced a right angle, but also that a whole class of triangle such as the 12–16–20, 5–12–13, and 8–15–17, generated the desired angle.

From clay tablets dated around 1500 B.C., there is evidence that the Pythagorean Theorem was at least understood in Babylonia. Again, the understanding was empirical and practical. As with the Egyptians and the Indians, there is no surviving evidence that they constructed a general proof.

About Pythagoras

An accurate depiction of Pythagoras' life (c.580–501 B.C.) is entangled with legend and deification. Most of our information about Pythagoras comes to us indirectly, or from sources such as Aristotle's *Metaphysics* and part of the book of Philolaus, a leading Pythagorean of the latter part of the fifth century B.C.

Estimates place Pythagoras' birth between 580 and 569 B.C. on the Aegean island of Samos, which would make him a contemporary of Lao-Tze, Budda, and Confucius. He was born during the Golden Age of Greece—the rebirth of culture and art. He probably left Samos around the age of eighteen to study in Phoenicia and Egypt. He reappeared after years of study and wandering and sought a place to form a school. Banned by the tyrant Polycarates from returning to the island of his birth, he settled in southern Italy at the Greek seaport of Croton.

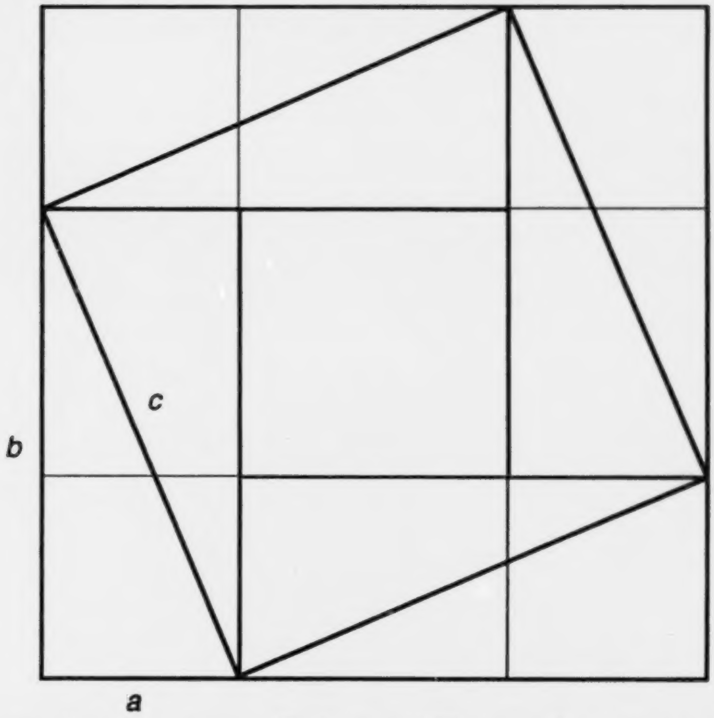
Piling up of rectangles

HO11

Follow the directions below to produce another proof of the Pythagorean Theorem based on a Chinese method, sometimes called 'piling up of rectangles'.

- On the grid provided draw a large square.
- Now draw a rectangle on the inside of one of the corners so that the sum of the two sides add to the length of your square. Name the sides a and b.
- Repeat the process to draw four rectangles altogether, each of side lengths a and b inside your square.
- Draw in the diagonals of each of the four rectangles in order to construct right-angled triangles in each corner of your large square. This should create a square in the middle of your outside square which is 'the square on the hypotenuse' of your corner right-angled triangles. Call this side c.

You should end up with a construction looking something like this:



- Use this construction to prove Pythagoras' Theorem for a corner triangle of sides a, b and c.

Hint: Shade in a square of area a^2 and a square of area b^2 and then try to make these equal the square on the hypotenuse, c^2.

The kou ku theorem

HO12
page 1

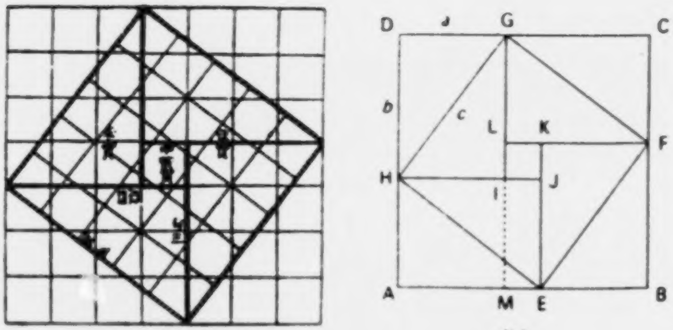
From Joseph 1991, *The Crest of the Peacock: non-European roots of mathematics*, pp. 180–2
© I. B. Tauris & Co., London.

In China too, the study of the right-angled triangle had a considerable impact on mathematics.

The earliest extant Chinese text on astronomy and mathematics, the *Chou Pei*, is notable for a diagrammatic demonstration of the Pythagorean (*or Kou ku*) theorem. Needham's translation of the relevant passage is illustrated by Figure 7.1(a), drawn from the original text.

The passage reads:

> Let us cut a rectangle (diagonally), and make the width 3 (units) wide, and the length 4 (units) long. The diagonal between the (two) corners will then be 5 (units) long. Now, after drawing a square on this diagonal, circumscribe it by half-rectangles like that which has been left outside, so as to form a (square) plate. Thus the (four) outer half-rectangles (of area 24) then (when this is subtracted from the square plate of area 49) the remainder is of area 25. This (process) is called 'piling up the rectangles'.



(a) (b)

Figure 7.1 The kou ku (Pythagorean) theorem:
(a) the original illustration from the Chou Pei; (b) the modern 'translation'

In terms of Figure 7.1 (b), the larger square ABCD has side 3 + 4 = 7 and thus area 49. If, from this large square, four triangles (AHE, BEF, CFG and DGH), making together two rectangles each of area 3 x 4 = 12, are removed, this leaves the smaller square HEFG and implicitly

$$(3 + 4)^2 - 2(3 \times 4) = 3^2 + 4^2 = 5^2$$

The extension of this 'proof' to a general case was achieved in different ways by Chao Chung Ching and Liu Hui, two commentators living in the third century AD. In modern notation, Chao's extension may be stated thus:

> if the shorter (kou) and the longer (ku) sides of one of the rectangles are *a* and *b* respectively, and its diagonal (shian) is *c*, then the above reasoning would produce:

$$\begin{aligned} c^2 = (b-a)^2 + 2ab &= \text{square IJKL} + \text{rect DGIH} + \text{rect CFLG} \\ &= \text{square AMIH} + \text{square MBFL} \\ &= a^2 + b^2 \end{aligned}$$

An alternative explanation is based on the identity:

$$\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ c^2 &= (a+b)^2 - 2ab \\ &= a^2 + b^2 \\ &= \text{square ABCE} - 4\Delta\text{DGH} \end{aligned}$$

A geometric interpretation of this identity is fairly easily established, and was certainly known to the authors of the *Sullbasutras* (c. 500 BC) of Vedic India ... There is also the possibility that it was known to the Babylonians of the Hammurabi dynasty. Later, geometric proofs of this identity are found in Euclid's *Elements* (c. 300 BC) and from Figure 7.1 a itself, where two squares of areas $a^2 = 4^2$ and $b^2 = 3^2$, together with two rectangles of area 'ab', together make up the large square of area $(a + b)^2$.

However, there is a third explanation, found in Liu's commentary, which does not refer to the diagram in the Chou Pei (i.e. Figure 7.1a), but is based on a principle of Chinese geometry well known at the time:

> the 'out-in complementarity' (dissection and reassembly) principle ...

This principle is based on two common-sense assumptions:

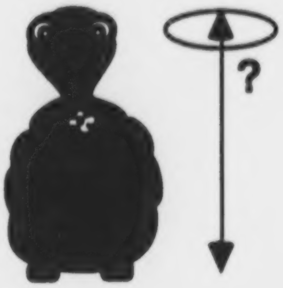
1. Both the area of a plan figure and the volume of a solid remain the same under rigid translation to another place.

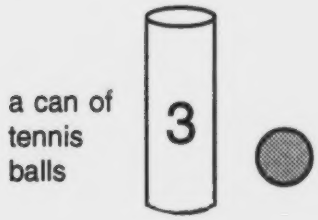
2. If a plane figure or solid is cut into several sections, the sum of the areas or volumes of the sections is equal to the area or volume of the original figure.

Station 1 ✣ AS12
Heads and tennis balls

How many times do you think a piece of string would have to go round your head to be as long as your body?

GUESS first, then try it ...




a can of tennis balls
3

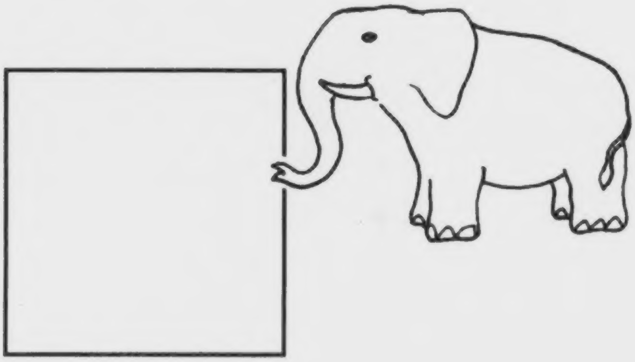
Now think about tennis balls ... if you cut a piece of string so that it just fits around the widest part of a tennis ball, how would the length of the string compare to the height of the 3-ball can?

GUESS first ... then try it ...

Station 2
Sheep in a hole

✣ AS13

Can you cut a hole in a piece of paper
this size
➾



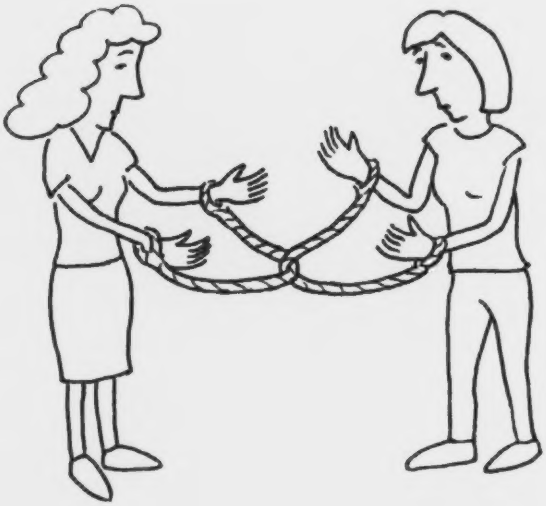
large enough for an
ELEPHANT
to walk through?

… for a start …
you could try cutting a hole,
in an A4 piece of paper,
large enough for a sheep to walk through …

Station 3
Handcuffs

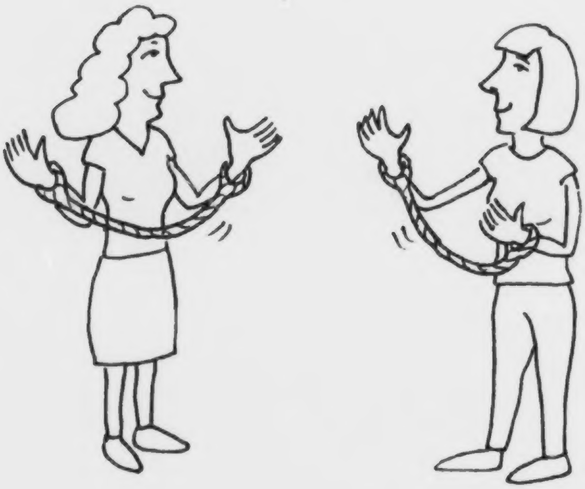
✣ AS14

Handcuff yourself
to a friend
like this: ➳

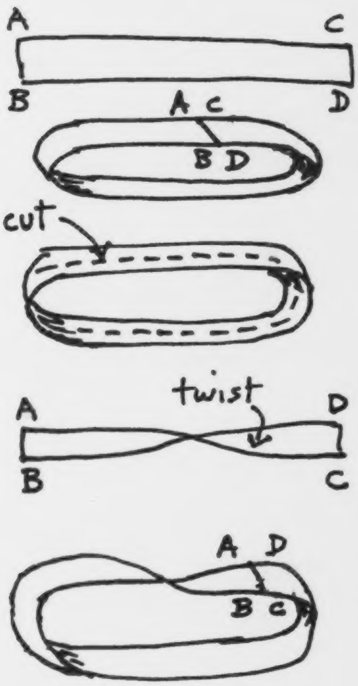


Now,
without untying the knots . . .
or taking the handcuffs off . . .
or cutting them . . .

can you free
yourselves
like this? ➳



Station 4 A twist a side ✣ AS15



Take a strip of paper.
Stick the ends together to make a ring.

How many surfaces does the ring have?
How many edges?
What would happen if you drew a line down the middle of the ring and cut along the line?
Try it.

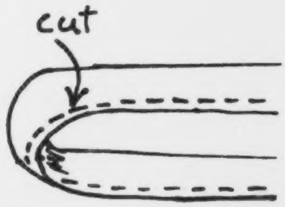
So far, not very surprising.

Start again with another strip of paper. This time twist one end with a half-turn before sticking the ends together. You are now holding a ...
MOBIUS BAND.

Let's have a look at it.

Draw a centre line along the 'inside' of the band.
What happens? How many edges does it have?
Trace with your finger along the edge to check.

What happens if you cut along the centre line until you come back to the starting point?
Guess first ... were you right?



Now, make other bands ... guess what will happen if you cut so that you keep a third of the distance from the edge ... try it ...
What if you had two twists? three? ... what if you cut a quarter of the distance from the edge? ...
... are you getting better at predicting?

IS THERE A PATTERN?

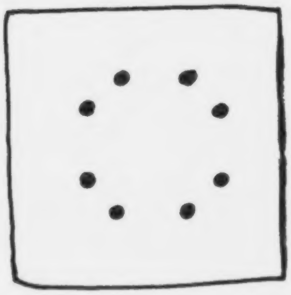
Can you think of answers to:

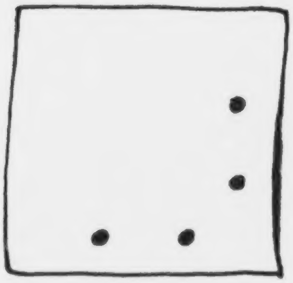
'why might engineers use this band in the construction of pulleys?
... of audio tapes?'
'whether the drive belt on a sewing machine is a Mobius band or not?'

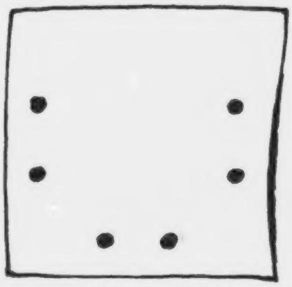
Station 5 Symmetry

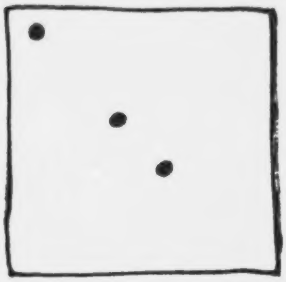
✣ AS16

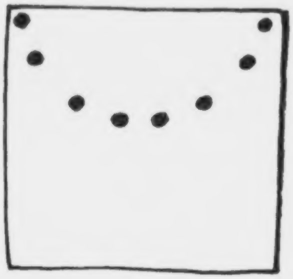
Try to make the patterns below by folding the square *no more than three* times, and punching only *one* hole through the paper. Make up new designs for others to try.

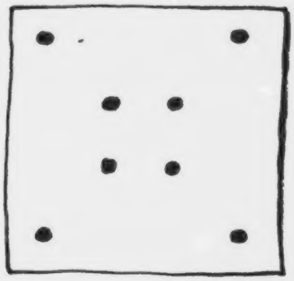










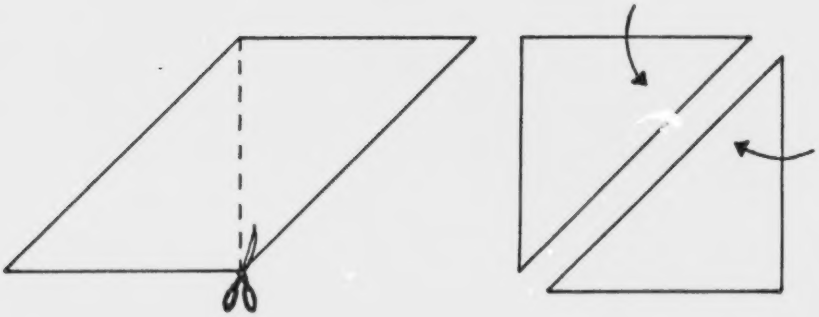


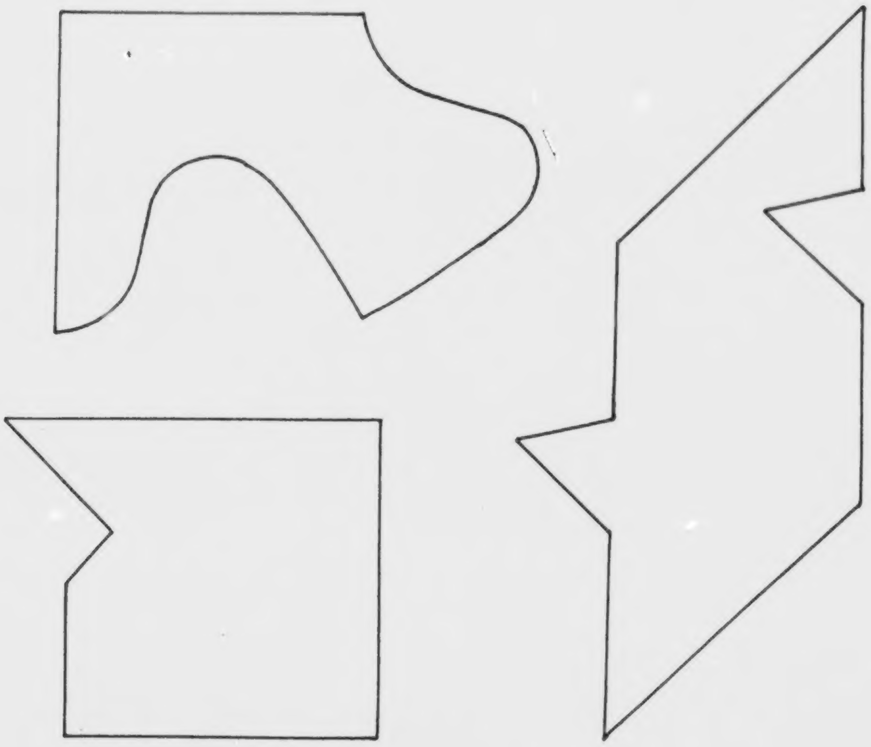
Station 6
Getting Square

✣ AS17

Cut out the 3 shapes below.
Then for each one try to make a straight cut so that
the two pieces fit together to make a square

Like this

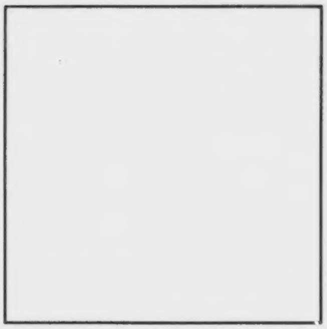


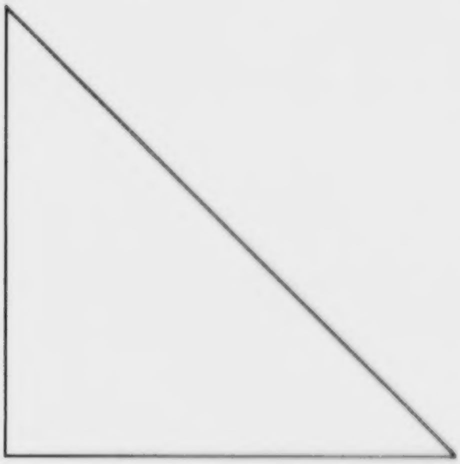


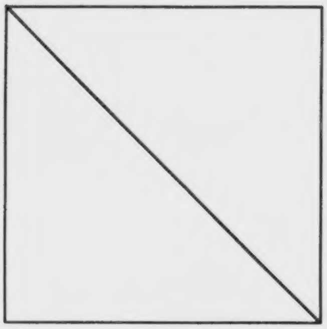
From B. Marr and S. Helme 1991 *Breaking the Maths Barrier*, DEET. Page 119.

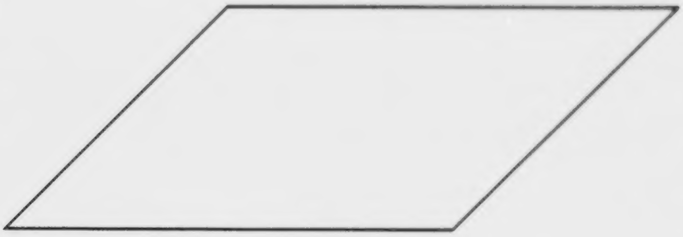
Squaring the circle shapes

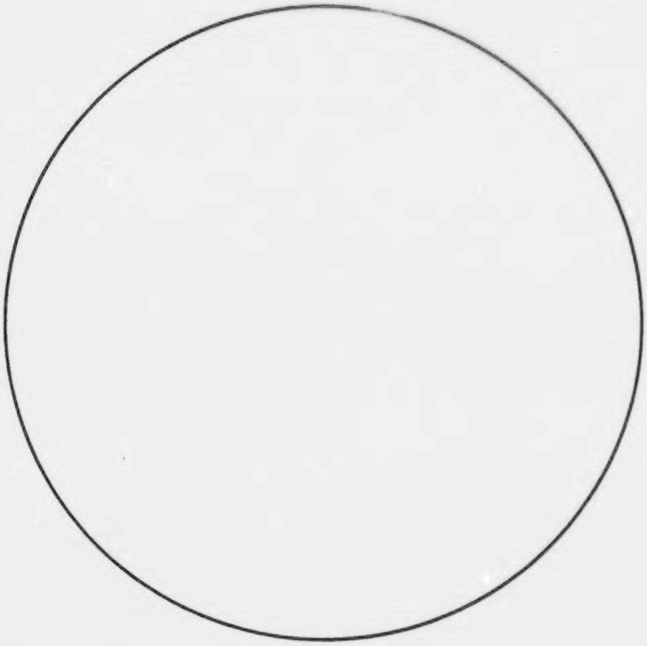
✣ AS18
page 1











Squaring the circle ✋ HO13

For centuries there was a perennial fascination with 'squaring the circle' (i.e. constructing a square whose area is exactly equal to that of a circle with a given diameter using only a straight edge and a compass). Here we will try to find some approximation of the area of a circle.

Look at the circle and sketch a square that you think might have the same area approximately. How many little squares would fit into your sketched square? Keep your sketch to refer to later.

Put the shapes in what you think might be their order of size (by area).

Then use the little triangles to measure them, as precisely as you can.
Were you right? Were any of your results a surprise?

How would you fill in these statements?

The small square has the same area as .... little triangles.

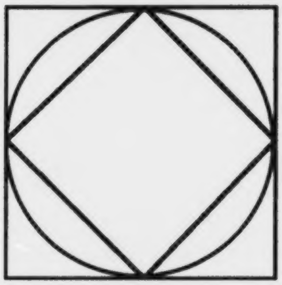
The medium square has the same area as .... little triangles

or .... small squares.

The large square has the same area as .... little triangles

or .... small squares.

Put the circle inside the large square, and the medium square inside the circle.



Can you estimate how many small squares would cover the space inside the circle?

More than .... how many?

Less than .... how many?

Does your sketch fit between the two squares?

Can you think of a way to make a more precise estimate of the area of the circle?

A detour: Can you make a square with the 3 triangles, the small square and the parallelogram?

Putting God back in math

P4
page 1

Extract from *The Kansas City Times*, copyright 1982, by Lyn O'Shaughnessy.

While creationists' beliefs are being weighed by an Arkansas judge, a sister organization has evolved, if you will, hundreds of miles away in the hallowed halls of Emporia State University.

A bold collection of free-thinking Kansas heretics has decided to continue the work started by the creationists, who want the biblical explanation of the beginning of man taught in the schools. About 100 professors, students and a few publicity shy Emporia ministers have formed the Institute of Pi Research.

Quite simply, the institute wants to put God back into mathematics. Or at least back into pi.

Pi is the symbol for 3.14159265 and so on to infinity, that impossibly awkward number that is used to multiply the diameter of a circle to obtain the circle's circumference. The contemporary pi was discovered by Anaxagoras, an ancient Greek, who was sentenced to die for his efforts. But members of the institute are clamouring for the return of the pi used by architects of King Solomon's Temple. According to the Bible, the builders used a pi of 3 to construct parts of the majestic structure. Measurements revealing the use of pi equalling three are mentioned in 1 Kings 7:23 and 2 Chronicles 4:2.

There is absolutely no mention of 3.14 and its subsequent, non-repeating digits anywhere in the Bible, the professors note.

If a pi of 3 is good enough for the Bible, it is good enough for modern man, concludes the institute's founder, Samuel Dicks, a professor of medieval history. Mr. Dicks attributes a great deal of the modern malaise to the Godless pi, which he contends is "an atheistic concept promoted by secular humanists."

"To think that God in his infinite wisdom would create something as messy as this (3.14 and on) is a monstrous thought," Mr. Dicks concludes.

Are these pi patrons really on the level?

"I think we deserve to be taken as seriously as the creationists," Mr Dicks replies bluntly, appearing to be not the least bit amused.

Only with a great deal of reluctance will members of the institute, with the motto, "Pi is 3 any way you slice it", admit they are full of baloney. It is their way of poking fun at the creationists by pointing out that even the Bible makes mistakes—or at least seems to.

In jest, the members say they want public schools to give the ancient pi equal time in the classrooms.

"If the Bible is right in biology, it's right in math," states Loren Pennington, an economic historian.

To lend some credibility to its crusade, the institute charmed an Emporia State mathematics professor, Marion Emerson, into its fold. Mr. Emerson, who realizes he is being exploited for the good of the institute, says he has paid a price among his colleagues for his conversion.

"Some of them think it's awful," he says.

Not content to stay within the ivory tower, the institute recently turned to the university's cable channel to spread the work. The show featured a string of professors explaining why their theory should not be discounted as "pi in the sky."

A physicist maintained that the glow of the constellation Perseus proves that pi 3 is correct. A historian blamed Arabs using oil barrels made with lesser pi for the energy crisis, and Mr. Emerson, the token mathematician, produced a wheel he made using pi 3. The wheel, however, had six sides.

"Of course it makes for a bumpy wheel," he conceded.

The institute's convoluted theories have won converts at several universities, including the University of Kansas, Iowa State University and the Texas Tech math department. The group even sent a letter to President Reagan asking for his support. The reply is long overdue, but Mr. Dicks believes the leader is a closet believer because he said in a recent speech: "The pi(e) isn't as big as we think."

The group now is trying to lend its taped show to any sympathetic television station, but the response so far has been underwhelming. Refusing defeat, pi enthusiasts are sending letters to Midwestern legislatures asking them to give pi equal time. If they succeed they have plenty of other targets.

Says Mr. Pennington, the historian: "Our next step is to replace the insidious secular humanistic meter with the biblical cubit."

Pi in the sky? ▤ OHT2

- What is the article claiming that the Bible is saying about π?
  Is the claim true?

- What does the Bible actually say?
  How is the conclusion drawn that 'according to the Bible, the builders used a π of 3 to construct parts of King Solomon's Temple'?

- What is the definition of π which is implicit in the above claim?

Pi in history

HO14

1 Experiments

Give a definition of π in terms of the diameter and circumference of a circle.

Conduct some experiments to answer the following questions:

- If a hexagon is inscribed in a circle, how close is its perimeter to the circumference of the circle?
- … how about the perimeter of a hexagon circumscribing a circle?
- Would an octagon be any closer?

  How do you get a sense of the 'goodness' of the approximation?
- How 'good' are the approximations $\frac{22}{7}$ or 3.14?
- What value of π would you get if the circumference of a circle was approximated by an equilateral triangle—and a square, hexagon, octagon or dodecagon—inscribed in and circumscribing a circle?
- How close is the area of the circle to the areas approximated by the various shapes?
- What is the maximum area that shapes inscribed in a circle can have?
- What is the minimum area that shapes circumscribing a circle can have?

2 History

Use available resources to conduct a brief historical research about π.

- How did the study about π evolve in different places throughout history?
- What were some early estimates for π ?
- How were better approximations made?
- Did anyone ever succeed in squaring the circle? why, or why not?
- What was Kepler's derivation for the area of a circle?
- What is the record number of decimal places so far worked out for π ? Will there be an end to them?
- Was the study of π focused in any particular part of the world?

What can you say about the construction of mathematical knowledge from your brief study of π?

The handshake problem

HO15

page 1

If everyone in this room shook hands with everyone else, how many handshakes would there be?

1 Read the question.
Are there ambiguities? If so, sort them out.

One handshake or two? It doesn't really matter what you decide, as long as it makes sense and you are consistent. It might be interesting to explore both outcomes, but we'll count it as a single handshake for now.

2 If there were only 4 people, say, that would be easy.
But what about if there are, say, 23 people? or 138?
So, *begin with a simpler case* ... let's say 4 people.

3 Draw it or model it; act it out.
You can't get very far without something concrete, diagrammatic or symbolic to react to.

4 people

or

4 Consider all possibilities systematically.

- Person 1 is first to arrive, so there is no-one else to shake hands with.
- Person 2 arrives, shakes hands with person 1 and so far, 1 handshake.
- Person 3 arrives and shakes hands with persons 1 and 2, so there are 2 more handshakes.
- Person 4 arrives and shakes hands with all the others:
person 1, person 2 and person 3, so there are 3 more handshakes.
- Everyone has now shaken hands with everyone else.

If we have 4 people there will be 1 + 2 + 3 handshakes, or 6 handshakes altogether.

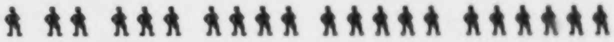
If another person comes into the room they will have to shake hands with all 4 persons already there—another 4 handshakes, i.e. 1 + 2 + 3 + 4, or 10 handshakes altogether.

5 How can we *organise and record* all this? Why would we want to?

Well, if we look carefully at what is happening in these cases we might see a *pattern* that would help us work it out for 23, or 138, or whatever.

no. of people	***how many handshakes? ... count ...***	***total no. of handshakes***
1	0	0
2	1	1
3	1 + 2	3
4	1 + 2 + 3	6
5	1 + 2 + 3 + 4	10
6	1 + 2 + 3 + 4 + 5	15
7	1 + 2 + ...	......

Any patterns? could you extend this to 23? to 138?
Could you describe a pattern in words so you could apply it to any number?
Could you make up some symbols—shorthand—to write it down?
and *extend* it to a larger number ...?


and so on

Your extended table might look like this:

no. of people	***how many handshakes? ... count ...***	***total no. of handshakes***
1	0	0
2	1	1
3	1 + 2	3
4	1 + 2 + 3	6
5	1 + 2 + 3 + 4	10
6	1 + 2 + 3 + 4 + 5	15
...	......	...
23	1 + 2 + 3 + 4 + ... + 21 + 22	...
...	......	...
number	add all the numbers from 1 to one less than the number of people	
n	1 + 2 + 3 + 4 + 5 + 6 + ... + (n–2) + (n–1)	

a pattern
to link
people and
handshakes

6 There might have been *a different way* of modelling or visualising the problem. For 23 people, you might have said:

> If I shake hands with all the other people here, that will be 22 handshakes, and that will be the same for everyone, so there will be 23 lots of 22 handshakes, but hang on a minute, I have counted both your handshake with me—and mine with you—so I've doubled up!

So there will only be *half* of that number of handshakes altogether, i.e. half of 23 x 22.

no. of people	***another pattern ... linking people and handshakes***	***total no. of handshakes***
1	1 times (1–1), then halved	0
2	2 times (2–1), then halved	1
3	3 times (3–1), then halved	3
4	4 times (4–1), then halved	6
5	5 times (5–1), then halved	10
6	6 times (6–1), then halved	15
...	......	...
23	23 times (23–1),then halved	...
.....	......	...
number	number is multiplied by (one less than the number), then halved	
n	$n \times (n-1) \times \frac{1}{2}$	$\frac{n(n-1)}{2}$

So you can find the solution in two ways:
either by adding up all the numbers from 1 to 22, or by halving 23 x 22
Incidentally this must mean that the sum of all the numbers from 1 to 22

$$= \frac{1}{2} \times 23 \times 22$$

7 Can you extend what you have found out? After all, we don't often want to count handshakes ... You can use this result for adding up numbers more *generally*. What is, for instance the sum of the first 100 whole numbers?

$1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100$ must equal $\frac{1}{2} \times 101 \times 100$, i.e...?

8 Try the following problems.
Can you see how they are similar to the one we have just done?

- You have 23 points on a circle. What is the maximum number of chords that you can draw joining those points?
  (Can you put this into a context that is more relevant to students?)
- Another problem:
  There are 31 ice-cream flavours. You can have a double cone using two flavours. How many choices do you have?

Could you describe in simple words what it is that all these problems have in common, or what their common structure is?

9 What strategies have we used:

- to start, to proceed and to finish solving this handshake problem?

Problem solving: a summary

HO16

What is a problem?

Just a question that you would quite like to solve …

… but you can't see how to, immediately.

Starting

- Begin with a positive attitude.
- Read the problem carefully.
- Keep an open mind.
- Familiarise yourself with the problem.
- Take risks.
- Relax.

Some problem solving strategies

- Act it out.
- Draw it, make a model.
- Change your focus—look at the problem a different way.
- Organise the problem.
- Work systematically to consider all possibilities.
- Use elimination.
- Look for patterns.
- Work with simpler cases.
- Do something else—have a coffee, go to bed.
- Break the problem into smaller parts.
- Visualise the problem.
- Work backwards.
- Guess and check.
- Use analogy … … and be persistent.

Finishing

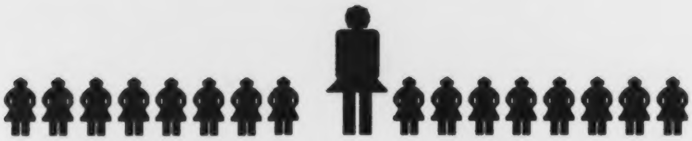
- Check that your answer actually fits the question.
- See if you can generalise from it, e.g. in the 9x table–up to 10x9–the digits in the answers all add to 9. Is this true for all multiples of 9?
- Have other questions emerged, e.g. what patterns are there in the other times tables?
- Can you solve them? (Let's look at patterns in the 8x table ...)

So, we begin the cycle again, and move from solving other people's problems to generating and expressing, and solving! our own.

Why do babies dehydrate faster than adults in summer?

HO17

page 1



How can you even start thinking about it?

Think about the problem solving strategies you have been collecting.

- You could make a model.
- You could take
  a simpler case
  a simpler shape—after all people are very complicated shapes
  and a simple size—say the adult is twice the scale of the baby.

Keep in mind the possibility of other strategies as you work through the question.

Doubling ...

- Use 3 small cubes to make a model of your baby.
- What is its volume? What is its surface area (skin), i.e. how many little squares would cover it?
- Now make a model of an adult that is double in scale. What is its volume? What is its surface area?
- Did surface area and volume increase by the same amount, or did one increase more than the other?

dehydration 1	*length*	*surface area*	*volume*
original model ... baby			
model doubled in scale ... adult			

- How many square units of skin for each cubic unit of volume allow water to be lost
  - by the baby?
  - by the adult?
  so which would lose water faster?

But why?

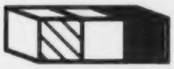
So we have the beginning of an answer, but let's try to go a bit further to understand what's happening...

See if you can fill in the table, by continuing to double the scale.
Can you see a pattern that lets you work out the last row or two without having actually to build the model?

HO17

page 2

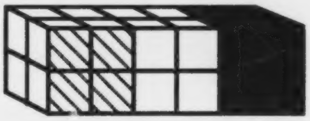
dehydration 2	*length*	*surface area*	*volume*
original model … baby			
model doubled in scale … adult			
model doubled in scale again …			
and again …			
and again …			



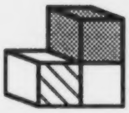
Why does the area go up by a factor of 4? and the volume by a factor of 8?

Look at the 2 models.

- Each single unit of skin in the little one is replaced by 4 units of skin in the larger one, i.e. a 1x1 square is replaced by a 2x2 square.
- Each single unit of volume in the little one is replaced by 8 units of volume in the larger one, i.e. a 1x1x1 cube is replaced by a 2x2x2 cube.



Discuss: would this be true for shapes other than our simple one?
Take a slightly more complex shape.



What happens when we double it in scale? Try it ...

But what if it's not just doubling?

We have begun to see what's happening when simple things are doubled in scale … but, an adult probably isn't simply double the scale of a baby—we just assumed that at the beginning to make life easier.

Can we predict what will happen if the adult is 3, or 10 or 100 times the scale of the baby?

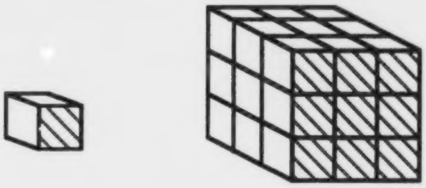
HO17

Use the blocks to treble the model.
Then try to use the pattern to fill in the rest of the table.

dehydration 3	*length*	*surface area*	*volume*	*surface area ÷ volume**
original model ... baby				
model doubled in scale ... adult				
model tripled in scale				
model x 10				
model x 100				

* The last column gives a measure of how many units of available skin are able to lose water compared to each unit of volume.

So what does this mean about dehydration when the scaling factor is 3?



Again, look at the two models.

Each single unit of skin in the little one
is replaced by 9 units of skin in the larger one
i.e. a 1x1 square is replaced by a 3x3 square;

and each single unit of volume in the little one
is replaced by 27 units of volume in the larger one
i.e. a 1x1x1 cube is replaced by a 3x3x3 cube.

So, as we treble, which is growing faster, the skin or the volume?

Which then, of these people would dehydrate fastest? The same one would also cool fastest, having the most skin in relation to its volume, and so losing heat as well as water.

Which person would cool the most slowly? Could there be problems?
Is it likely that such a giant could exist? Why?

224

HO17
page 4

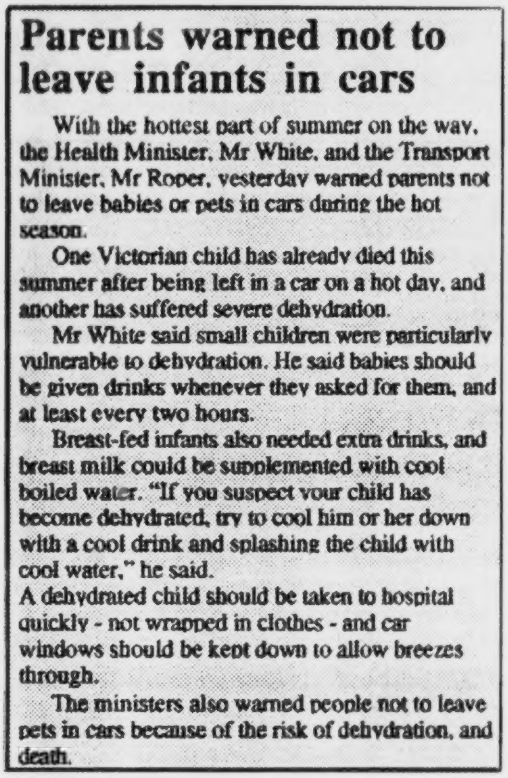
So, what does all this mean?

It means that ...
a large object has more volume in relation to its surface area
(or less surface area in relation to its volume)
than a similar small one.

So, lets return to our original question! Discuss ...

Why do babies dehydrate faster than adults in summer?

Some newspaper extracts



Parents warned not to leave infants in cars

With the hottest part of summer on the way, the Health Minister, Mr White, and the Transport Minister, Mr Roper, yesterday warned parents not to leave babies or pets in cars during the hot season.

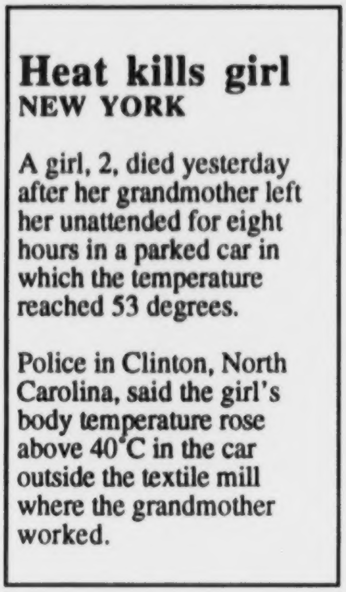
One Victorian child has already died this summer after being left in a car on a hot day, and another has suffered severe dehydration.

Mr White said small children were particularly vulnerable to dehydration. He said babies should be given drinks whenever they asked for them, and at least every two hours.

Breast-fed infants also needed extra drinks, and breast milk could be supplemented with cool boiled water. "If you suspect your child has become dehydrated, try to cool him or her down with a cool drink and splashing the child with cool water," he said.

A dehydrated child should be taken to hospital quickly - not wrapped in clothes - and car windows should be kept down to allow breezes through.

The ministers also warned people not to leave pets in cars because of the risk of dehydration, and death.



Heat kills girl

NEW YORK

A girl, 2, died yesterday after her grandmother left her unattended for eight hours in a parked car in which the temperature reached 53 degrees.

Police in Clinton, North Carolina, said the girl's body temperature rose above 40°C in the car outside the textile mill where the grandmother worked.

Some related questions ...

- If King Kong is seven times the scale of an ordinary gorilla, will he need seven times the stuffing and seven times the fur?
- Why do engineers use mercury instead of water in small-scale models of dams?
- Why do beached whales die?
- Why do polar bears tend to be bigger than other bears?
- Why are large buildings often taller and more slender than small ones?

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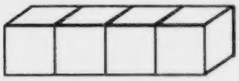
Why do chips fry faster than whole potatoes?

HO18

Arrange 4 cubes in each of the ways shown below:

a)



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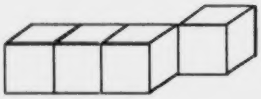
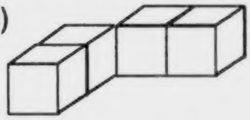
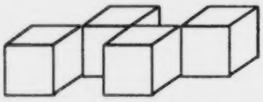


c)



d)



- Which model is the bumpiest? which is the smoothest?
- Using the models, find the volume and surface area of each of the figures.
- What is the mass of each figure if each cube weighs one gram?

	volume	*surface area*	*mass*
a			
b			
c			
d			

- Does the mass of each model depend on: its surface area or on its volume?
- Does the amount of paint needed to cover each model depend on:
  its surface area or on its volume?
- Does the bumpiness of a model affect its surface area or its volume?

Discuss

If each of the models above was in fact an earthworm that breathed through its skin:
which would require the most oxygen to live?
which has the most skin to take in the most oxygen?
which would dehydrate fastest in a hot summer?

Some related questions

- Why do chips fry faster than whole potatoes?
- Why do penguins cluster together in large groups to keep warm?
- Why did some dinosaurs have fins on their backs?

Floating or drowning HO19

**A fly can sit on the surface of water, but a person will often sink.
On the other hand, a person covered by water hardly notices it,
though a fly covered by water cannot move.
Why?**

First,
does the mass of anything, person or fly, depend on surface area or volume?

And what about the amount of water covering something, person or fly?
Does that depend on surface area or volume?

Using the following information, see if you can fill in the gaps in the table below and work out why the fly floats on a lot of water and drowns in a little.

1 the water covering the fly or person is about 0.05 cm (or $\frac{1}{2}$ mm) thick.

2 1 cubic centimetre of water weighs about 1 gm. (Did you know that?)
How much, then, do 1000 cubic centimetres of water weigh?
What is another name for this amount of liquid?

	fly	*person*
mass	0.01 g	70 kg
volume	0.04 cm^3	80 000 cm^3
surface area	0.645 cm^2	9675 cm^2
volume of water (cm^3) on the …		
mass of water (grams) on the …		
ratio of mass of 'covering' water to the mass of the …		
would water feel heavy to the …?		

So, is it sink or swim for the fly? and why?

The tale of the polyploid horse

P5

page 1

From Bateson 1979, *Mind and Nature: a necessary unity*, pp. 54–57, © E.P. Dutton, New York.

Sometimes small is beautiful

Perhaps no variable brings the problems of being alive so vividly and clearly before the analyst's eye as does size. The elephant is afflicted with the problems of bigness; the shrew, with those of smallness. But for each, there is an optimum size. The elephant would not be better off if he were much smaller, nor would the shrew be relieved by being much bigger. We may say that each is addicted to the size that is.

There are purely physical problems of bigness or smallness, problems that affect the solar system, the bridge, and the wristwatch. But in addition to these, there are problems special to aggregates of living matter, whether these be single creatures or whole cities.

Let us first look at the physical. Problems of mechanical *instability* arise because, for example, the forces of gravity do not follow the same quantitative regularities as those of cohesion. A large clod of earth is easier to break by dropping it on the ground than is a small one. The glacier grows and therefore, partly melting and partly breaking, must begin a changed existence in the form of avalanches, smaller units that must fall off the larger matrix. Conversely, even in the physical universe, the very small may become unstable *because* the relation between surface area and weight is nonlinear. We break up any material which we wish to dissolve because the smaller pieces have a greater ratio of surface to volume and will therefore give more access to the solvent. The larger lumps will be the last to disappear. And so on.

To carry these thoughts over into the more complex world of living things, a fable may be offered.

> *They say the Nobel people are still embarrassed when anybody mentions polyploid horses. Anyhow, Dr. P. U. Posif, the great Erewhonian geneticist, got his prize in the late 1980s for jiggling with the DNA of the common cart horse (Equus caballus). It is said that he made a great contribution to the then new science of transportology. At any rate, he got his prize for creating —no other word would be good enough for a piece of applied science so nearly usurping the role of deity—creating, I say, a horse precisely twice the size of the ordinary Clydesdale. It was twice as long, twice as high, and twice as thick. It was a polyploid, with four times the usual number of chromosomes.*
>
> *P. U. Posif always claimed that there was a time, when this wonderful animal was still a colt, when it was able to stand on its four legs. A wonderful sight it must have been! But anyhow, by the time the horse was shown to the public and recorded with all the communicational devices of modern civilization, the horse was not doing any standing. In a word, it was too heavy. It weighed, of course, eight times as much as a normal Clydesdale.*
>
> *For a public showing and for the media, Dr. Posif always insisted on turning off the hoses that were continuously necessary to keep the beast at normal mammalian temperature. But we were always afraid that the innermost parts would begin to cook. After all, the poor beast's skin and dermal fat were twice as thick as normal, and its surface area was only four times that of a normal horse, so it didn't cool properly.*
>
> *Every morning, the horse had to be raised to its feet with the aid of a small crane and hung in a sort of box on wheels, in which it was suspended on springs, adjusted to take half its weight off its legs.*
>
> *Dr. Posif used to claim that the animal was outstandingly intelligent. It had, of course, eight times as much brain (by weight) as any other horse, but I could never see that it was concerned with any questions more complex than those which interest other horses. It had*

> *very little free time, what with one thing and another—always panting, partly to keep cool and partly to oxygenate its eight-time body. Its windpipe, after all, had only four times the normal area of cross section.*
>
> *And then there was eating. Somehow it had to eat, every day, eight times the amount that would satisfy a normal horse and had to push all that food down an oesophagus only four times the calibre of the normal. The blood vessels, too, were reduced in relative size, and this made circulation more difficult and put extra strain on the heart.*
>
> *A sad beast.*

The fable shows what inevitably happens when two or more variables, whose curves are discrepant, interact. That is what produces the interaction between change and tolerance. For instance, gradual growth in a population, whether of automobiles or of people, has no perceptible effect upon a transportation system until suddenly the threshold of tolerance is passed and the traffic jams. The changing of one variable exposes a critical value of the other.

Of all such cases, the best known today is the behaviour of fissionable material in the atom bomb. The uranium occurs in nature and is continually undergoing fission, but no explosion occurs because no chain reaction is established. Each atom, as it breaks, gives off neutrons that, if they hit another uranium atom, may cause fission, but many neutrons are merely lost. Unless the lump of uranium is of critical size, an average of less than one neutron from each fission will break another atom, and the chain will dwindle. If the lump is made bigger, a larger fraction of the neutrons will hit uranium atoms to cause fission. The process will then achieve positive exponential gain and become an explosion.

In the case of the imaginary horse, length, surface area, and volume (or mass) become discrepant because their curves of increase have mutually nonlinear characteristics. Surface varies as the square of length, volume varies as the cube of length, and surface varies as the 2/3 power of volume.

For the horse (and for all real creatures), the matter becomes more serious because to remain alive, many internal motions must be maintained. There is an internal logistics of blood, food, oxygen, and excretory products and a logistics of information in the form of neural and hormonal messages.

The harbour porpoise, which is about three feet long, with a jacket of blubber about one inch thick and a surface area of about six square feet, has a known heat budget that balances comfortably in Arctic waters. The heat budget of a big whale, which is about ten times the length of the porpoise (i.e. 1000 times the volume and 100 times the surface), with a blubber jacket nearly twelve inches thick, is totally mysterious. Presumably, they have a superior logistics system moving blood through the dorsal fins and tail flukes, where all cetaceans get rid of heat.

The fact of growth adds another order of complexity to the problems of bigness in living things. Will growth alter the proportions of the organism? These problems of the limitation of growth are met in very different ways by different creatures.

A simple case is that of the palms, which do not adjust their girth to compensate for their height. An oak tree with growing tissue (cambium) between its wood and its bark, grows in length and width throughout its life. But a coconut palm, whose only growing tissue is at the apex of the trunk (the so-called millionaire's salad, which can only be got at the price of killing the palm), simply gets taller and taller, with some slow increase of the bole at its base. For this organism, the limitation of height is simply a normal part of its adaptation to a niche. The sheer mechanical instability of excessive height without compensation in girth provides its normal way of death.

Maps, models, theories

OHT3

Like a mathematician considering more general kinds of maps, every map-maker must compromise by deciding which features of representation are most important for particular purposes.

Senechal 1990, 'Shape' in Steen (ed.) *On the Shoulders of Giants: new approaches to numeracy*, p. 163

My belief—theory?—about theories is that they are like maps that help us to understand and navigate new waters. They are powerful tools for action, they can help us develop and reflect on practice. But just as different maps of the same waters stress and ignore—or even distort—different features, so any theory is necessarily partial, incomplete and interested (even this one!).

Johnston 1995, 'Which map shall I use?', *Numeracy in Focus*, No. 1

The language of nature

P 6

page 1

From Peterson 1988, *The Mathematical Tourist: snapshots of modern mathematics*, pp. 115–6,
© Ivars Peterson, 1988.

The flickering flames of a campfire highlight the jagged forms of nearby rocks. The smell of frying fish weaves through the still air. Clusters of gnarled pines crouch on the ragged, stony shore. Patches of golden wildflowers, scattered across a meadow, glow in the last light of a weary sun. At the far end of the lake, a saw-toothed range of towering mountain peaks, crowned with agglomerated ice, tear into the sky. From distant, billowy clouds, a flash of lightning zigzags through the air.

This mountain landscape, like many natural scenes, has a roughness that's hard to capture in the classical geometry of lines and planes, circles and spheres, triangles and cones. Euclidean geometry, created more than 2,000 years ago, best describes a human-made world of buildings and other structures based on straight lines and simple curves. Although smooth curves and regular shapes represent a powerful abstraction of reality, they can't fully describe the form of a cloud, a mountain, or a coastline. In the words of mathematician Benoit B. Mandelbrot, "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."

The language of nature

A close look shows that many natural forms, despite their irregular or tangled appearance, share a remarkable feature on which a new geometry can be hung. Clouds, mountains, and trees wear their irregularity in an unexpectedly orderly fashion. Nature is full of shapes that repeat themselves on different scales within the same object.

A fragment of rock looks like the mountain from which it was fractured. Clouds keep their distinctive appearance whether viewed from the ground or from an airplane window. A tree's twigs often have the same branching pattern seen at the tree's trunk. Elms, for instance, have two branches coming out of most forks. In a large tree this repeated pattern, on every smaller scale, may go through seven levels, from the trunk to the smallest twigs. Similar branching structures can be seen in the human body's system of veins and arteries, and in maps of river systems.

In all these examples, zooming in for a closer view doesn't smooth out the irregularities. Instead, the objects tend to show the same degree of roughness at different levels of magnification. Mandelbrot, the first person to recognise how extraordinarily widespread this type of structure is in nature, introduced the term self-similar to describe such objects and features. No matter how grainy, tangled, or wrinkled they are, the irregularities are still subject to strict rules.

In 1975, Mandelbrot coined the word *fractal* as a convenient label for irregular and fragmented self-similar shapes. Fractal objects contain structures nested within one another. Each smaller structure is a miniature, though not necessarily identical, version of the larger form. The mathematics of fractals mirrors this relation between patterns seen in the whole and patterns seen in parts of that whole.

Fractals turn out to have some surprising properties, especially in contrast to geometric shapes such as spheres, triangles, and lines. In the world of classical geometry, objects have a dimension expressed as a whole number. Spheres, cubes, and other solids are three-dimensional; squares, triangles, and other plane figures are two-dimensional; lines and curves are one-dimensional; and points are zero-dimensional. Measures of size—volume, area, and length—also reflect this fundamental classification.

Fractal curves can wriggle so much that they fall in the gap between two dimensions. They can have dimensions anywhere between one and two, depending on how much they meander. If the curve more closely resembles a line, then it is rather smooth and has a fractal dimension close to 1. A curve that zigzags wildly and comes close to filling the plane has a fractal dimension nearer to 2.

Similarly, a hilly fractal scene can lie somewhere between the second and third dimensions of classical geometry. A landscape with a fractal dimension close to 2 may show a huge hill with tiny projecting bumps, whereas one with a fractal dimension close to 3 would feature a rough surface with many medium-size hills and a few large ones. A higher fractal dimension means a greater degree of complexity and roughness. But a fractal dimension is never larger than the Euclidean dimension of the space in which the fractal shape is embedded: a hilly scene would never have a dimension greater than 3. In general, fractal geometry fills in the spaces between whole-number dimensions.

Maps of a rugged coastline illustrate another curious property of fractals. Finer and finer scales reveal more and more detail and lead to longer and longer coastline lengths. On a world globe, the eastern coast of the United States looks like a fairly smooth line that stretches somewhere between 2,000 and 3,000 miles. The same coast drawn on an atlas page showing only the United States looks much more ragged. Adding in the lengths of capes and bays, its extent now seems more like 4,000 or 5,000 miles. Piecing together detailed navigational charts to create a giant coastal map reveals an incredibly complex curve that may be 10,000 or 12,000 miles long. A person walking along the coastline, staying within a step of the water's edge, would have to scramble more than 15,000 miles to complete the trip. A determined ant taking the same shoreline expedition but staying only an ant step away from the water may go 30,000 miles. Tinier coastline explorers even closer to the shoreline would have to travel even farther.

This curious result suggests that one consequence of self-similarity is that the simple notion of length no longer provides an adequate measure of size. Although it's reasonable to consider the width of a bookcase as a straight line and to assign to it a single value, a fractal coastline can't be considered in this way. Unlike curves of Euclidean geometry, which become straight lines when magnified, the fractal crinkles of coastlines, mountains, and clouds do not go away when observed closely. If a coastline's length is measured in smaller and smaller steps or with shorter and shorter measuring sticks, its length grows without bound. Because it wiggles so much, the true length of a fractal coast is infinite. Length, normally applied to one-dimensional objects such as curves, doesn't work for objects with a fractal dimension greater than 1.

Fractal geometry doesn't prove that Euclidean geometry is wrong. It merely shows that classical geometry is limited in its ability to represent certain aspects of reality. Classical geometry is still a handy way to describe salt crystals, which are cubic, or planets, which are roughly spherical and travel around the sun in elliptic orbits. Fractal geometry, on the other hand, introduces a set of abstract forms that can be used to represent a wide range of irregular objects. It provides mathematicians and scientists with a new kind of meter stick for measuring the exploring nature.

Rugged coastlines

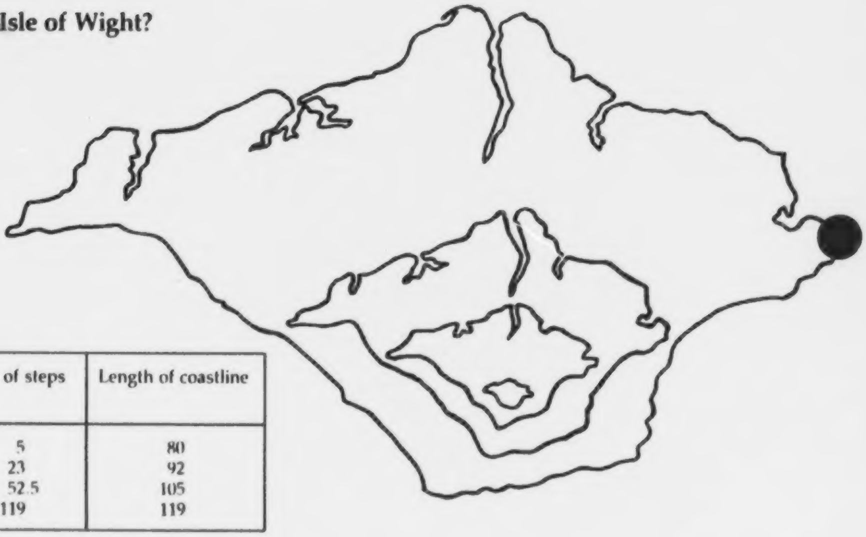
P7

From Hemmings & Tahta 1984, *Images of Infinity*, p. 76, © Leapfrogs, London.

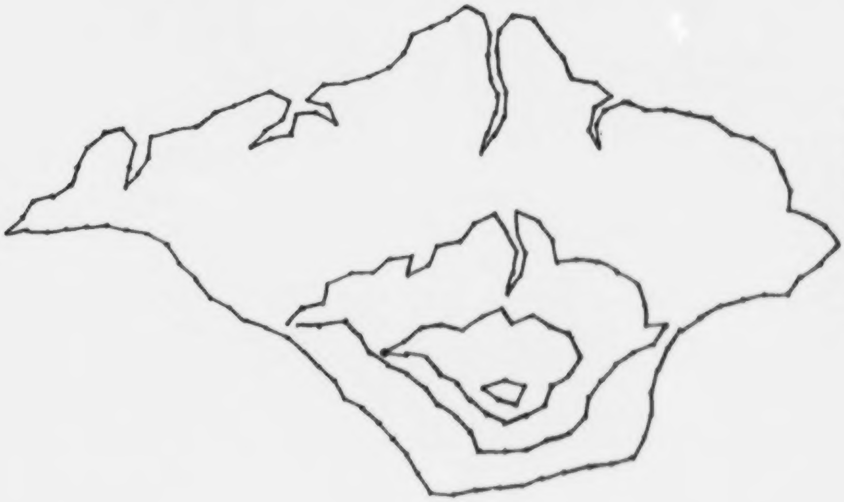
How long is the coastline of the Isle of Wight?

The coastline has been measured off maps of various scales by 'stepping' round with steps of constant length (0.4 cm).

The conversions to actual lengths (in km) are shown in the table below.



Scale of map (1 cm =)	Length of a 0.4 cm step	No. of steps	Length of coastline
40 km	16 km	5	80
10 km	4 km	23	92
5 km	2 km	52.5	105
2.5 km	1 km	119	119



The smaller the steps, the closer we get to the actual coastline, so the measurement gets more accurate.
But the table suggests that as the length of the steps gets smaller the length of the coastline gets steadily greater.
Is there a limit to this—or is the coastline infinitely long?

Why your tape measure isn't long enough for mother nature

Graham Phillips, © *The Sydney Morning Herald*

You might think it's fairly easy to work out the length of Australia's coastline. All you need is a nice big map and a piece of string.

Lay the string carefully on the map and then measure the length of the string. It sounds simple but your answer would be quite wrong.

Think about what would happen if you took a much larger map and did the same thing. Of course, your string would need to be longer, but once you took the scale of the map into account you might think you would end up with the same answer.

The problem is you don't. On the bigger map, Australia's coastline seems to be much longer.

This is because more detail is shown on a larger map: there are a lot more wiggles and bumps. On a smaller map the string cuts across this detail.

The larger the map, the greater the variation and the longer the coastline will measure.

Even if you took an enormous tape measure and walked around Australia's coastline you still wouldn't come up with an absolutely accurate figure.

You can see why by examining the coastline through a magnifying glass: you discover there is still more intricate detail you have missed.

By using stronger and stronger magnifications you can make the coastline as long as you like. In effect it is infinitely long.

You may think this is being pedantic, but countries often disagree on the length of common borders. It all depends on the scale on which the countries do their measuring.

Obviously, this causes headaches for geographers. Standard high school geometry just can't deal with shapes that have infinitely long edges.

For this reason mathematicians have invented a radically new type of geometrical shape, called the fractal.

Fractals are among the weirdest shapes imaginable. Just like coastlines, their edges are infinitely long and their detail is infinitely intricate. The closer you look, the more detail you see.

When they were first invented, fractals were considered to be nothing more than weird mathematical oddities. But in recent years scientists have discovered that fractal shapes are extremely common in nature—the shape of a cloud is best described as a fractal; the path boiling water follows as it seeps through the coffee in your coffee-maker is the shape of a fractal.

In fact, fractal geometry is playing a vital role in the development of an exciting new frontier in physics—the study of complexity.

This new frontier has developed from a recent break through in physics which has shown that some seemingly complex phenomena may in fact have exceedingly simple explanations.

For example, it may be possible to explain the complicated motion of turbulent waterfalls by a few simple rules. The turbulent eddies themselves may show fractal behaviour.

Fractals come in many varieties. Just as there are different shapes in standard geometry, there are different fractal shapes.

Try constructing the following simple fractal. Draw a triangle with equal sides. Rub out the middle third of each side and replace it by a smaller equal-sided triangle, as in the illustration at the bottom of the [next page]. Then rub out the middle third of each side of the figure you have constructed and draw even smaller triangles in the gaps.

Imagine you could repeat these steps over and over again for ever. The shape you would create, the Koch curve, is a simple fractal.

Just like a coastline, the outer edge of the Koch curve is infinitely long, and the more you magnify it the more detail you see.

Another interesting feature of a fractal is that the pattern is the same at higher and higher magnification. No matter what strength magnifying glass you use, the Koch curve always looks the same.

One very practical application of fractals is in the oil industry. To enhance recovery from an oil field, water is injected into the middle of the field. This bubble of water helps drive the oil to the surface where it can be easily extracted. You might think this water bubble would just be a shapeless blob, or at best a sphere. Instead, the water forms a beautiful and intricate pattern of long fingers and branches. Magnifying any of the fingers reveals even more intricate fingers and branches. The shape of the water bubble is a fractal.

The more we learn about fractals, the more we discover just how much mother nature seems to like them. They appear everywhere. The task now is to discover just why they occur.

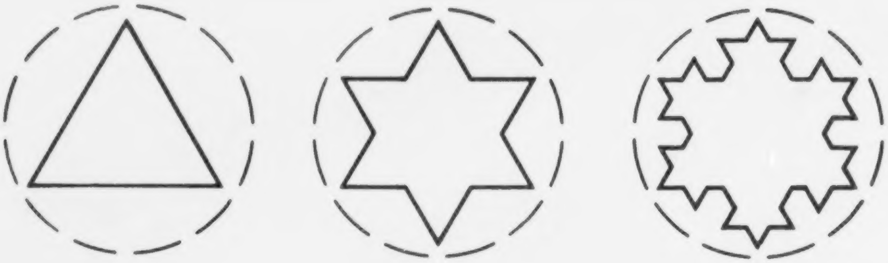
The pathological snowflake HO20

Adapted from Perl 1978, *Math Equals: biographies of women mathematicians and related activities*, pp. 146–7, © Addison-Wesley, California.

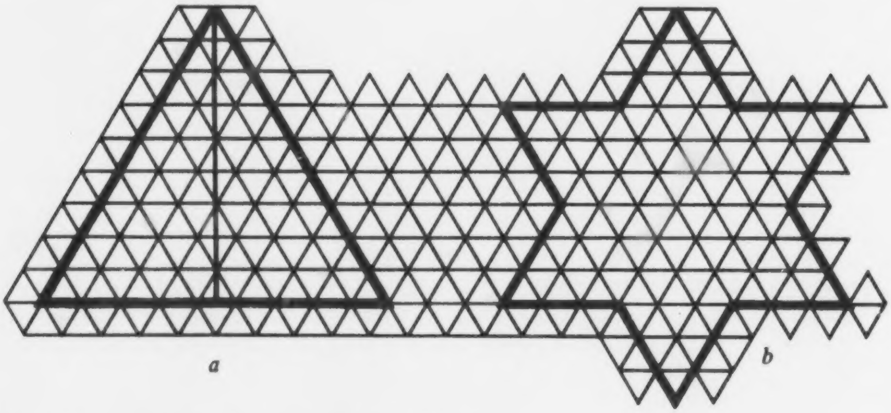
The snowflake is generated in the following way:

1 Start with an equilateral triangle.
2 Divide each edge of this triangle into thirds.
Build, off each middle third, another equilateral triangle of which the original middle third is one edge.
3 Continue this process ad infinitum.

The first three stages of this sequence will look like this.



The figure below shows the first two stages of the snowflake as drawn on isometric paper.



Try this yourself, at least until the fourth division.
Isometric paper is available on AS20.

Isometric paper

✣ AS20

Invariants

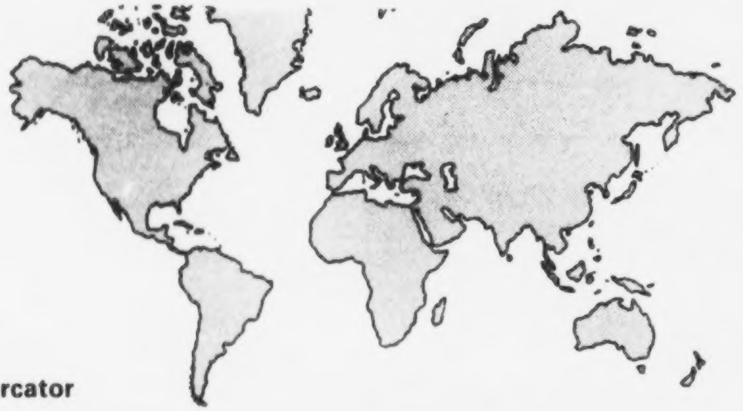
P9

page 1

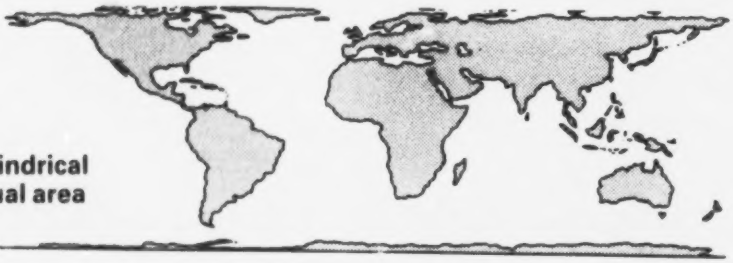
From Wells 1988, *Hidden Connections, Double Meanings: a mathematical exploration*, pp. 55–56,
© Cambridge University Press, Cambridge.

An experiment with an old hollow rubber ball and a knife or scissors will suggest, correctly, that the surface of a sphere cannot be flattened into a sheet without some distortion. Indeed, even a small piece of the surface, such as one panel of a football, will be distorted a little if forced flat. It will either be wrinkled, or stretched, or may be torn as well.

The entire surface of the earth must also be distorted before it can be displayed flat. Cartographers therefore face a tricky problem. Some features of their maps are going to be wrong. How many? What features can possibly be preserved? Although the news for mapmakers is grim, it could be worse. It turns out that they have a choice of which features they will preserve. The maps below illustrate two possibilities.



Mercator



Cylindrical equal area

The Mercator projection was introduced in 1567. It is probably the best known of all projections and is widely used in atlases and wall charts. It has the immense advantage for navigators that courses of constant bearing, called loxodromes or rhumb lines, become straight lines on the map. The navigator can draw a straight line from the ship's present position to its destination, measure the bearing, set the automatic compass, and go to sleep.

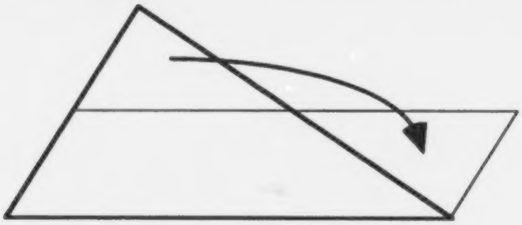
(This does not entirely solve the navigator's problems. The shortest distance between two points at sea is an arc of a great circle which is not a course of constant bearing. However, a great circle course can easily be closely approximated by several shorter courses of constant bearing.)

Mercator's projection inevitably has disadvantages also. The comparison of areas is especially difficult, because the lines of latitude move further apart as they approach the poles. A well-known absurdity concerns the relative sizes of Greenland and South America. On a Mercator map, Greenland appears rather the larger of the two. In actual fact is only about one tenth the area of the South American continent. Loxodromes are an invariant of the transformation which produces Mercator's projection. Area is not invariant.

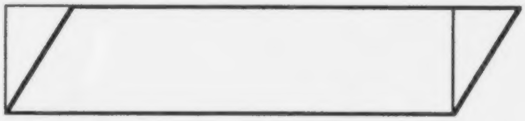
The cylindrical equal area projection does preserve areas, though it changes their shape. Loxodromes are no longer invariant and this cylindrical projection would be a navigator's nightmare. Greenland now appears accurately small, while South America which straddles the equator is little changed. Europe itself is somewhat reduced in size because on Mercator it is far enough from the equator to be slightly enlarged. In contrast Africa which also crosses the equator is about the same size, and so appears relatively larger than Europe, as it should do. The atlases of my and many readers' childhoods made Europe larger than it is, and also more important by placing it in the centre of the map.

Mathematicians are continually searching for ways to transform one object into another, or one problem into another. If some features remain their old familiar selves, so much the simpler.

One of the simplest invariants in all of elementary mathematics is area. When a plane shape is cut up and reassembled in a different way the area does not change. This is how mathematicians have always calculated the area of triangles and parallelograms and other simple shapes.



Here is one way to turn a triangle, any triangle, into a parallelogram. The area has not changed, it has merely been rearranged.



The parallelogram can in turn be transformed into a rectangle by dissection. Here a right-angled triangle is cut off one end and placed at the other.

The original parallelogram and the new rectangle have the same area.

Curricular issues

From Senechal 1990, 'Shape' in Steen (ed.), *On the Shoulders of Giants: new approaches to numeracy*, pp. 171–174, © National Academy Press, Washington, DC.

Curricular issues

Students should learn to recognise the patterns of shape, to understand the principles that govern their construction, and to be able to move easily back and forth between shapes and their images. Although the study of shapes seems to fall between the cracks of traditional subjects, the new *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics reflect an emerging consensus that this situation must be improved.

The study of shape must be more than the sum of its parts; an integrated view of shape can help accentuate the whole subject. One possible approach is illustrated by the chart in Figure 30.

A Structure for Shapes

Identification and Classification

ELEMENTARY:	INTERMEDIATE:	ADVANCED:
Circles	Spheres	Surfaces
Plane Polygons	Zig-zag and star polygons; knots	Helices, spirals, cylinders, tori, Mobius bands
Polyhedra	Polyhedra	Polyhedra
Puzzles	Tiling the plane with polygons	Escher-like tilings
	Networks	Simple crystal structures
Congruence; similarity		
Soap bubbles	Soap bubble clusters	Orientation, genus texture

Analysis

ELEMENTARY:	INTERMEDIATE:	ADVANCED:
Mirror symmetry; rotational symmetry	Two-mirror kaleidoscopes	Polyhedral kaleidoscopes
Congruence	Symmetry of finite figures	Symmetry as an organizing principle; transformation geometry
Paper-folding; patterns	Dissection; puzzles	
Similarity	Rep-tiles; fractals	Exploring fractals
	Natural patterns	Scale in biology
Constructing and deconstructing polyhedra	Regular and semiregular polyhedra	Euler's formula for polyhedra
Linear/volume measurement	Angle measurement	Fundamentals of plane and 3-D geometry
Making quilts and mosaics	Tiling the plane with polygons	Lattices; elementary tiling theory

Representation and Visualization

ELEMENTARY:	INTERMEDIATE:	ADVANCED:
Model-making	Model-making	Model-making
Drawing, reading, and using simple maps	Relief maps and level curves	Cross-sections of 3-D shapes structures
	The globe	Geometry of the sphere; projections; maps
Shadows	Shadow geometry	Images and image reconstruction; impossible figures
Drawing	Perspective drawing	Technical drawing; stereoscopes
Scale projectors	Telescope and microscope	Lens geometry; the camera
	Plane coordinates	3-D coordinates
Turtle geometry	Exploring geometry with the computer	More computer graphics

Figure 30
An arrangement of topics related to shape that provides structure and coherence to what might otherwise appear as an arbitrary collection of quite disparate topics.

Forging connections

Rethinking the subject as a whole provides us with an opportunity to forge substantive connections between the study of shape and the role of shape in the real world. We can take seriously Arnheim's plea for integrating art and science. We can also reduce the mystery of some of our contemporary technology. The principles of the electron microscope, the radio telescope, and ultrasound are not wholly beyond the scope of the K–12 curriculum; high school students can, if we wish, learn the foundation necessary to understand the action of these and other modern imaging techniques.

Indeed a focus on shape makes many aspects of modern technology much more accessible than is commonly supposed. Here are just three examples of important shapes whose key features could easily be taught in our schools.

The *silicon chip*, which has transformed the industrialised world in just a few decades, is based on a structure that is a carrier of incredibly miniaturised circuits. Although the circuits themselves are complex, the crystal structure of the silicon that houses them is a simple modular structure.

For example, crystalline silicon is built of linked zig-zag hexagonal rings (Figure 31), which are easy to make and instructive to study. In the silicon structure the rings are linked to form cage-like polyhedra. Elementary school children can learn to build and identify these substructures, middle school children can learn to put them together, and high school children can study the relation between the silicon structure and the properties that make it so useful.

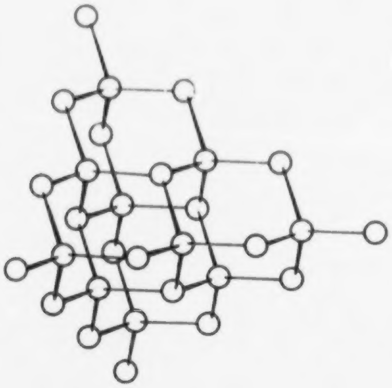


Figure 31 The structure of crystalline silicon. It is made entirely of zig-zag hexagons. This is also the structure of diamond (with carbon atoms at these positions instead of silicon).

The *CAT scan* and other forms of computer-assisted image reconstruction have revolutionised medical diagnosis in recent years. While diagnosis by X-ray is an exercise in reading shadows, diagnosis by CAT scan is an exercise in reconstructing images from their cross sections. Like the circuitry on a silicon chip, the image reconstruction used in this technology is a complex process, but the simplest geometrical principles that underlie it are easily understood.

Here again we find that the same geometric principles are central to many fields. For example, the construction of shapes from sections and shadows has been the task of architects and builders for centuries. While it is not feasible to bring a CAT scan machine or a construction site into the classroom, many projects suitable for school can help students understand the relation between shadow or cross section and shape.

Snowflakes, especially the feathery ones, are enchanting. Children often learn to make paper snowflakes in school, an exercise that can easily be extended to a study of symmetry. The hexagonal symmetry of the snowflake provides an introduction to the symmetry of polygons; it is an ideal subject for the elementary classroom.

But the snowflake has much more to teach us. In the first place a snowflake looks like a pattern we might see in a kaleidoscope, and so it is an application of the principles of mirror geometry. These same reflection principles undergird contemporary technology: one need only think of the reflection beams of burglar alarms and lasers or of radar and sonar. Middle school children can easily understand and appreciate such applications. At the high school level the emergence of hexagonal symmetry from aggregates of water molecules can be explored and so can the crystals' dendritic growth, or branching.

The branching of the snowflake is as characteristic as its symmetry and is equally significant in the study of shape. First, corners of the snowflake sprout beyond a hexagonal "core." Then these branches themselves sprout branches, the branches of the branches branch, and so forth (Figure 32). The result is a structure in which a certain feature—branching—is increasingly repeated on a smaller and smaller scale. If this process could be repeated indefinitely, the result would be a self-similar structure; indeed, the snowflake is a fractal at an early stage of its development.

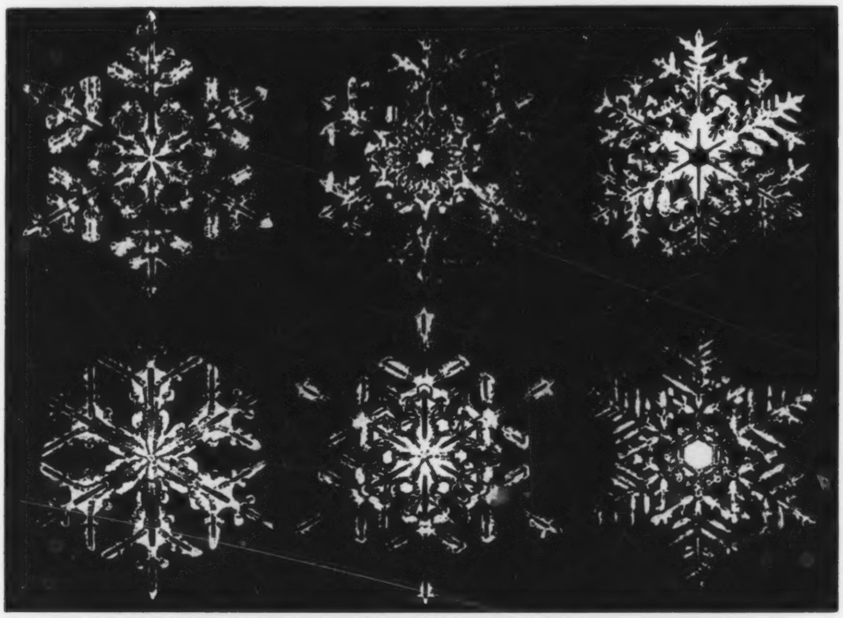


Figure 32 Branched snowflakes reveal the familiar hexagonal symmetry of ice crystals repeated fractal-like at every scale.

A map of the journey

HO21
page 1

a conducted tour and some possible detours
the first stage

- circles can be seen as polygons with an infinite number of sides ... investigate infinity

we talked about polygons and other shapes in Module A
what are these names, and where did they come from?
shaping our world
what shapes do we name?

- some numbers are known as square numbers, others as triangular ... find out about them ... what about pentagonal numbers ...?
  - find out about Johann Kepler and his six spheres

making polygons by
playing with mirrors
and finding
angles, hexagons, patterns, symmetry
... and why is symmetry interesting?

- angles: a whole turn is 360 degrees ... why 360?
  - what's so special about right angles?
- use a single mirror to investigate line symmetry
- how does symmetry happen in the real world?

- how are shape and space important in Aboriginal or Inuit or Navajo cultures?
  - investigate navigation techniques

where do we find these shapes?
and what shapes don't we name?
different worlds
different needs, different categories

- find out about mosaics in other cultures
  - make yourself some Escher-type tiles
  - what about combinations of regular polygons? which combinations will tessellate? ... squares with triangles? ... others?

those hexagons
bees cells:
why hexagonal?
angles are interesting here...

- why are most Australian houses made up of rectangular or square rooms?
- why do some African houses consist of a single round room?
  - look at patchwork

OK, so they fit, but so do triangles and squares.
why not them?
'a sense of frugality'
and back to the question of boundaries

'any quadrilateral can be used as a tile.' is this true?... why?
- investigate how or whether polyhedra can fill space
- what are the Archimedean solids?

A map of the journey

HO21
page 2

the second stage

our society likes to measure
measuring flat spaces
we measure angles, areas, edges

- what don't we measure in our society? others? collect and investigate a variety of units of measurement

if area is to do with covering,
then let's use mosaics
getting square
use what you know—rectangles—
to find what you don't know

why don't we use circles as a unit of area to cover surfaces ? (not just because they leave spaces between)

- starting from rectangles, derive formulas for the areas of triangles, parallelograms, trapeziums
- reverse the process in *Getting Square* and make your own Escher-type tile by distorting the rectangle ...

and talking of squares
there's all this stuff about
right-angled triangles
who was Pythagoras
anyway?
and was he the first to use this
relationship?

- teach yourself to draw perpendicular and parallel lines
- who first used the Pythagorean relationship and why? and is it still used?

- find out about the widespread use of the Pythagorean relationship in decorative arts
- investigate the toothed square ... and its use in yet another proof of Pythagoras

well, that takes care of shapes
with straight edges
but how about the not so straight ...?
what about circles and pi?
and time for a bit of body maths

- is pi rational?
- what is an ellipse? in what sense is it a special case of a circle? where do we find ellipses in nature?
- what is curve-stitching? try it ...
- find out about conic sections ... make some

just a reminder about the
relationship that exists—or
doesn't!—between area and
perimeter
sheep in a hole
and could it be an elephant?

- another strange number like pi is the Golden Ratio ...what is it and how is it connected with the Fibonacci sequence?
- well, could it be an elephant? is there a limit to the size of the hole you could make? (this connects to coastlines ... see later)
- a good chance to take off to infinity ... where have we already encountered it in this course? could we do without it?

and what if you weren't
interested in measuring
insides and outsides–
how else can they possibly be
interesting?
try twisted handcuffs
and see

- find out about networks and trees ...
- can you walk through all the doors of your house before walking through any a second time?

If not all,
then draw a plan of a house
where it is possible ...

- investigate African and other sand- drawings
- E and F are topologically equivalent letters... and equivalent to 3 or 4 more capital letters (depending how you draw them) ... which ones?

A map of the journey

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page 3

the third stage

talking about sheep in holes,
and land inside fences, what
about?
measuring coastlines
– a surprisingly impossible task

- investigate the Koch snowflake and fractals
  - is there a 3-D equivalent of measuring coastlines?

- what different ways have been used to map the earth onto a plane?
- find out about the kinds of maps traditionally used by Australian Aborigines or Inuit hunter-gatherers

what was the problem of the seven bridges of Konigsberg?

- read *Flatland*

so, what do maps tell us?
distorted maps
well, find a map that isn't distorted ...
or a model for that matter

- find how squares grow
- how big were the giants that Gulliver met in his travels? is that possible?
- why do polar bears tend to be bigger than other bears?
  - fold a piece of A4 paper in half ... is it similar to the original? i.e. is it a scale model of the original? fold it again ... does the same thing happen? is this the same with foolscap paper?

some maps and models don't bother
about scale
but what are the effects if they do?
the strange case of the
dehydrating earthworms

next:
how to be a cool earthworm
arranging space
to suit skin and flesh
or vice versa

- why does a sleeping child adopt a foetal position when the covers fall off?
  - why do chips fry faster than whole potatoes? investigate the most common shapes of containers found in supermarkets ... why these?

in conclusion
making a map about maps
map upon map
reflecting on where we have been and
where we might go

A map of the journey

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references
for some possible detours

circles can be seen as polygons with an infinite number of sides ... investigate infinity	Hemmings & Tahta
some numbers are known as square numbers, others as triangular ... find out about them ... what about pentagonal numbers ...?	Jacobs ch. 2
angles- a whole turn is 360 degrees ...why 360?	Hogben
what's so special about right angles?	
use a single mirror to investigate line symmetry	Jacobs ch. 5, Fuys & Tischler ch. 3
how does symmetry happen in the real world?	Senechal, Stevens
how are shape and space important in Aboriginal or Inuit or Navajo cultures?	Ascher, Kearins & Harris P.
investigate navigation techniques	Ascher
find out about patchwork	
'any quadrilateral can be used as a tile.'is this true?... why?	Jacobs ch. 5
investigate how or whether polyhedra can fill space	Jacobs ch. 5
find out about Johann Kepler and his six spheres	Jacobs ch. 5
what are the Archimedean solids?	Jacobs ch. 5
why are most Australian houses made up of rectangular or square rooms? why do some African houses consist of a single round room?	Zaslavsky
make yourself some Escher-type tiles ...	Jacobs ch. 5
find out about mosaics in other cultures	Nelson, Joseph & Williams ch. 6, Shan & Bailey ch. 5 & 6
what about combinations of regular polygons? which combinations will tessellate? ... squares with triangles?... others?	Jacobs ch. 5

A map of the journey

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what don't we measure in our society? others? collect & investigate a variety of units of measurement	Kearins
why don't we use circles as a unit of area to cover surfaces? (not just because they leave spaces between)	Stevens ch. 8
starting from rectangles derive formulas for the areas of triangles, parallelograms, trapeziums	common textbooks, Wells p. 56
reverse the process in *Getting Square* and make your own Escher-type tile by distorting the rectangle ...	look at Escher prints
teach yourself to draw perpendicular and parallel lines	common textbooks
who first used the Pythagorean relationship and why? and is it still used?	Joseph (see index)
find out about the widespread use of the Pythagorean relationship in decorative arts	Gerdes 1988, Shan & Bailey pp. 110–113
investigate the toothed square ... and its use in yet another proof of Pythagoras	Wells pp. 24–26
is pi rational?	Joseph (see index), textbooks, Bergamini pp. 43–4
what is an ellipse? in what sense is it a special case of a circle? where do we find ellipses in nature?	Wells ch. 4
what is curve-stitching? try it ...	
find out about conic sections ... make some	Wells ch. 4, Jacobs ch. 6, Perl ch. 2
another strange number like pi is the Golden Ratio ...what is it and how is it connected with the Fibonacci sequence?	Perl pp. 144–5, Jacobs pp. 97–109
well, could it be an elephant? is there a limit to the size of the hole you could make?	... later, Perl pp. 146–7
a good chance to take off to infinity ... where have we already encountered it in this course? could we do without it?	Hemmings & Tahta
can you walk through all the doors of your house before walking through any a second time? If not all, then what is the most? draw a plan of a house where it is possible ...	Burns pp. 22–23, Jacobs ch. 10/2
find out about networks and trees ...	Jacobs ch. 10
investigate African and other sand- drawings	Ascher ch. 2, Shan & Bailey pp. 114–5, Jacobs p. 619
E &F are topologically equivalent ... and equivalent to 3 or 4 more capital letters (depending how you draw them) ... which ones?	Jacobs pp. 576–579

A map of the journey

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investigate the Koch snowflake and fractals	Perl pp. 146–7, Hemmings & Tahta pp. 54–5
is there a 3-D equivalent of measuring coastlines?	
what different ways have been used to map the earth onto a plane?	Wells ch. 7, Stevens pp. 9–10, Hogben (see index)
find out about the kinds of maps traditionally used by Australian Aborigines or Inuit hunter-gatherers	Ascher ch. 5 (Inuit)
what was the problem of the seven bridges of Konigsberg?	Jacobs p. 590
read Flatland	Abbott
find how squares grow	
how big were the giants that Gulliver met in his travels? is that possible?	Swift, Stevens pp. 16–34
why do polar bears tend to be bigger than other bears?	Stevens pp. 16-34
fold a piece of A4 paper in half- is it similar to the original? i.e. is it a scale model of the original? fold it again ... does the same thing happen?... is this the same with foolscap paper?	Dunkels
why does a sleeping child adopt a foetal position when the covers fall off?	
why do chips fry faster than whole potatoes?	
investigate the most common shapes of containers found in supermarkets ... why these?	Stevens ch. 8

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page 1

the first journey and detours

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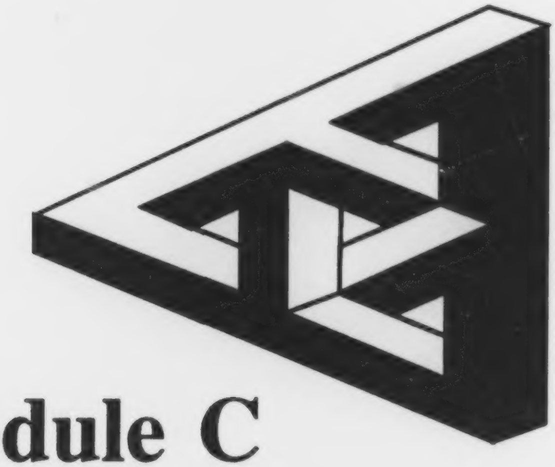

Module C
Mathematics as a critical tool

Presenter's notes

Module C: Mathematics as a critical tool

Nominal time: 27 hours

Brief description

Participants focus on the use of maths as a critical tool in the process of:
- exploring ideas of number including size, averages, sampling, measurement, estimation and graphs
- becoming familiar with available resources
- excavating mathematics from a range of contexts.

Rationale/aims

This section consists of two strands, woven together: a series of discussions about learners, their needs and learning contexts, and a Mathematical Journey in three parts. As an integral part of the whole process, the presenter and participants engage in some negotiation about what topics will be explored, both in the Group situation and in the individual or small group Curriculum Project.

The framework:
This section has been written with a particular framework of numeracy in mind. According to this framework, numeracy is a critical awareness which enables us to build a bridge between mathematics and the real world, with all its diversity. Being numerate also involves the personal responsibility of reflecting that same critical awareness in one's social practice. Thus, being numerate means being able to situate, interpret, critique, use and perhaps even create maths in context, taking into account all the mathematical as well as social and human complexities which come with that process. This framework recognises that we live in a world where mathematics is a body of knowledge much valued for its inaccessibility to all but a privileged few; thus bridges have to be built across the gulf between the 'bits' of the teachers' mathematical knowledge rationed out to us via school maths, and the realities and values which we need to live with and negotiate. We call this bridge 'numeracy'. Numeracy helps us to activate the mathematical knowledge within us and to make meaning of mathematics as presented explicitly or in disguised forms in our lives.

Strand 1: Learners, their needs and learning contexts
In this strand participants have an opportunity to think about the meaning of the word 'critical' particularly in relation to adult teaching and learning of mathematics. They will discuss reasons for—and against—learning maths and being numerate, and consider how different individual and institutional contexts influence what can be taught and learnt.

Strand 2: The Mathematical Journey
The three-part Mathematical Journey gives participants a chance to examine and explore mathematics in a variety of contexts and to consider issues about the mathematical and social appropriateness of its use.

The first part is a guided tour over three terrains: numbers as metaphors; counting as a cultural activity; and guessing, estimation and precision. The presenter will act as a tour guide and introduce the participants to these terrains by conducting three activities:
- *Food for thought*
- *Estimation as strategic guessing*
- *The average house price.*

The second part of the Journey will be along a negotiated route through a set of sites selected from such prepared activities as:

- *Your metric conversion chart*
- *A taste of sampling*
- *Precision: what does it mean?*
- *Percentages and loans: in whose interest?*
- *Graphs: information and distortion*

and others chosen from available resources such as *Numeracy and How We Learn, Starting Points, Mathematics: a Human Endeavour* and *Breaking the Maths Barrier.*

The final part of the Journey will involve small groups, each engaged in an activity called *Mathematical excavations*. Each group will first excavate the mathematics, as well as broader assumptions and issues embedded in a particular case, around a theme chosen by the presenter. Participants will later use newspaper articles, or any other material of their own choice, to engage with the maths involved; to feed it back into the chosen context; and to discuss how their understanding of the maths influences the way they understand the original article or material. There will also be sharing of teaching and learning resources.

Presenter's preparation

The presenter should read the following extracts before preparing and delivering these sessions. The first two are supplied as pre-readings, pages 238–240. The third was used as P4, Module B.

1 *Numeracy as a critical awareness of maths*, Presenter's pre-reading 1, on pages 6–7
2 Brown, R. 1989, 'Carpentry: a fable', *Mathematical Intelligencer*, Vol. 11, No. 4, p. 37, which is Presenter's pre-reading 2, on page 8
3 O'Shaughnessy, L. 1983, 'Putting God back in math'

Learning outcomes

Participants will be able to:

- explain how mathematics is used or misused to influence opinions and decision-making processes in terms of political, social, work, cultural, gender and racial issues;
- plan and develop a numeracy curriculum activity for a particular context and student group;
- analyse and apply specific areas of mathematics; and
- identify teaching and learning strategies and resources for these specific areas.

Assessment requirements

1 Maintain a written journal of reflections on the content of the module and any teaching implications.
The journal consists of personal reflections on the course and is to be done out of course time, consisting of at least 2 brief informal entries for the module.

2 Complete Curriculum Project 3 (CP3) – Developing a critical view.
A contract for the content of CP3 is to be negotiated between the participants and presenter. CP3 will involve identifying the maths embedded in a context of the participants' choice and developing a critical teaching activity or curriculum outline from this context, but related to their work context. It will include planning the topic, exploring the topic and making a presentation to the Group. CP3 is preferably to be done as a team activity.
Each participant must write their own report (200–2500 words) either of this curriculum project, or of Curriculum Project 2.

Module C outline

section	*time*	*development of issues and activities*	*maths involved*
C1 – Being critical about maths			
C1.1 Being critical about number	2.5 h	– *the three terrains* – *food figures for thought* – *estimation as strategic guessing* – *the average house price*	estimating, averages, statistics, mean, median and mode
C1.2 Reflecting on and negotiating your learning	30 min	– *discussion*	
C2 – Being critical about learning			
C2.1 Looking at learners	2 h	– *adults as critical learners* – *our adult learners* – *dealing with difference*	
C2.2 Street maths, school maths	1 h		percentages
C3 – A negotiated interval			
C3 A negotiated interval	3 h	– *negotiating a beginning* – *the negotiated tour* – *the journey so far*	percentages, decimals, graphs, estimation, precision, sampling, metric conversion
C4 – Maths myths and realities			
C4.1 Why learn maths? Why be numerate?	1 h	– *some myths about maths* – *consequences of low adult numeracy*	
C4.2 Curriculum planning and meeting student needs	2 h	– *the wider context: state and national* – *developing a program* – *program constraints and problems*	participants' choice

C5 – Excavating maths			
C5.1 Who learns? Who benefits? A maths excavation	2 h	– *a sample excavation* – *possible directions, mathematical and social*	percentages, fractions, decimals, statistics
C5.2 Resourcing numeracy	1 h	– *analysing resources* – *sharing results*	
C5.3 Critical literacy, critical numeracy	1 h	– *an excursion into literacy* – *constructing a critical mathematics*	
C5.4 Excavating mathematics	2 h	– *an excavation:* – *possible directions, mathematical and social*	participants' choice
C6 – Curriculum Project 3 – Developing a critical view			
C6 CURRICULUM PROJECT 3	9 h	– *the excavation of some maths from a context*	participants' choice

References

Davis, P. & Hersh, R. 1986, *Descartes' Dream*, Penguin, especially pp. 15–17.

Hofstadter, D. 1986, 'On Number Numbness' in *Metamagical Themas: questing for the essence of mind and pattern*, Penguin, pp. 115–135.

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A full bibliographical list of references for this module is given at the end of Resources C [HO23].

Presenter's pre-reading 1

Numeracy as a critical awareness of maths

What is the nature of numeracy?

According to the framework we are using, numeracy is a critical awareness which enables us to build bridges between mathematics, and the real world with all its diversity. Being numerate also involves the responsibility of reflecting that critical awareness in one's social practice. Thus, being numerate means being able to situate, interpret, critique, use and perhaps even create maths in context, taking into account all the mathematical as well as social and human complexities which comes with that process. As such, the awareness and practice to which we refer cannot be called numeracy unless it's political. Unlike Mathematics, numeracy doesn't pretend to be objective and value-free.

This framework recognises that we live in a world where mathematics is a body of knowledge much valued for its inaccessibility to all but a privileged few, and bridges have to be built across the gulf between the 'bits' of the teachers' mathematical knowledge rationed out to us via school maths, and the realities and values which we need to live with and negotiate. This bridge, we call numeracy. Numeracy needs to help activate the mathematical knowledge within us and to make meaning of more mathematics as it presents itself explicitly or in disguised forms in our lives.

What does it mean to teach numeracy, or can it be taught at all?

Given that numeracy can be understood as a consciousness reflected by responsible social practice, it would be inappropriate to talk about 'teaching' numeracy. More appropriate would be to say that learners and teachers can engage in the process of engendering numeracy. This is in fact a much harder task than teaching the most complicated mathematical result.

From one perspective, teaching and learning mathematics in a mathematics class has a safety factor both for the teacher and the students. The 'content' can be objectified, and the teacher can take the role of a *transmitter* of knowledge, and control the rate, content, and level of transmission. The learners, can take the role of *receivers*, and occupy themselves totally in receiving and deciphering the transmitted signals. The teacher assumes full control of what goes through the channel; the learners do not have to (nor do they have the time to) question what is the value of the transmitted signal. If learners do not demonstrate their understanding, the teacher assumes a fault in the receiver; the receivers, may blame the rate or the level at which the signals are being transmitted, or perhaps blame their own capacity of reception, but neither parties question the nature of the channel (or that a channel actually exists!) nor do they discuss the validity of what is transmitted. These questions would be noise in the communication channel which this type of system would be adept at filtering out altogether.

What are some of the political dimensions in a numeracy classroom?

The transmission model of teaching mathematics can give a sense of satisfaction to the teachers and the students. At the end of a class or a course, a list of signals which had been transmitted can be made. The longer the list, the greater the sense of having 'covered a lot of content', and the false impression that knowledge and meanings have been created. It also carries with it a 'safety' factor for both the teacher and the learners; what the teacher does is dictated by what is presented as the 'objective' and 'value free' content. It is a risk-free environment. There is no room to challenge values and validity of the content which has been chosen, because in the manner that they are presented, they may have all come from god.

In contrast to the latter, engendering numeracy is not necessarily content driven. The aim of engendering numeracy is not to 'cover' but to 'excavate' and 'uncover' what lies beneath the surface and the facade. What assumptions and value systems went into constructing this model of reality? What is valid, and what is being misrepresented? Whose interests are being served by representing this reality in this form rather than another reality in another form? Engendering numeracy involves drawing connections between what is on the surface and what is beneath, and identifying what is unknown or unstated. It is a process which activates the learners' past mathematical knowledge as well as their experience and knowledge in the wider social realities in which they live.

The process of uncovering assumptions and challenging prevailing value systems leaves no room for any pretence that numeracy is objective and value-free. Numeracy is subjective. Numeracy makes different meanings of the same things for different people. Both the teacher and the learners have to understand the political nature of their classroom engagement, and have to negotiate a means of sharing knowledge and viewpoints, and accepting differences of value systems and realities. This requires all parties to recognise the power that they have, and discuss what power must be relinquished and what power can be shared.

In addition to the power relationships between teachers and learners, there is the wider context in which the learning takes place. This may influence the extent to which teacher and learners' ideologies and views about their learning and of numeracy can be enacted in the classroom. These may include the bureaucracies of the teaching institution, the learners' personal circumstances, standards and syllabi dictated by external bodies, and so on. These constraints are very real contexts, but because they are real, both teachers and learners need to bring these to the surface and be engaged in a negotiation of how they will deal with them; ignore them, make compromises, or challenge those who are imposing the constraints.

Some general aims in numeracy education

Numeracy is centrally concerned with social justice and as such, is needed by people in all walks of life. Irrespective of particular needs or levels, the broad aims of numeracy education could be outlined as follows:

1. Make explicit the concept of mathematics as a social construction through engaging both presenter and participants in a negotiated process of constructing a blue print to realise the educational objectives.
2. Develop an appreciation that individual human construction of knowledge is a sub-process of, but not equivalent to, social construction of knowledge.
3. Reinforce the value of reflective practice as a way of enriching personal knowledge as well as the learning environment.
4. Develop an awareness of the political nature of numeracy, and the sensitivity to power which is required in the practice of engendering numeracy in and out of a classroom.
5. Critically examine the politics of 'content and coverage' versus 'meanings and connections' in a teaching and learning environment.
6. Critically analyse the relationships and interactions between teacher's and learners' view of mathematics and numeracy, their espoused and their enacted models of learning and teaching, and the factors which influence these.
7. Become familiar with more areas of mathematics as they emerge, and become resourceful in the process of learning maths.

What we found most memorable in our experience of this kind of numeracy was finding that the critical awareness was waiting for an outlet. The shift in our role from that of 'god' to that of facilitator allowed learners to test out and develop their critical stance towards maths and the broad contexts in which maths resides.

Extracts from K. Yasukawa, B. Johnston & W. Yates 1995,
'Numeracy as a critical constructivist awareness of maths:
case studies from Engineering and Adult Basic Education',
Regional Collaboration in Mathematics Education,
Proceedings of ICMI Conference, pp. 815–825.

Presenter's pre-reading 2

Carpentry: a fable

Recently I attended a carpentry course. It was pretty tough.

All the students (or almost all) were eager to learn. The first three weeks we learned to drill holes. We found out about curious kinds of drills and bits, and how to make holes at odd angles. We got pretty good and accurate at drilling holes.

The next six weeks were involved in cutting wood. We used all kinds of saws, found out how they interacted with different kinds of wood, and learned to cut accurately and smoothly. I got pretty good at cutting wood.

The next four weeks we learned how to plane wood. We used all kinds of planes, on many different kinds of wood. I got pretty good at planing wood.

'Joints' was a different course. It took eight weeks, and we learned many kinds of joints. I was quite good at making joints.

We did courses on other things too: sanding, turning, polishing, gluing, and so on.

Finally, we had an examination. We had to use some of these skills. I did reasonably well, fifth in the class.

After the course ended, I went to see the Director. I told him I quite liked the course in a way, though some of the students were turned off by it all. 'But really,' I said, 'I took the course because I wanted to make a table.' He said that only the top two or three in the course went on to do things like that. I began to get mad. I said, 'What did we learn all that stuff for?' He said, 'Our course prepares students for making tables.' His face got larger and larger. He began to fill the room. I got scared. Then I woke up.

This was very worrying. I discussed it with my colleagues. A psychiatrist took me back to my childhood. But no one could explain why a professor of mathematics should have a nightmare like that.

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Brown, R. 1989 'Carpentry: A Fable', *Mathematical Intelligencer*, Vol. 11, No. 4, p. 37.

C1 Being critical about maths

C1.1 Being critical about number

Brief description

Participants engage in a short discussion and three activities focusing on the use of numbers in relation to averages, estimation and the nature of statistics.

Rationale/aims

The activities in this section provide a way of raising issues and mathematical concepts which can be addressed in a numeracy teaching situation. The Journey will continue in C3, where all parties can, to some extent, negotiate specific mathematical terrains to visit. The presenter has to keep in mind and tell participants that the purpose of this part of the Journey is to provide a way of identifying a range of issues and maths which can be addressed; that the first part will be based on a story of some connections made by the presenter, but that the next two parts will demand greater involvement from participants in determining the paths of the Journey.

Preparation

Presenter
Read
Huff (1954), Hofstadter (1986) and Davis & Hersh (1986)

Participant
Read
Hofstadter (1986) and Davis & Hersh (1986)

Time: 2.5 hours

Materials needed

Concrete materials/paper
calculators
real estate sections of newspapers, 1 per group of 5 (Check that the information includes not only auction notices, but details of selling prices and locations so that an analysis of house prices by suburbs is possible. Local papers sometimes do not have this information.)
clear OHT and marker

Handouts/OHTs
The three terrains of the mathematical journey [OHT1]
On number numbness [HO1]
Are we drowning in digits? [HO2]
Food figures for thought – China [AS1]
Food figures for thought – USA [AS2]
Food figures for thought – discussion sheet [AS3]
Food figures for thought – summary sheet [OHT2]
State of the world 1994 [HO3]
Estimation as strategic guessing: three tasks [AS4]
The average house price [AS5]

References

Davis, P. & Hersh, R. 1986, *Descartes' Dream*, Penguin, especially pp. 15–17.

Hofstadter, D. 1986, 'On Number Numbness' in *Metamagical Themas: questing for the essence of mind and pattern*, Penguin, pp. 115–135.

Huff, D. 1954, *How to Lie with Statistics*, Penguin.

Summary of activities for C1.1

The table below sets out the activities which you will use in the first part of the Journey. The depth to which the presenter chooses to explore the topics is to some extent negotiable between presenter and participants, but at the least an attempt should be made to visit the various issues and related maths which are listed in the table.

Some of the maths topics may have been touched upon previously. You may or may not choose to revisit them.

Activity	**Rationale, Aims and Context**	**Maths and Mathematical Concepts**
Food for thought	– ice breaker – establishing a context for the rest of the Journey	– estimation – averages – correlation – cause and effect
Estimation as strategic guessing	– estimation as an 'everyday' process and tool – the process of estimation how do we estimate – personal and collective knowledge and reasoning – relationship with context – need for a comparator and reference	– comparisons – equivalence – approximation – measurement – prediction and forecasting
The average house price	– varieties of averages and misleading information – vested interests and use of maths	– mean – median – mode

C1.1 Being critical about number

Detailed procedure

This session consists of a short discussion and three activities:

– **The three terrains**	(15 min)
– **Food figures for thought**	(45 min)
– **Estimation as strategic guessing**	(45 min)
– **The average house price**	(45 min)

The three terrains

1 Show *The three terrains of the Mathematical Journey* [OHT 1].

2 Explain that these are three terrains that the course writers have imagined for this Journey.
Ask each participant to jot down 2 or 3 questions or concepts which they think might be associated with those terrains.
Ask them to keep what they have written to put in their own Journal.
Some ideas are indicated in Figure 1 on the next page which you may like to use as stimulus material when negotiating topic selection in section C1.2.

3 Tell participants that you are going to be a guide for the first part of this Journey and name the areas, *Food for thought*, *The average house price*, and *A taste of sampling*, which you have chosen as destinations.
Briefly explain the rationale for this choice.
Check with the participants if they are happy with your plan.
If they are not, engage in negotiation, but make explicit the constraints you recognise, including:

- what you have managed to prepare
- what you think is particularly important
- what you have been advised to cover.

Figure 1 The three terrains of the Mathematical Journey: some questions

numbers as metaphors

What lies beneath the concise symbolism of numbers?
Why are statistics controversial?
Is there anything objective left in this world?
How can statistics be a human creation if the data are there to be collected?
If pictures convey a million words, do graphs tell a million numbers?

quantification

enumeration

census

demography

class and gender

data and statistics

measurement

money

IQ and tests

graphs

---

counting as a cultural activity

If there are 3 stones in front of you, what difference does the cultural association of the 'counter' make?
Who counts and why?
Why do we find different enumeration systems (and sometimes none) in different cultures?
Does a culture which counts always compute?
Are whole numbers any more real than fractions?
Why is it more 'rational' to build a flat roofed home than an igloo?
Are negative numbers what you get when you count backwards?

orders of magnitude

language and maths

maths: an abstraction

maths: a dehumanising language

counting and social power

maths and economics

counting and abstractions

imagination and complexity

maths and art

---

guessing, estimation and precision

How big is BIG and how small is small?
If the metric system is the 'standard', why not 'metricise' time?
Does inaccuracy mean you've made a mistake?
How accurate is 'accurate enough'?
What is the difference between a guess and an estimate?
Can everyone guess and estimate?
What is an approximation?

abstraction, comparators and standards

invariance and variance of standards

technological control of our imagination

Food figures for thought

1. Have 2 sets of question sheets ready. (If fewer than 15 students, use China sheets only.) One set will have questions about China, the other about the USA. Both sets of questions will ask about:
   - population in 1990
   - per capita annual grain consumption
   - per capita annual egg consumption in 1990.

2. Give each participant a *Food figures for thought Question Sheet* [AS1 or AS2] from one of the sets, and ask them to estimate the figures for these quantities.
   Be aware that there may be some uncertainty about what per capita annual grain consumption actually means. In this case it includes grain used for all purposes. (See China/USA Source Material *State of the World 1994* [HO3]).

3. Collect the answers into clusters of between 5 and 7 for each country. Divide the Group into small groups of 5 to 7. Assign each group one set of answers to tally.

4. Give each group a *Food figures for thought Discussion Sheet* [AS3] to work on.

5. Get each group to report back on the figures they have calculated, and as they do so, to complete the table on *Food figures for thought: summary sheet* [OHT2].
   Highlight the range of numbers, high and low, that come up.
   Ask them what they think would be 'the most likely' figure for each of the estimates.
   Write up the 'real' figures using *State of the World 1994* [HO3].

6. Handout copies of the China/USA source material, *State of the World 1994* [HO3].

7. Lead a discussion on issues arising from this activity. These might include:
   - the difficulty of estimating these figures, and why
   - the magnitude of the numbers, what magnitudes we are comfortable with and what we're not comfortable with (See *On Number Numbness* [HO1].)
   - 'million' and 'billion' as metaphors for largeness
   - the sources of the figures: reliability, interests of the authors, intended readership
   - the 'owners' of these statistics and their reasons for ownership
   - correlations which can be seen in these numbers, and the ease and danger of drawing causal relationships
   - the potential users and uses of these numbers and
   - the culture of this form of quantification, and the possible origins.

Be aware that this activity can lead to discussions about estimation, averages, sampling, measurement, precision, exponential growth, and graphing.

There are prepared activities on all these topics in the rest of this section.

Estimation as strategic guessing

1 Ask participants in groups of three or four to work out how they would go about the tasks in *Estimation as strategic guessing: three tasks* [AS4].

2 Debrief, by getting each group to share their ideas with the whole Group.

You might like to have in mind estimation techniques such as:

- using known weights, e.g. 1 kg bags of rice, to put together a simple balance scale
- using the past year's utility bills to predict what the next year's might be
- looking at how many people are currently in the room to visualise how comfortable the room would be for 30 people.

3 Tabulate the ideas on the whiteboard.

Try to address the following ideas through the discussion:

- if you don't know what the figures represent, you can't begin to estimate
- if you can situate the question in a context, you may have comparators to help you make a guess
- if you can make comparisons, you've established a reference
- if you want 'everyone' to be able to make comparisons the way you do, you need to standardise your reference.

Be aware that this activity can be used as motivation for discussions and prepared activities about:

- averages as one form of estimating the value of a particular set or collection of numbers, figures and/or quantities
- precision
- units of measurements.

Some other paths which have not been developed specifically to work with in this Journey, but which are equally valid, include:

- the role of algebra, functions, and formulas in making predictions
- the spreadsheet as a budgeting tool
- an exploration of space and visualisation.

You may like to identify these as critical signposts.

The average house price

1 Give groups of 3 or 4 a copy of the real estate section from the newspaper.

Get each group to choose a suburb, or a group of comparable neighbouring suburbs, and find the selling prices of houses sold. Choose a suburb or groups of suburbs in order to:

- have at least 15 to 20 house prices
- have one or more localities that are of similar socio-economic status.

2 Ask the group to calculate averages: mean, median and mode, of the house prices for that suburb or area. Make sure all participants know the meanings of mean, median and mode.

Note: You could get participants to write individual house prices on cards. This allows flexibility—the cards can be ordered to calculate the median; they can be used to create pictographs or bar graphs.

3 Using *The average house price* [AS5] ask participants to role play of each of these characters in turn, and decide which 'average' would be most useful for their purpose:

- a real estate agent, discussing house prices with someone who is trying to sell
- a real estate agent, discussing house prices with someone who is trying to buy
- a property developer, writing an advertisement for the homes being built in a new estate in that suburb and
- an individual, thinking of buying a home so as to live in that suburb.

4 Get each group to report back and discuss the differences in the numerical values of the three averages, and also the meanings they convey (or not).

Get them to consider how that informs them about which 'average' is most useful—under what conditions and for what purposes.

5 List a few items to which the participants can respond, as to whether a mean, a median, or mode would be the most informative 'average' these items. These may include:

- average level of congestion on a certain road
- average amount of orange juice consumed in a household each week
- average score on a school maths exam
- average alcohol content in a glass of wine
- average unemployment rate of school leavers in Australia.

Be aware that this activity can be used as motivation for discussions and activities about:

- sampling: how the sampling size and method affects the averages
- precision
- estimation as strategic guessing: averages as estimates.

C1.2 Reflecting and negotiating your learning

Brief description

Participants debrief the previous activities and negotiate further activities on data and number.

Rationale

In the previous Section (C1.1) participants have explored topics chosen by the presenter from the course materials. This section allows some negotiation of the directions they would like to follow a little later in Module C, when more areas related to number will be explored.

Preparation

Presenter
Familiarise yourself with the rest of Module C.

Time: 30 minutes

Materials needed

Handouts/OHTs/paper
Some critical questions [HO4]

C1.2 Reflecting and negotiating your learning

Detailed procedure

This session consists of a Group discussion (30 min).

1 Ask participants to write a summary of their experience in the Journey so far. (5 minutes)

2 Hand out the quotes: *Does a larger sample mean greater representation? Why convert?*, *Precision: what does it mean?*, and *If a picture can convey a thousand words* ... which are all part of *Some critical questions* [HO4].

Identify some directions that may have come up as possible ways of continuing the Journey when you get to Section C3. These may include topics which have been developed as activities for this Journey, for example:

- sampling: how sampling size and method affects the averages
- measurement: units, standardisation and precision
- percentages
- graphs.

There may be other directions which have not been developed specifically to work within this Journey, but which are equally valid, for example:

- the role of algebra, functions, and formulas in making predictions
- the spreadsheet as a budgeting tool
- an exploration of space and visualisation.

It may also be appropriate to conduct activities from other resources such as: *Numeracy and How We Learn, Starting Points, Mathematics: a Human Endeavour*, or *Breaking the Maths Barrier.*

4 Explain that it will not be possible to do all these in one 3 hour session. (Remember that there is a small selection of activities prepared for use in Section C3.)

Discuss available options and constraints: what activities are available, what the next presenter wants to do, time etc. Reach consensus on how the Journey will proceed.

5 Preview the third part of the Journey (Sections C5 and C6) where participants will need to bring in their own materials to use as a context for a mathematical excavation.

Explain that these materials may include

- newspaper articles with mathematical or statistical references
- technical material relevant to particular trades or crafts
- books of statistical and other information.

C2 Being critical about learning

C2.1 Looking at learners

Brief description

Participants reflect on who their students are.

Rationale

Until this point in the course, learning and teaching have been considered in fairly general contexts, or in relation to the participants own experience. In this section participants consider more closely who it is they are teaching and have a chance to tease out commonalities and differences in the groups, and to review some ideas about adult learning.

Preparation

Presenter
Read
Johnston (1990) and Newman (1993)

Participants
Domains of learning [P1]
Dealing with difference [P2]

Time: 2 hours

Materials needed

Concrete materials
butcher's paper
felt pens
Handouts/OHTs/paper
Meeting individual needs [HO5]
Domains of learning [P1]
Dealing with difference [P2]

References

Jarvis, P. 1983, 'Adults learning: some theoretical perspectives', in P. Jarvis, *Adult and Continuing Education: theory and practice*. Croom Helm, New York, Chapter 4.

Johnston, K. 1990, 'Dealing with difference', *Education Links*, No. 38, pp. 26–29.

Marr, B. & Helme, S. 1991, *Breaking the Maths Barrier: a kit for building staff development skills in adult numeracy*, DEET, Canberra.

McCormack, R. & Pancini, G. 1990, 'Current explanations of learning', in R. McCormack & G. Pancini, *Learning to Learn*, DFE, Melbourne.

Newman, M. 1993, *The Third Contract: theory and practice in trade union training*, Stewart Victor Publishing, Sydney.

Newman, M. 1994, *Defining the Enemy: adult education in social action*, Stewart Victor Publishing, Sydney.

C2.1 Looking at learners

Detailed procedure

This session consists of three activities:

– **Adults as critical learners**	(45 min)
– **Our adult learners**	(30 min)
– **Dealing with difference**	(45 min)

Adults as critical learners

1 Point out that we have been using the word 'critical' fairly freely, and brainstorm what meanings participants have themselves, or have heard others use.

2 Participants will have read *Domains of learning* [P1], which includes a particular meaning of the word 'critical' used in a particular context: that of union training.
In order to tease out this meaning, get each small group to
- take one of the domains of learning described by Newman
- summarise it and
- try to work out what it might mean in the context of their own work.

3 Groups report back. Try to get some consensus about how Newman uses 'critical'.

4 Ask:
Does the article argue that adult and child learning are different? How? Do you agree?

Our adult learners

1 Ask: *Who are our learners?*

2 To discuss the question, organise participants into pairs and get them to draw up, on butcher's paper, student profiles of one or two groups that they are currently teaching.
Factors to be taken into consideration include:
- student backgrounds
- their reasons for being there
- confidence
- competence
- verbal ability
- goals
- familiarity and use of English.

3 Get pairs to report back to the Group. Collate and categorise student group profiles.

Ask:

Are there any overall commonalities among the groups?

What are the most important differences among the groups?

Important factors for the presenter to bring out in the discussion will include:

- the wide variety of student backgrounds and goals
- the wide variety of contexts including traditional ABE
- gender, race, class and cultural factors
- the assumption of many teachers that all learners are 'adult learners', i.e. motivated (and as more and more learners are not voluntary, motivation is now a larger issue)
- the wealth of learners' previous learning, the importance of becoming aware of what people already know, if they are to build on that
- the existence of different learning styles, e.g. visual, concrete, written, physical (doing), oral.

Dealing with difference

1 Give out *Dealing with difference* [P2]. Either in small groups, or as a whole Group, read and discuss the extract. You may like to ask questions like the following.

- what does the author see as the dangers and problems of what he calls 'difference thinking', e.g. stereotypes; alternative, less powerful knowledge; restricted capacity for action?
- What solution(s) does he suggest, e.g. making social distinctions work for the disadvantaged; being aware of what the context teaches; decoding social and cultural contexts of learning?
- The author says, '*all knowledge, even the most abstract mathematical relationship, is context-dependent*'. How? Think of some examples that appear decontextualised and try to tease out ways in which they in fact reflect the context, e.g. sexist examples in maths problems, the 'either right or wrong' message given by most test situations, a frequent emphasis on profit rather than debt, the assumption that maths began with the Greeks, talking is cheating ...

2 Ask: *What are some implications of this argument for teaching adult numeracy?*

3 Here we had a focus on social difference. Other more individual differences also need to be catered for, and we will be returning to this issue from time to time.

You may like to discuss the examples listed in *Meeting individual needs* [HO5].

C2.2 Street maths, school maths

Brief description

Participants explore the effects of context on how and what maths is learnt.

Rationale/aims

Much adult numeracy learning goes on in non-formal situations. There is evidence to show that such learning is markedly different from formal school learning of mathematics. In Curriculum Project 1 participants have had a chance to realise that the four arithmetical operations are approached in a wide variety of ways. This section gives participants a glimpse into the ways in which context may be involved in such differences.

Preparation

Presenter
Read
Evans & Harris (1991)

Time: 1 hour

Materials needed

Handouts/OHTs/paper
The practice of mathematics [OHT3]
Work contexts [HO6}
Everyday contexts [HO7]
Street contexts [HO8]
Language contexts [HO9]
A farming context [HO10]

References

Department of School Education, 1985, *Real Maths, School Maths* (video), Department of School Education, Victoria.

Evans, J. & Harris, M. 1991, 'Theories of practice', in M. Harris (ed.) 1991, *Schools, Mathematics and Work*, Falmer Press, London.

Numes, T., Schliemann, A.D. & Carraher, D.W. 1993, *Street Mathematics and School Mathematics*, Cambridge University Press, Cambridge.

The Open University, 1992, *Learning to Learn*(video), UK, available from Educational Media Australia, 7 Martin Street, South Melbourne, Vic. 3205, Ph: (03) 9699 7144.

C2.2 Street maths, school maths

Detailed procedure

1 Show and discuss briefly *The practice of mathematics* [OHT3].

2 Organise participants into 4 small groups and hand out the four short readings—HO6, HO7, HO8 and HO9 (not HO10)—so that each group has one passage to read and discuss. After 10 or 15 minutes get each group to:

- describe briefly the content of the reading
- highlight what points the reading makes about learning maths and doing maths in the particular context.

3 Ask participants to calculate 16% of $220 000, by themselves, and then to compare their method with their neighbour's. Hand out *A farming context* [HO10] and get participants to go through the method used there for working out 16% of $220 000.
Get them to try it on another example. Is it different from their own methods? How? Could it be helpful with their own students?

4 Try to summarise with the participants what Soto, Lave, Scribner, Carraher & Walkerdine—and the participants—think are the differences between formal and informal learning of mathematics.

5 This may be a good opportunity to watch the video *Real maths, school maths* which addresses many of these issues in the Australian context.
Look in detail at the process that the boy uses to multiply 17 times 6, and discuss what this has in common with the Chilean farmers' percentage calculation.
Also the video, *Learning to Learn,* contrasts two views on the significance of the context of learning. One of the case studies is of the way Brazilian street kids acquire mathematical skills without going to school.

C3 A negotiated interval

Brief description

Participants explore areas of numeracy negotiated in Section C1.2.

Rationale/aims

This part of the Journey is intended to have an element of negotiation between participants and presenter. At the end of the first part of the Journey, a negotiation should have taken place where the parties discussed possibilities and decided on the paths to pursue in this session. There is a selection of activities from which to choose. In the following two or three sessions there will be time to explore what terrains of interest have not been visited from the participants' perspective, and to discuss the resources and strategies which they may consider should they wish to visit these terrains on their own. A central aim of this Journey is to give the participants experience of numeracy as a critical awareness, reflected in social practice, enabling them to bridge the gap between academic maths and the diverse realities of their lives.

Preparation

Presenter

Read *Module C: rationale and aims*, and the Presenter's Notes for C1.1, C1.2. and C2.2. Think about what is involved in a critical constructivist approach to teaching maths .

Make sure you know what program has been negotiated for this session and that you are happy with it.

Decide whether you will use the stimulus extracts HO5 from Section C1.2 as OHTs, handouts, or as ideas for generating your own questions or motivational tools.

Bring newspapers as resource material for graphs.

Time: 3 hours

Materials needed

Concrete materials

See materials listed on the next page for the different activities.

Handouts/OHTs/paper

Your metric conversion chart [HO11]
Putting God back into math [P4 from Section B3.2]
Why I like numbers.[P4]
Unemployment rates [OHT4]
Percentage scenarios [HO12]
Loans: whose interest? [HO13]

Concrete materials for the activities for C3

A taste of sampling
- real estate figures used in C1.1

Precision: what does it mean?
- calculators
- newspaper ads or junk mail ads for supermarket weekly bargains
- rulers, tape measures
- TaxPack booklet or pages

Your metric conversion chart
- measuring cups with various types of gradation (mL, oz, fractions)
- calculators
- metric/imperial rulers, tape measures
- a kitchen scale
- various size milk cartons
- thermometer
- floor tiles/bricks
- Dienes blocks
- poster papers
- coloured pens, glue and sticky tape

Percentages and loans: in whose interest?	*Graphs: information and distortion*
brochures or leaflets from banks about loans and mortgage schemes calculators graph paper Dienes blocks	newspaper graphs of any sort calculators plain paper linear graph paper log linear graph paper rulers Dienes blocks

Summary of activities for C3

The table below sets out possible activities which the presenter may choose to use in the second part of the Journey. The selection of activities should be negotiated with the participants at the end of part 1 of this Journey—see C1.2—taking into account participants' needs, availability of resources, and presenter's expertise. The depth to which the presenter and participants choose to explore the topics is to some extent negotiable between the parties, but we would like the presenter to at least highlight the various issues and related maths which are listed in the table. Some of the maths topics are those touched upon previously; you may or may not choose to revisit them.

Activity	*Rationale, Aims and Context*	*Maths and Mathematical Concepts*
A taste of sampling	– the nature of statistics – subjectivity in sampling	– sample size – parameters in sampling
Your metric conversion chart	– standardised units – the process of standardisation: pros and cons – non-standard units	– metric system – units for lengths, area, volume, weight, and temperature – conversion – place values – algebra
Precision: what does it mean?	– judgment of precision – imprecision due to ways of representing numbers – precision, approximation and estimation – discounts and mark ups	– rounding – truncation – rational and irrational numbers – average of averages – percentages
Graphs: information and distortion	– use and abuse of graphs – relationships, cause and effect, and correlations – high tech graphs	– pie chart – histograms – scales – different types of growth – functions
Percentages and loans: in whose interest?	– misleading precision – demystification of bank loans – language ambiguities	– percentages – reference groups – ratio and proportion

C3 A negotiated interval

Detailed procedure

This section consists largely of several pre-negotiated activities:

– **Negotiating a beginning**	(15 min)
– **The negotiated tour**	(2.5 hours)
– **The Journey so far**	(15 min)

There is a small selection of activities prepared for use in **The negotiated tour**:

- A taste of sampling
- Your metric conversion chart
- Precision: what does it mean?
- Graphs: information and distortion
- Percentages and loans: in whose interest?

Each of the prepared activities can take up to 1 hour, or longer if there is sufficient interest in the group. It may also be appropriate to widen the selection to include activities from other resources such as *Numeracy and How We Learn, Starting Points, Mathematics: a Human Endeavour,* or *Breaking the Maths Barrier.* What activities will be conducted will have been determined at the end of Session C1.2.

Negotiating a beginning

1 Briefly remind participants that an aim of this Journey is to explore what it means to be mathematically critical.

2 Put up an agenda for the session which shows the areas negotiated between the presenter and the participants at the end of the first part of the Journey (C1.2).

3 Ask the participants to spend 2 or 3 minutes jotting down their objectives and expectations for the session.

The negotiated tour

You should keep the critical constructivist framework in mind, and keep alive the question of connections to the 'real world'.

A taste of sampling

1 Using the real estate figures from *The average house price* again (Section C1.1), get participants in groups to calculate the averages using different selections and numbers of the listings. Ask them to take out any extreme figures. How do the averages vary?

2 It may be useful to do the activity, *Average wages* from *Numeracy on the Line* (p 29). This activity gives a clear demonstration of the differences between the median and the mean, and how data can be distorted by extreme figures.

3 Ask: *How many eggs did each participant eat yesterday?*
What was the mean consumption of eggs for the group?
If this average were multiplied by 365 and divided by the number in the group, would this be a 'good' estimate of the annual per capita consumption of eggs in Australia?
Why or why not?

4 Get each group to draw some conclusions about the processes of sampling, and the way they affect the averages that are calculated from the samples. Get each group to think about how the 'real' figures in the *Food for Thought* activity might have been estimated.

5 Debrief the activity by relating their conclusions to other contexts.

- Is the level of traffic congestion on a particular road the same over the whole day? If not, what time period should we average over?
- Is the unemployment rate for an age group uniform across a particular city, state, and country? How finely should we focus our attention?

6 Lead to a conclusion, that the more representative of the 'truth' you want the statistics to be, the further you get from the act of generalisation and into that of specification—that is paying attention to the particular and the individual. Following from that, discuss the importance of clarifying boundaries within which the statistics are meant to apply.

7 Be aware that this activity can be used as motivation for discussions and an activity about

- units of measure and
- standardisation.

Just as sampling is used to create averages which are used as implicit standards of measuring certain social phenomenon or trends, standard units of measurement are used as explicit standard references for measuring physical and other quantities.

Your metric conversion chart

1 Get participants to work in small groups to draw up a metric conversion chart, according to the ideas outlined in *Your metric conversion chart* [HO11].

2 Debrief the activity by displaying the conversion charts where the Group can see them.

3 Highlight recurring or similar everyday references in the different charts. Discuss how culturally, socially, gender, or age-specific they might be. Ask the participants what familiar references might be useful if they were explaining the metric units to young children, or a group of people working in a very specific and common area, such as sewing or carpentry.

4 Discuss what units or scale of units they use the least, and why. Point out that there are conversions within a system, e.g. cm to km; and between systems, e.g. cm to inches or width of a finger. Discuss the meaninglessness of measurements given without units; and conversely, how units can be helpful in identifying the nature of the quantity which was measured.

5 Ask participants to illustrate some of the patterns they could see in the relationships between different units using the Dienes blocks.

6 If any of the groups presented a conversion between the imperial system and the metric, or between Fahrenheit or Kelvin and Celsius, you may like to introduce the role of algebra as a tool in doing conversion between one standard unit and another, and may illustrate the point by examples. The concepts of functions and formulas may be introduced, with caution.

7 Be aware that this activity can lead on to a prepared activity about precision: why do we measure distance on the road in km rather than mm?

It could also lead on to discussions and investigations about

- other quantities such as speed, acceleration, density, population growth, toxicity, chemical concentration
- units of measurement which are used in more specialised contexts such as volts, watts, ohms, farads, henrys, joules, newtons, moles, hertz, Kelvin, Fahrenheit, GNP, GDP, bytes, bits
- measurement and the culture of measuring anything and everything, its social impact, and its cultural foundation.

Precision: what does it mean?

1 In groups, get participants to tabulate the number of siblings each has. Ask them to use calculators to find the mean exactly.

2 Get each group to report back. If the mean is 2.5, what does that mean?

Ask: *If your purpose was to design homes for the 'average' size nuclear family with a bedroom for each child in the family, what would you do with a mean of 2.5?*

3 Introduce the concept of rounding, rounding up and rounding down.

Get participants to think of examples where one is more appropriate than the other.

4 Pose the following question:

A shop rounds your total bill to the nearest 5 cents. If a particular product costs $1.99 and you want to buy several of them, will it make any difference if you buy them together or pay for them separately?

5 Get participants to develop a hypothetical grocery budget using the supermarket ads they brought in. Get them to investigate if different shopping strategies, e.g. buying the week's groceries all in one hit, or buying in strategic little lots—work better in saving money if the shop always rounds to the nearest 5 or 10 cents. Would it make any difference if the shop always rounded *down* to the nearest 5 or 10 cents?

6 Debrief by getting each group to report back to the Group. What is the effect of rounding? Do people use rounding as they shop in order to estimate their total bill as they shop? Do they tend to round up or down? Why?

7 Hand out copies of the TaxPack, or photocopies of some pages from it, and get participants to glance through and find where the tax payer is told to give whole dollar figures only. Whose money is being saved there? Is it clear if the tax payer is expected to round up or down to the nearest dollar?

Precision as a social construction ... or, is maths a frivolous activity?

1 Using a tape measure, have participants measure the diameter and circumference of a cake or pie dish. Ask them what they measured, and ask them to calculate the circumference using a formula. Tell them, if they do not come up with this themselves, that one formula is

$$\text{circumference} = \text{diameter} \times \pi$$

Ask them to do the calculation both by hand and with the aid of a calculator.

2 What value of π did they use? Why? What would happen if they only used 3? Or if they used 3.1? Is the claim that π is equal to $\frac{22}{7}$ a myth? How would we know?

3 Read again the article 'Putting God Back into Math' (used earlier in Module B). What is the article saying? Who are the authors? What is their stance? Where did the article appear?
Discuss how the mathematicians' stance on the value of π compares with different epistemologies of maths, e.g. maths as eternal truth, maths as socially constructed. How should society at large regard the mathematicians' quest for the millionth decimal place approximation to π ?

4 Using *Why I like numbers..* [P4] you may like to introduce the concept of different categories of numbers:

- natural (or counting)
- rational
- irrational
- real
- imaginary
- complex

and π is a transcendental number (a special type of irrational number).

Graphs: information and distortion

'If a picture can convey a thousand words...'

1 Using the table below, available on *Unemployment rates* [OHT4], ask participants to draw a line graph of the figures, with a vertical scale from 0% up to 10%.
Make a Group decision about how many squares will represent one percent on the vertical axis, and how many will represent one year on the horizontal axis.

Males in the labour force: unemployment rates, Australia, 1978 to 1984

year	1978	1979	1980	1981	1982	1983	1984
percentage	5.2	5.1	5	4.8	4.8	8.9	9.3

approximated from I. Castles, 1992, *Surviving statistics*, Australian Bureau of Statistics, Sydney, p. 32.

2 Now suggest the following:

- one group draws the graph for the same set of numbers, using twice as many squares to represent one unit on the vertical
- another group draws the graph using half as many squares to represent one unit on the vertical.

Get each group to show their result. Discuss what effect the scaling has.
How can you make growth seem more rapid? How can you make it seem less rapid?

3 Get half the Group to draw the graph again, but this time to include the whole vertical scale from 0% to 100%. Get the other half to redraw it with the original scale, but starting from 4% rather than 0%, and omitting the labelling numbers.
What is the effect of these strategies on the picture conveyed by the graph?

4 Get each group to look through the newspapers brought in for the session and to critically analyse any graphs that they find for informative and distorted content. Ask them to build up a set of criteria for what makes a good graph.

5 Post the groups' work along the wall, and get participants to walk around and look at the others' work.

6 Get each group to report back to the Group. Discuss the range of graphs which may have been encountered such as pie charts and histograms, and what they show.
A useful reference for this whole activity is Castles (1992). Discuss what sorts of language, text, maths, and graphs are informative, and what things cause confusion.

7 Explore further, if some are still confused by any of the graphs, what is causing the confusion. Is the graph deliberately trying to mislead and confuse?
Discuss whether or not the graphs add to the information of the articles or not.
Collate Group criteria for a good graph.

Graphing different types of growth

1 Ask participants if they could give examples of different types of growth.
Write them up on the board and ask them if they can rank them in terms of growth rate.
If they do not have many ideas, get them to suggest particular things which grow in size or amount: your age, height, interest in your bank account, a tree in the garden, national debt in the 90s, literacy rate.

2 Give each group a type of growth to investigate. Leave it to the group members to work out how they would carry out their investigation, but if they look really stuck, suggest that they tabulate numbers, and draw some graphs.
Group 1
The circumference of a circle as you increase the diameter, starting with diameter of 1 cm.
Group 2
The area of a circle as you increase the diameter, starting from a diameter of 1 cm.
Group 3
The area of a side of a cube as the length of the side grows, starting from a length of 1 cm.
Group 4
The volume of a cube as the length of its side is increased, starting from a length of 1 cm.

3 As each group reports on their findings, illustrate the different types of growth using Dienes blocks:

- *linear*: adding the same number of cubes with each increment—the increase is proportional to a growth in 'length'
- *quadratic*: for each increment, the increase is proportional to a growth in 'area'
- *cubic*: for each increment, the increment is proportional to a growth in 'volume'

4 As a Group, do a graphing activity where you graph the various growths on the board to compare linear, quadratic and cubic. Show by way of illustration how eventually quadratic becomes faster than linear, and cubic overtakes quadratic.

How decimal numbers grow

1 Get participants in pairs to tabulate how many times the number 1 has been multiplied by 10 to get the numbers 1, 10, 100, 1000, 10 000, 100 000, up to a billion.
Get them to draw a graph where they have the 'number of times multiplied by 10' on the horizontal axis, and 'the numbers they get' from the multiplication on the vertical.
How many points can they graph? Why?

2 Discuss what exponents mean, and the shorthand for writing numbers which are powers of 10. Get them to suggest shorthand for writing the sequences

1, 2, 4, 8, 16, 32, 64, ... and
1, 5, 25, 125, 625, ...

3 Show them how these are related to place value and the bases of number systems by constructing a base 2 Dienes block (cube, 2^3) using centicubes.

4 Now ask them how they might graph the base 10 numbers (as described in point 1 above) without using reams of paper. Give a brief sketch of how log-linear graph papers are used. Inform them that they are used extensively in the sciences and engineering.

Percentages and loans: in whose interest?

1 Get participants to discuss in small groups *Percentage scenarios* [HO12].

2 Discuss the issues that arise, including

- the total of the parts cannot exceed 100%
- language can be confusing: 'increase by 200%', '200% increase' and 'increase 200%' ... are they the same?
- 25% (or any percentage) of a small number will be less than 25% of a larger one
- for useful comparison of percentages, we need to know the size of the reference groups.

Bank repayments

1 Get participants in groups to develop an understanding of how bank repayments on a loan change over time, using *Loans: whose interest?* [HO13]. As they begin to do Question 2, try to make sure that at least one pair or small group is working on each of the variants.

Calculators are obviously useful, but working with Dienes blocks can also help to show physically what is happening.

2 Debrief by discussing results with Group.
Get participants to describe in words what is happening in the graph for the loan under the original conditions, in particular, how it decreases very slowly for a long time, and then quite quickly in the last few years.
Try to get them to say why this is so.
What happens to the interest each year? What happens to the repayments?

3 Elicit conclusions that the loan will be repaid substantially faster (how much faster?) by making:

- more frequent interest payments
- higher monthly repayments (how much higher?)
- an extra one-off payment early in the process.

4 Get participants to examine the mortgage rate information that was brought in.
Can they work out what a 'reducible' interest rate means? When do loans actually require you to pay your first interest repayment, and does it matter?
Get them to develop a one page information sheet for bank customers using the most informative language for this purpose.

The Journey so far

1 Get each participant to take 5 minutes to write a summary of their personal experience in the Journey so far.

2 In small groups, get participants to make a metaphor, draw a mind map, or choose another picture or diagram to bring together ideas about what *maths as a critical tool* might mean.

3 Debrief with the Group.
Facilitate a discussion about *maths as a critical tool,* including the issue of negotiated versus non-negotiated curriculum.
What are the implications in terms of power, responsibility and meaning making for the presenter, the participants as a Group, and the participants as individuals?

C4 Maths myths and realities

C4.1 Why learn maths? Why be numerate?

Brief description

Participants discuss the questions, 'why learn maths?' and 'why be numerate?'

Rationale/aims

Learning mathematics is too often assumed to be self-evidently worthwhile. In this session, participants are given a chance to tease out some of the myths surrounding the question of 'why learn maths?' and to go on to look at what might be the consequences of a low level of numeracy for individuals and society.

Preparation

Participants
Read
Social practice of mathematics [P5]
The politics of numeracy [P6]

Time: 1 hour

Materials needed

Handouts/OHTs/paper
Social practice of mathematics [P5]
The politics of numeracy [P6]

References

Chevallard, Y. 1989, 'Implicit mathematics: its impact on societal needs and demands', in J. Malone, H. Burkhardt & C. Keitel (eds), *The Mathematics Curriculum: towards the year 2000*, Science and Mathematics Education Centre, Curtin University, Perth.

*Evans, J. 1989, 'The politics of numeracy', in P. Ernest (ed.) *Mathematics Teaching: the state of the art.*, Falmer Press, London.

Keitel, C. 1989, 'Mathematics education and technology', *For the Learning of Mathematics*, Vol. 9, No. 1, pp. 7–13.

Thiering, J. & Barbaro, R. 1992, *Numeracy and How We Learn*, TNSDC, Sydney, pp. 31–32

Walkerdine, V. 1990, *Schoolgirl Fictions*, Virago, London.

C4.1 Why learn maths? why be numerate?

Detailed procedure

This session consists of a group discussion followed by a reading and analysis of an article:

– **Some myths about maths**	(30 min)
– **Consequences of low adult numeracy**	(30 min)

Some myths about maths

1 Brainstorm the question: *Why should people learn maths?*

Include any reason you have ever heard, whether or not you believe it yourself.

Write the reasons on a whiteboard or OHT. Reasons will probably be able to be grouped under headings such as the following:

- it is the pinnacle of human achievement
- it is useful, a tool
- it is a gatekeeper, you can't go on without it
- it teaches you to think
- it is fun
- it is demystifying.

2 Ask: *Which of these reasons do you agree with?* Some possible responses could include:

reason	***responses***
it is the pinnacle of human achievement	– a mathematician's reason – rubbish! – it is a pinnacle – read Valerie Walkerdine
it is useful, a tool	– some of it, for some people – up to Year 7 or 8 perhaps – why should everyone learn algebra? how much algebra do you see in the papers? – read Christine Keitel – work through the Thiering & Barbaro reference
it is a gatekeeper, you can't go on without it	– useful as a gatekeeper when in fact you need it for further study, but often it is never referred to again – falsely seen to be 'objective' (why isn't it?) – find another gate
it teaches you to think	– too often it teaches you *not* to think in context – too often, it teaches you to obey, and not question
it is fun	– for some people, sometimes
it is demystifying	– at its best, it can be

3 To conclude, participants could discuss the issues raised by *Social practice of mathematics* [P5]. Ask, for instance:

Do you agree with Christine Keitel's quote (from Yves Chevallard)?

> 'no modern society can exist without mathematics, but the overwhelming majority of people in a modern society can and do live quite well while doing hardly any mathematics.'

Consequences of low adult numeracy

1 Participants will have read *The politics of numeracy* [P6]. Ask:

What is Evans' understanding of numeracy?

On p. 203, Evans points out three 'noteworthy features of this definition of numeracy: confidence, practicality, and its critical potential.'

2 Next get participants to talk in groups, and using Evans' section on *Consequences* (pp. 210–214) get each group to summarise one of the four questions, with an example of what is meant in each case—either from the article, or from their own experience:

i) What are the material consequences for the individual?
ii) What are the material consequences for society?
iii) What are the ideological consequences for the individual?
iv) What are the ideological consequences for society?

Small groups prepare their summary for reporting back to the Group.

3 A possible summary follows:

	individual	***societal***
material	– restriction on freedom of access to further education and training – restriction on access to jobs (and so to rewards of income, companionship, satisfaction.) – self-restriction of subject choice and job – less ability to perform in and enjoy one's job and everyday life	– loss of production – waste of resources – production of inaccurate or useless information – threats to life and limb
ideological	– lack of competence – low level of confidence in constructive skills and critical insights – leading to mystification and dependence on 'experts' – failure to consider evidence ... through reluctance to seek it or mistaken mistrust or misinterpretation of it	– failure to consider evidence ... through reluctance to seek it, or mistaken mistrust or misinterpretation of it ... leading to myths which influence the beliefs and practices of society

4 Comment on the fact that this is an article written in England.

Ask: *Do you think its arguments apply to Australia?* Discuss.

C4.2 Curriculum planning and meeting student needs

Brief description

Participants consider the factors that influence program development, from individual needs to state and national frameworks, and undertake a small curriculum planning exercise.

Rationale

Program planning is an important part of the life of a teacher in Adult Basic Education. Some teaching must be done to accord with specific curriculum outlines. Other teaching is done within the guidelines of a framework. Still other situations are purely responses to an individual need. This session aims to give participants an opportunity to do some planning in the context of their own teaching situation and on a topic of their choice but hopefully linked to the topics and areas being investigated in this mathematical journey.

Preparation

Presenter

You can use the student profile descriptions developed during session C2.1, so bring these materials along if necessary.

Also decide who is to present on state and/or national curriculum frameworks—you or a guest speaker—and make arrangements accordingly.

Time: 2 hours

Materials needed

Handouts/OHTs/paper

Any relevant state or national curriculum guidelines
Planning a program [HO14]
The best laid plans [HO15]

References

Boomer, G. 1992, 'Negotiating the curriculum reformulated', in G. Boomer, N. Lester, C. Onore & J. Cook (eds), *Negotiating the curriculum: educating for the 21st century*, Falmer Press, London, pp. ??.

Breen, B. 1987, 'Contemporary paradigms in syllabus design', in *Language Teaching*, No. 20, pp. 2–3.

*Burton, L. 1990, 'Passing through the mathematical critical filter—implications for students, courses and institutions.', in B. Johnston, 1992, *Reclaiming mathematics*, DEET, Canberra, pp. 137–144.

Coates, S. (ed.) 1994, *National Framework of Adult English Language, Literacy and Numeracy Competence*, ACTRAC, Melbourne.

Cook, J. 1992, 'Negotiating the curriculum: programming for learning.', in G. Boomer, N. Lester, C. Onore & J. Cook (eds) *Negotiating the Curriculum: educating for the 21st century*, Falmer Press, London, pp. ??.

and state and territory curriculum frameworks and accredited courses.

C4.2 Curriculum planning and meeting student needs

Detailed procedure

This session has three sections:

– **The wider context: state and national**	(30 min)
– **Developing a program**	(60 min)
– **Program constraints and problems**	(30 min)

The wider context: state and national

1 In this section of the course it is important for the participants to get a clear overview of national and State frameworks that inform their teaching and program development. It might be useful to invite a guest speaker involved with, or experienced in, such frameworks and policies. Because each State has such different structures, it is important that the speaker should be familiar with local frameworks and contexts as they affect numeracy teaching in that State.

2 The central questions to be addressed are:

- How do national and State curriculum frameworks and guidelines affect the work of numeracy teachers?
- What do they say about what we teach our students?
- How can we use them to plan our teaching?

3 Remember that the focus here is to be on using the state or national curriculum frameworks to plan a teaching program. It is also important to remember that the focus should be on how such frameworks can be used to support the approach that we have been promoting in the teaching of numeracy, particularly in this section where the aspect of critical numeracy is being introduced.

Developing a program

1 Ask the participants who have already worked with each other on developing a student profile in Section C2.1 to again get together or ask them to work in pairs or small groups with colleagues who are teaching similar students with similar curriculum focuses. Ask them to choose a topic of mutual interest to themselves and their chosen student group and possibly one that they have investigated during this mathematical journey.
Remind them that this could also be the start of their own mathematical investigation or excavation and even lead on to the third curriculum project.

2 Let participants work in their pairs or small groups on the worksheet *Planning a program* [HO14]. Remind them that they should still be considering incorporating into their curriculum planning the critical aspects of numeracy teaching. Use the Planning Grid provided as part of *Planning a program* [HO14] or use one that you have developed yourself to fit your state or territory curriculum framework or any other curriculum planning device that you feel is appropriate to the activity.

3 Ask each group to quickly report back on the work they have done with a focus on the relationship between the curriculum framework they were using and the teaching activities they were planning.

Program constraints and problems

1 Ask:

What factors intervene between planning a curriculum and its implementation?

What limitations do externally set curricula have on your teaching?

What can you do to ensure that you still have the interests of your students at heart?

What happened to the negotiated curriculum?

Can it, or does it still occur within the state or national framework?

What can you negotiate, what is set?

Refer participants to *The best laid plans...* [HO15] for discussion points.

Also remember that the issue of assessment, which can be one of the major issues arising from the implementation of state or national frameworks, will be discussed in Module D.

2 Briefly discuss the nature of a process of negotiated curriculum as described in the reading. Ask:

How does such a notion of curriculum fit the process we have just gone through in developing a program?

Is the model proposed appropriate for all the situations in which you teach?

The notion of a 'continuum of negotiation'—from totally prescribed to entirely negotiated—might be useful in thinking about what is appropriate in different situations.

3 Suggest that participants might like to bring in examples of program planning and resources to share next time.

C5 Excavating maths

Time: 6 hours

Brief description

Participants engage in a number of mathematical excavations uncovering and activating maths embedded in context. They address issues such as maths in context, who learns and who benefits from maths learning and knowledge, whilst highlighting the critical nature of numeracy.

C5.1 Who learns? who benefits? a maths excavation

Brief description

Participants engage in a mathematical excavation into a selected topic.

Rationale

In this session, in session C5.4 and in Curriculum Project 3 which follows, participants will be involved in using a range of everyday materials to uncover and activate maths embedded in context. Before choosing their own topics to excavate in the later sessions, the Presenter offers the participants a prepared set of materials on a theme, so that the Group can jointly develop and share ideas about possible strategies, benefits and pitfalls associated with this approach to learning mathematics. As an essential part of the exercise, issues about access to maths education are raised and explored.

Preparation

Presenter
Read
Lee L (1992) and Helme (1994)
Photocopy 5 or 6 sets of materials putting each set in a large envelope. Choose from those provided in *Gender and maths: some issues* [AS6] or use materials on the theme you have chosen.

Participants
Read
'Real girls don't do maths' [P8]

Time: 2 hours

Materials needed

Concrete materials
as many learning resources as possible, including books and articles, concrete materials, mathematicians —particularly ones that might be useful for learning and teaching about percentages and statistics
a selection of present and past texts
a package of resources on the theme you have chosen, or from *Gender and maths: some issues* [AS6] which is a package of resources on gender and access to maths consisting of M1 to M13.

Handouts/OHTs/paper
Maths embedded in context [P7]
'Real girls don't do maths' [P8]
A mathematical excavation [HO16]

Preparation

Presenter
Read
Lee L (1992) and Helme (1994)
Photocopy 5 or 6 sets of materials putting each set in a large envelope. Choose from those provided in *Gender and maths: some issues* [AS6] or use materials on the theme you have chosen.

Participants
Read
'Real girls don't do maths' [P8]

Time: 2 hours

Materials needed

Concrete materials
as many learning resources as possible, including books and articles, concrete materials, mathematicians —particularly ones that might be useful for learning and teaching about percentages and statistics
a selection of present and past texts
a package of resources on the theme you have chosen, or from *Gender and maths: some issues* [AS6] which is a package of resources on gender and access to maths consisting of M1 to M13.

Handouts/OHTs/paper
Maths embedded in context [P7]
'Real girls don't do maths' [P8]
A mathematical excavation [HO16]

References

Alpers, J. 1985, 'Sex differences in brain asymmetry: a critical analysis', *Feminist Studies*, Vol. 11, No. 1, pp. 7–37.
Assessment of Performance Unit, 1982, *Mathematical Performance, Primary Survey Report*, No. 3, HMSO, London.
Bellisari, A. 1989, 'Male superiority in mathematical aptitude: an artefact', *Human Organisation*, Vol. 48, No. 3, p. 276.
Cockburn, C. 1985, *Machinery of Dominance*, Pluto Press, London.
*Evans, J. 1989, 'The politics of numeracy', in P. Ernest (ed) *Mathematics Teaching: the state of the art*, Falmer Press, London.
Hanna, G., Kundiger, E. & Larouche, C. 1990, 'Mathematical achievement of Grade 12 girls in fifteen countries', in L. Burton (ed) *Gender and Mathematics: an international perspective*, Cassell, London.
Helme, S. 1994 'Maths embedded in context: how do students respond?' in *Numeracy in Focus*, pp. 24–32.
Leder, G. 1992, 'Mathematics and gender: changing perspectives', in D. Grouws (ed.) *Handbook of Research on Mathematics Teaching and Learning: a project of the National Council of Teachers of Mathematics*, Macmillan, New York.
Lee, L. 1992, 'Gender fictions', *For the Learning of Mathematics*, Vol. 12, No, 1, pp. 28–37.
Shuard, H. 1986, *Primary Mathematics Today and Tomorrow*, Longman, London.
Walkerdine, V. 1989, *Counting Girls Out*, Virago Press, London.
*Willis, S. 1989, *'Real Girls Don't Do Maths': gender and the construction of privilege*, Deakin University Press, Geelong.

C5.1 Who learns? who benefits? a maths excavation

Detailed procedure

This section consists of a short introductory discussion, a longer small group activity, and a concluding discussion:

–	**What is maths in context?**	(15 min)
–	**A sample excavation**	(1 h 15 min)
–	**Possible directions, mathematical and social**	(30 min)

What is maths in context?

1 Ask: *What does it mean to teach maths in context?*

Get participants to give a few examples, and to suggest benefits and problems.

2 Ask:

If the context was 'gender and access to mathematics', what sort of maths might it be possible to learn from an investigation of this issue?

Briefly brainstorm a few ideas.

3 Further discussion about the issue of teaching maths in context and the implications for teachers could be based around the article by Sue Helme, 'Maths embedded in context: how do students respond?' [P7]. This small study raises some very pertinent issues. However, point out that the author states that her results are not statistically significant.

A sample excavation

1 Explain that in the next session (C5.2) and in Curriculum Project 3 which follows, participants will be involved in using a range of everyday materials to uncover and activate maths embedded in context. Before choosing their own topics to excavate in the later sessions, they will excavate in this session a prepared set of materials, so that the whole Group can jointly develop and share ideas about possible strategies, benefits and pitfalls associated with this approach to learning mathematics. Resources to create a set of materials on the theme of gender and mathematics is provided in AS6. You may choose to use these, or choose your own theme and create an appropriate set of materials for excavating.

2 Ask them also to consider in this situation the role that maths plays in conveying information, and the interplay between maths, language and context, including the political context.

3 Let them know that there is a range of resources available in the room to assist in the excavation and exploration of the mathematics, including books and concrete materials. Encourage them to use these and also to view collaboration as a resource for maths learning.

4 Hand out *A mathematical excavation* [HO16] and the package of materials you have prepared and let them get on with it. Tell them that they have about 1 hour.

Possible directions, mathematical and social

1 When they have resurfaced from their excavations, get each group to report back to the whole Group, on what they set about to excavate and how successfully or otherwise they activated some mathematics to gain a better understanding of their contexts. Encourage them to speak about:

- the process of excavation and grappling with maths
- how they felt at various points
- what decisions had to be made
- how the maths shed light on the social issues involved.

2 Conclude by getting the participants to spend 10 minutes or so writing in their Journals about some of these issues.

C5.2 Resourcing numeracy

Brief description

Participants examine a variety of resources for numeracy teaching.

Rationale

One of the outcomes for which the course aims is heightened resourcefulness on the part of the participants. In the session (C5.1) preceding this one, participants will have been engaged in a sample mathematical excavation involving, amongst other possibilities, statistics and percentages. In this session they will examine the resources they used then, and others, to identify and analyse whether and how they would be useful in such an activity, and if not, in what other ways they might be useful for numeracy activities.

Preparation

Presenter
Collect a range of teaching resources.
Read
Marr & Helme (1994) and Kindler & Tout (1991)

Participants
Bring a resource that you think might be useful in teaching statistics or percentages. This could range from concrete materials to newspaper articles to student worksheets to ancient or modern maths texts; from ready-to-use to needing radical adaptation.

Time: 1 hour

Materials needed

Concrete materials
as many learning resources as possible, including books and articles like the ones included in References, concrete materials, mathematicians, present and past texts—particularly materials that might be useful for learning and teaching about percentages and/or statistics

Handouts/OHTs/paper
User-friendly numeracy resources [P9]
Criteria for good Adult Basic Education resources [HO17]

References

Banwell, C., Saunders, K. & Tahta, D. 1986, *Starting Points: for teaching mathematics in middle and secondary schools*, Stradbroke, Tarquin Publications, UK.

Castles, I. 1992, *Surviving Statistics: a user's guide to the basics*, Australian Bureau of Statistics.

Goddard, R., Marr, B. & Martin, J. 1991, *Strength in Numbers: a resource book for teaching adult numeracy*, Division of Further Education, Victoria.

Harris, M. 1991, *Schools, Mathematics and Work*, Falmer Press, London.

Hogben, L. 1936, *Mathematics for the Million*, Allen & Unwin, London.

Irvine, J., Miles, I. & Evans, J. (eds) 1979, *Demystifying Social Statistics*, Pluto Press, London.

Jacobs, H. 1982, *Mathematics: a human endeavor*, Freeman & Co, New York.

Joseph, G.G. 1991, *The Crest of the Peacock: non-European roots of mathematics*, Tauris, London.

Kindler, J. & Tout, D. 1991, 'Criteria for good adult basic education resources', *ARIS Bulletin*, Vol. 2, No. 4, pp. 12–13.

Marr, B. 1992, *An Annotated Bibliography of Basic Maths and Numeracy Resources for Adults*, Adult Community and Further Education Board, Victoria. (Available free from ARIS)

Marr, B. & Helme, S. 1987, *Mathematics: a new beginning*, State Training Board, Victoria.

Marr, B. & Helme, S. 1991, *Breaking the Maths Barrier: a kit for building staff development skills in adult numeracy*, DEET, Canberra.

Marr, B. & Helme, S. 1994, 'User-friendly numeracy resources (or how we overcame the tyranny of the traditional text)', in S. McConnell & A. Treloar (eds), *Voices of Experience*, Book 4, 'Reframing Mathematics', DEET, Canberra, pp. 36–44.

Marr, B., Tout, D. & Anderson, C. 1994, *Numeracy on the Line: language-based activities for adults*, National Automotive Industry Training Board, Victoria.

Nelson, D., Joseph, G. & Williams, J. 1993, *Multicultural Maths: teaching mathematics from a global perspective*, Oxford University Press, Oxford.

Nunes, T., Schliemann, A. & Carraher, D. 1993, *Street Mathematics and School Mathematics*, Cambridge University Press, Cambridge.

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Taylor, E. & Giffard Huckstep, S. 1992, *Taking on Technology: a manual for teachers*, Women in Engineering, UTS, Sydney.

C5.2 Resourcing numeracy

Detailed procedure

In this session, participants work individually or in pairs and then share their findings with the whole Group.

– **Analysing resources**	(45 min)
– **Sharing results**	(15 min)

Analysing resources

1 With participants brainstorm the questions:

What information would be helpful to know about a numeracy resource?

What criteria and aspects make a successful adult numeracy teaching resource?

On whiteboard or OHP write points raised, then develop a Reporting Sheet, by organising these points into a short series of questions or a checklist, including:

- Name, author, date of publication, publisher, availability, price
- Is the resource useable as it is? How?
- How could it be adapted? In what sorts of ways?
- Is it aimed at adults or children?
- What methodology does it promote?
- Is it culturally biased?
- What is its reading level?
- Is its presentation appropriate and relevant?
- Is it a resource for:
  a) mathematical routines?
  b) mathematical concepts? which ones?
  c) historical contexts?
  d) social or cultural contexts?
  e) critical analysis of the use of maths?

2 Participants select one of the books or articles they used for the sample excavation in the previous session (C5.1). Ask them to choose one that they found useful. They may choose to work individually on a single resource or in pairs on several. After a quick reading, get them to use the above questions to identify and analyse:

- whether and how the selected article or book and the resource they have brought with them would be useful in teaching percentages and/or statistics.
- and in what other ways the resources might be useful for numeracy activities that they would like to know more about teaching.

3 In preparation for reporting, participants write up their findings on the Reporting Sheet devised earlier. You could collect these and later collate and return the results to participants as a Resource Audit Checklist.

Sharing results

1 Participants report to Group, each participant briefly describing the resource, its uses and problems for teaching percentages and/or statistics, as well as for teaching numeracy more generally.

2 Give out *User-friendly numeracy resources* [P9] and/or *Criteria for Good Adult Basic Education resources* [HO17] as reading material about the use of resources in ABE teaching.

C5.3 Critical literacy, critical numeracy

Brief description

Participants develop questions to foster critical mathematics awareness.

Rationale/aims

During Module C, the participants have explored mathematics in relation to a number of social issues. In this session, they use work done on critical literacy awareness to reflect on what they have been doing and to develop questions which might be fruitful to use with their students in developing the practice of critical mathematics. The questions will be of immediate application as they go on to the excavation in the next session, and to the following Curriculum Project.

Preparation

Presenter
Read:
Freebody & Luke (1990) also available as P10.

Time: 1 hour

Materials needed

Concrete materials
large sheets of paper
textas
Handouts
Reader roles [P10]
Literacy AND numeracy? [HO18]
Critical literacy awareness [HO19]
Teaching critical maths [HO20]
Questions for critical numeracy [HO21]

References

Barnes, M. 1995, Teaching critical mathematics in secondary schools, in M. Barnes, B. Johnston & K. Yasukawa, 'Critical Numeracy: a poster', presented at the Regional ICMI conference, Monash University.
Freebody, P. & Luke, A. 1990, 'Literacies programs: debate and demands in cultural context', *Prospect*, Vol. 5, No. 3, pp. 7–16.
Johnston, B. & Yasukawa, K. 1995, Becoming critically numerate, in M. Barnes, B. Johnston & K. Yasukawa, 'Critical Numeracy: a poster', presented at the Regional ICMI Conference, Monash University.
Lee, A., Chapman, A. & Roe, P. 1993, *Report on the Pedagogical Relationships between Adult Literacy and Numeracy*, DEET, Canberra.
Wallace, C. 1992, 'Critical language awareness in the EFL classroom', in N. Fairclough (ed.) *Critical Language Awareness*, Longman, London.
Yasukawa, K. 1995, *Notes on Critical Numeracy for the Graduate Diploma of Adult Basic Education*, UTS, Sydney.

C5.3 Critical literacy, critical numeracy

Detailed procedure

This session begins with a brainstorm and shifts into several small group tasks:

–	**An excursion into literacy**	(45 min)
–	**Constructing a critical mathematics**	(15 min)

An excursion into literacy

1 Ask participants to think back over the mathematics explored in Module C (*Maths as a critical tool*) from the perspective of how they might teach such material to their students. Ask them to think of questions they might use to increase their students' critical mathematical awareness. Brainstorm these and collect them on the board.

2 Point out that there has been considerable work done on critical literacy awareness. Can we use any of it to clarify our thinking about critical numeracy? Start by reading the article by Freebody & Luke, *Reader roles* [P10]. Divide the Group into 4 groups, each focusing on one 'reader role' and ask the question:
Could these roles be translated into equivalent roles for 'doing maths'? How?

3 Get the groups to report back, explaining the reader role and any possible numeracy or mathematics adaptation of it.

4 Hand out to everyone *Literacy AND numeracy?* [HO18], *Critical literacy awareness* [HO19], and *Teaching critical maths* [HO20]
Using *Questions for critical numeracy* [HO21], the ideas from Freebody and Luke and the list of questions brainstormed earlier, ask the participants to work in small groups on the task of developing a short, practical, concise list of questions that could guide them in developing critical mathematics awareness in their students. Ask them to write the questions clearly on a large sheet of paper.

Constructing a critical mathematics

1 Debrief with the Group. Share the questions, perhaps by displaying them as posters on the walls of the room. (Leave the posters on the wall, to be resources in the next session, C5.4).

2 Go around the room discussing what each group made of the notion of critical mathematics or critical numeracy. Is there a difference between these two (critical mathematics, critical numeracy)? What is the role of the 'human' and the 'social'? Why is

negotiation a key element? Talk to them about how you, as a presenter, have felt about 'teaching' with this (critical constructivist) approach. What does it mean in terms of the power relationships? ownership of knowledge?

3 Conclude by discussing also the usefulness and difficulties of such an approach in the participants' own teaching contexts.

Preparation for the next session

1 Warn participants that in the next session they will work in small groups (or individually if preferred) to excavate the maths from materials they need to bring in themselves. This excavation may, if they choose, form the knowledge base for the following Curriculum Project. You may feel it appropriate to hand out the instructions for the next session, *Mathematical excavations* [HO22], at this point to give participants some idea and warning of the next session's excavations. Make some suggestions about the sorts of materials that they might like to excavate, including art and craft materials and resources, training or trade manuals, newspaper articles, gambling information, bills and forms.

C5.4 Excavating mathematics

Brief description

Participants engage in a mathematical excavation into a topic of their own choice.

Rationale

This Session gives participants greater scope for negotiation than before. Using the experience they have gained from the mathematical explorations undertaken in Module B, the excavations they have done in this section and the questions they developed in the last session, participants will work in small groups (or individually if preferred) to excavate the maths from materials they have chosen to bring in. The effort in this excavation may, if they choose form the knowledge base for the following Curriculum Project.

Preparation	Materials needed
Participants Bring a set of materials to be excavated mathematically. **Time:** 2 hours	**Concrete materials** as many learning resources as possible including books, articles, concrete materials, mathematicians and a selection of present and past texts **Handouts/OHTs/paper** *Mathematical excavations* [HO22]

C5.4 Excavating mathematics

Detailed procedure

The procedure here is similar to that in Session C5.1, except that this time the materials used are brought in by the participants, rather than supplied by the Presenter.

- **An excavation** (1 h 30 min)
- **Possible directions, mathematical and social** (30 min)

An excavation

1 Briefly remind participants that an aim of Module C is to understand how maths can be used as a critical tool, and go over the aims of this current section as stated in *Mathematical excavations* [HO22].

2 Ask the participants to spend 2 or 3 minutes jotting down their objectives and expectations for the session.

4 Quickly go around the room and ask each participant who they are working with, and what contexts they are excavating. Remind them that they should be viewing collaboration as a resource for maths learning.

5 If you haven't done so already, hand out *Mathematical excavations* [HO22] (which is similar but not the same as *A mathematical excavation* [HO16] in Section C5.1) and let them get on with it. Tell them that you will ask them to present a short report on their excavation during the last half hour of the session.

Possible directions, mathematical and social

1 When they have resurfaced from their excavations, get each group to report back to the whole Group, on what they set about to excavate and how successfully or otherwise they activated some mathematics to gain a better understanding of their contexts. Encourage them to speak about:

- the process of excavation and grappling with maths
- how they felt at various points
- what decisions had to be made
- how the maths shed light on the social issues involved.

2 Remind participants that they may like to use this excavation as the basis for Curriculum Project 3. They should however feel free to use a different starting point if they wish.

C6 Curriculum project 3 – Developing a critical view

Brief description

Participants choose a topic and develop teaching materials to include a critical perspective of both the social and mathematical contexts.

Rationale/aims

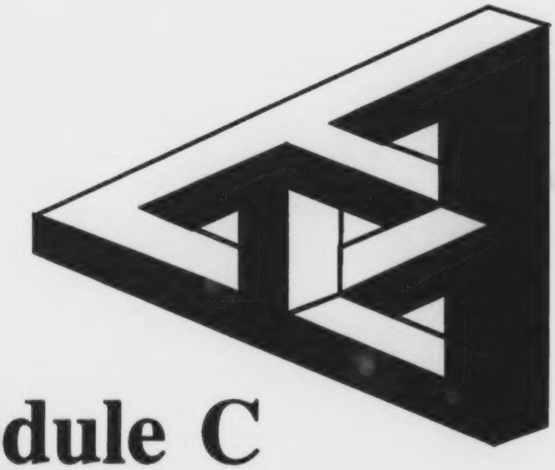
'Developing a critical view' is the third of the four Curriculum Projects in the course. Like the second project, it is a more substantial project than the others, where participants develop teaching materials, around a topic or context of their own choosing. The focus chosen should be one that will allow them to generate activities that will help their students to activate and develop mathematical knowledge and to become more critical of the role that maths plays in the given context.

Preparation	Materials needed
Presenter Read Brown (1984) and Lerman (1989) **Participants** Read *Curriculum project 3: Developing a critical view* [CP3–1] *Developing a critical view: contract* [CP3–2] *Mathematics through problem-posing* [CP3–3] *User-friendly numeracy resources* [P9] **Time:** 9 hours over several weeks	**Handouts/OHTs/paper** Bring as many learning resources as possible—including books, articles, newspapers, journals (e.g. *New Internationalist*), trade course outlines and manuals, craft books, concrete materials, mathematicians. *Curriculum project 3: Developing a critical view* [CP3–1] *Developing a critical view: contract* [CP3–2] *Mathematics through problem-posing* [CP3–3] *User-friendly numeracy resources* [P9] *Poverty traps* [CP3–4]

Procedure

This Project should be timed to start towards the end of the second Mathematical Journey (Module C) and should extend over at least three weeks. The first quite difficult task is to negotiate contracts with the participants.

Detailed instructions for this Curriculum Project will be found in the **Curriculum Projects and Numeracy Journal** section.



Module C
Mathematics as a critical tool

Resources

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List of readings

Make copies of the following readings and distribute them before the course begins, at the beginning of Module C or when indicated in Module C Presenter's Notes.

P1	Domains of learning	304
P2	Dealing with difference	306
P3	Why I like numbers....	319
P4	Social practice of mathematics	324

All the readings above are supplied in Resources C, on the pages indicated.

The following readings are not supplied and must be obtained before they are needed for copying and distribution to participants. Alternatively, direct participants to obtain copies themselves.

P5	Evans, J. 1989, 'The politics of numeracy', in P. Ernest (ed.) *Mathematics Teaching: the state of the art*, Falmer Press, London, pp. 203–205 & 210–214.[1]
P7	Willis, S. 1989, *'Real girls don't do maths': gender and the construction of privilege*, Deakin University Press, Geelong, Chapters 1 & 5.[2]
P8	Marr, B. & Helme, S. 1994, 'User-friendly numeracy resources (or how we overcame the tyranny of the traditional text)', in *Reframing Mathematics*, Deakin University and DEET, pp. 36–44.
P9	Freebody, P. & Luke, A. 1990, 'Literacies programs: debate and demands in cultural context', *Prospect*, Vol. 5, No. 3, pp. 7–16.

The following reading is not supplied as it is available in *Numeracy in Focus* , which participants were advised to purchase from ALIO or ARIS before the course began.

P6	Helme, S. 1995, 'Maths embedded in context: how do students respond?' *Numeracy in Focus* , No. 1, pp. 24–32.

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[1] Available in Johnston (ed.) 1991, *Reclaiming Mathematics*, University of Technology, Sydney, pp. 44–45

[2] Available in Johnston (ed.) 1991, *Reclaiming Mathematics*, University of Technology, Sydney, pp. 71–78

The three terrains of the Mathematical Journey

OHT1

numbers as
metaphors

counting as a
cultural activity

guessing,
estimation and
precision

On number numbness

HO1
page 1

From Hofstadter 1986, 'On Number Numbness', from *Metamagical Themas: Questing for the Essence of Mind and Pattern*, © Penguin, pp. 129–130 and pp. 117–118.

As an aid to numerical horse sense, I thought I would indulge in a small orgy of questions and answers. Ready? Let's go! How many letters are there in a bookstore? Don't calculate—just guess. Did you say about a billion? That has nine zeros (1,000,000,000). If you did, that is a pretty sensible estimate. If you didn't, were you too high or too low? In retrospect, does your estimate seem far-fetched? What intuitive cues suggest that a billion is appropriate, rather than, say, a million or a trillion? Well, let's calculate it. Say there are 10,000 books in a typical bookstore. (Where did I get this? I just estimated it off the top of my head, but on calculation, it seems reasonable to me, perhaps a bit on the low side.) Now each book has a couple of hundred pages filled with text. How many words per page—a hundred? A thousand? Somewhere in between, undoubtedly. Let's just say 500. And how many letters per word? Oh, about five, on the average. So we have 10,000 x 200 x 500 x 5, which comes to five billion. Oh, well—who cares about a factor of five when you're up this high? I'd say that if you were within a factor of ten of this (say, between 500 million and 50 billion) you were doing pretty well. Now, could we have sensed this *in advance*—by which I mean, *without calculation?*

We were faced with a choice. Which of the following twelve possibilities is the most likely:

(a) 10
(b) 100
(c) 1,000
(d) 10,000
(e) 100,000
(f) 1,000,000
(g) 10,000,000
(h) 100,000,000
(i) 1,000,000,000
(j) 10,000,000,000
(k) 100,000,000,000
(l) 1,000,000,000,000?

In the United States, this last number, with its twelve zeros, is called a *trillion*; in most other countries it is called a *billion*. People in those countries reserve 'trillion' for the truly enormous number 1,000,000,000,000,000,000—to us a 'quintillion'—though hardly anyone knows that term.

What most people truly don't appreciate is that making such a guess is very much the same as looking at the chairs in a room and guessing quickly if there are two or seven or fifteen. It is just that here, what we are guessing at is the number of zeros in a numeral, that is, the logarithm (to the base 10) of the number. *If we can develop a sense for the number of chairs in a room, why not as good a sense for the number of zeros in a numeral?* That is the basic premise of this article.

HO1
page 2

These large and small numbers are so far beyond our ordinary comprehension that it is virtually impossible to keep on being more amazed. The numbers are genuinely beyond understanding—unless one has developed a vivid feeling for various exponents. And even with such an intuition, it is hard to five the universe its awesome due for beings so extraordinarily huge and at the same time so extraordinarily fine-grained. Number numbness sets in early these days. Most people seem entirely unfazed by words such as 'billion' and 'trillion'; they simply become synonyms for the meaningless 'zillions'.

This hit me particularly hard a few minutes after I had finished a draft of this column. I was reading the paper, and I came across an article on the subject of nerve gas. It stated that President Reagan expected the expenditures for nerve gas to come to about $800 million in 1993, and $1.4 billion in 1984. I was upset, but I caught myself being thankful that it was not $10 billion or $100 billion. Then, all at once, I really felt ashamed of myself. That guy has some nerve gas! How could I have been *relieved* by the figure of a "mere" $1.4 billion? How could my thoughts have become so dissociated from the underlying reality? One billion for nerve gas is not merely lamentable; it is odious. We cannot afford to become number-number than we are. We need to be willing to be jerked out of our apathy, because this kind of 'joke' is in very poor taste.

Survival of our species is the name of the game. I don't really care if the number of mosquitoes in Africa is greater or less than the number of pennies in the gross national product. I don't care if there are more glaciers in the Dead Sea or scorpions in Antarctica. I don't care how tall a stack of one billion dollar bills would be (an image that President Reagan evoked in a speech decrying the size of the national debt created by his predecessors). I don't care a hoot about pointless, silly images of colossal magnitudes. What I *do* care about is what a billion dollars *represents* in terms of buying power: lunches for all the schoolkids in New York for a year a hundred libraries, fifty jumbo jets, a few years' budget for a large university, one battleship, and so on. Still, if you love numbers (as I do), you can't help but blur the line between number play and serious thinking, because a silly image converts into a more serious image quite fluidly. But frivolous number virtuosity, enjoyable though it is, is far from the point of this article.

What I hope people will get out of this article is not a few amusing titbits for the next cocktail party, but an increased passion about the importance of grasping large numbers. I want people to understand the very real consequences of those very surreal numbers bandied about in the newspaper headlines as interchangeably as movie stars' names in the scandal sheets. *That's* the only reason for bringing up all the more humorous examples. At bottom, we are dealing with perceptual questions, but ones with life-and-death consequences!

Are we drowning in digits?

HO2
page 1

From Davis & Hersh 1986, 'Are We Drowning in Digits',
in *Descartes' Dream*, © Penguin, London, pp. 15–17.

The Post Office has recently added four digits to its zip numbers. They promise better service, but cannot guarantee it. To call England I must dial fifteen digits (but then I have the thrill of crossing the ocean myself). Institutions installing tricky new phone systems are sending their secretaries to seminars to teach them how to call the office down the hall. For instant money, available twenty-four hours a day, I am encouraged to get a magic card and follow a simple program. I have no doubt that within a few short years, I will have to do some preliminary programming in order to use a public convenience. Putting a nickel in the slot will be listed among the Holy Simplicities of the Past. Are we drowning in digits? Is the end in sight?

Yes, we are, and no, it is not.

What underlies all the digits is that our civilisation has been computerised. We are in the grips of the symbol processors and the number crunchers. The nature of this slavery is often misunderstood. It is not thraldom to an individual computer; rather it is the total computerisation of the sources of information and communication. Every time a dentist fills a cavity a computer, somewhere, finds out about it and sends a bill. Unplug the computer network? No way. Your son-in-law may have a good job programming the billing system. The dentist himself owns IBM stock.

Numbers and symbol processing; this is mathematics. 'Study mathematics! It keeps your options open.' Mathematics has joined mechanism and money. Some people think this combination is the monstrosity of the age. Others say, on the contrary, it is the road to salvation. In the new Jerusalem, people speak FORTRAN or BASIC. A computer game can be the new theophany. 'I compute, therefore I am' is the new assertion of existence.

We all see the benefits of computers: trips to the moon, pacemakers, intractable mathematical problems solved in a jiffy. We do not yet see the price that will be paid for a state of super-digitalisation.

There is occurring today a mathematisation of our intellectual and emotional lives. Mathematics is not only applied to the physical sciences where successes have been thrashed out over the centuries but also to economics, sociology, politics, language, law, medicine. These applications are based on the questionable assumption that problems in these areas can be solved by quantification and computation. There is hardly any limit to the kind of things to which we can attach numbers or to the kinds of operations which are said to permit us to interpret these numbers. We are awash in questionnaires, statistics.

Standard deviations and correlation coefficients are spat out by computers held in the hands of the uncritical and used as hammers to pulverise us into compliance with the conclusions of the investigator. (Do you think of yourself as deprived? Yes: 17%. No: 48%. Don't understand what deprived means: 12%. Other 23%.) The Criterion Makers tell us that society should move so that such and such a norm is optimised, and they base policy on this, but no one can say why the criterion is itself appropriate.

HO2

page 2

Excessive computerisation would lead to a life of formal actions devoid of meaning, for the computer lives by precise languages, precise recipes, abstract and general programs wherein the underlying significance of what is done becomes secondary. It fosters a spirit-sapping formalism.

The computer is often described as a neutral but willing slave. The danger is not that the computer is a robot but the humans will become robotised as they adapt to its abstraction and rigidities.

The problem in the coming years is that of establishing meaning in a sea of neutral symbols.

Food figures for thought Question sheet – China

✣ AS1

Please write down your answer to each of the following questions on this sheet.

1 What is your estimate of the population of China in 1990?

2 What is your estimate of the per capita consumption of grain in China in 1990? Give your estimate in kilograms.

3 What is your estimate of the per capita consumption of eggs in China in 1990? Give your estimate in kilograms.

Food figures for thought Question sheet – USA

✣ AS2

Please write down your answer to each of the following questions on this sheet.

1 What is your estimate of the population of the USA in 1990?

2 What is your estimate of the per capita consumption of grain in the USA in 1990? Give your estimate in kilograms.

3 What is your estimate of the per capita consumption of eggs in the USA in 1990? Give your estimate in kilograms.

Food figures for thought Discussion sheet

✣ AS3

1 From the answer sheets you have, find the highest, lowest, and the average estimates for the population, grain consumption and egg consumption for the country.

Population
  Highest
  Lowest
  Average

Grain consumption
  Highest
  Lowest
  Average

Egg consumption
  Highest
  Lowest
  Average

2. What are some of the personal resources we call on when we try to estimate figures such as these?

Food figures for thought Summary sheet

OHT2

population (1990)		
	China	USA
Group 1	Highest	Highest
	Lowest	Lowest
	Average	Average
	Number of responses	Number of responses
Group 2	Highest	Highest
	Lowest	Lowest
	Average	Average
	Number of responses	Number of responses
per capita consumption of grain in kg (1990)		
Group 1	Highest	Highest
	Lowest	Lowest
	Average	Average
	Number of responses	Number of responses
Group 2	Highest	Highest
	Lowest	Lowest
	Average	Average
	Number of responses	Number of responses
per capita consumption of eggs (1990)		
Group 1	Highest	Highest
	Lowest	Lowest
	Average	Average
	Number of responses	Number of responses
Group 2	Highest	Highest
	Lowest	Lowest
	Average	Average
	Number of responses	Number of responses

State of the world 1994

HO3
page 1

Excerpts from L.R. Brown et al. 1994, 'Facing Food Insecurity' in *State of the World 1994 - A Worldwatch Institute Report*, Earthscan Publications, © W.W. Norton & Co., New York.

… and with rangelands widely overgrazed in most countries, there is an urgent need for national assessments of carrying capacity. Otherwise, there is a real risk that countries will blindly overrun their food carrying capacity, developing massive deficits that will collectively exceed the world's exportable supplies. Recent data showing the level at which the rise in grain yield per hectare is slowing or levelling off in countries with a wide range of growing conditions provide all governments with the reference points needed to estimate the population carrying capacity of their croplands.

China, which already has one of the slowest population growth rates in the developing world, is projected to add 490 million people over the next four decades, increasing to 1.6 billion in 2030. (See Table 10-4.) Currently it is adding 14 million people per year. Meanwhile, its economy is expanding at 10 percent or more annually, fuelling steady rises in consumption of pork …

Table 10.4 Population growth 1950-90, with projections to 2030 for the most populous countries

				increase	
country	***1950***	***1990***	***2030***	***1950-90***	***1990-2030***
			(million)		
Slowly Growing Countries					
United States	152	250	345	98	95
Russia	114	148	161	34	13
Japan	84	124	123	40	-1
United Kingdom	50	58	60	8	2
Germany	68	80	81	12	1
Italy	47	58	56	11	-2
France	42	57	62	15	5
Rapidly Growing Countries					
Philippines	21	64	111	43	47
Nigeria	32	87	278	55	191
Ethiopia and Eritrea	21	51	157	30	106
Iran	16	57	183	41	126
Pakistan	40	118	260	78	142
Bangladesh	46	114	243	68	129
Egypt	21	54	111	33	57
Mexico	28	85	150	57	65
Turkey	21	57	104	36	47
Indonesia	83	189	307	106	118
India	369	853	1443	484	590
Brazil	53	153	252	100	99
China	563	1134	1624	571	490

Census Bureau data are used because they are updated more often than the UN medium-range projection; the two series are usually similar, but small differences do exist. SOURCE: US Bureau of the Census, in Francis Urab and Ray Nightingale, World Population by Country and Region, 1950–90 and Projections to 2050 (Washington, DC, US Department of Agriculture, Economic Research Service, 1993).

314

Treading more lightly

In low-income countries, where diets are often dominated by a single starchy staple, rises income quickly translate into consumption of more livestock products. This rise in animal protein intake, which both improves nutrition and adds variety to an otherwise monotonous diet, is widely seen as an early sign of progress. When asked by a *New York Times* reporter if living conditions were improving, a Chinese villager responded, 'Overall life has gotten much better. My family eats meat maybe four or five times a week now. Ten years ago we never had meat.'

Thus as incomes rise, so does grain use. In low-income countries, grain consumption per person averages some 200 kilograms a year, roughly one pound per day. (See Table 1-2 in Chapter 1.) At this level, diets are high in starch and low in fat and protein, with up to 70 percent or more of caloric intake coming from one staple, such as rice.

By contrast, individuals in affluent societies such as the United States consume some 800 kilograms of grain a year, the bulk of it indirectly in the form of beef, mutton pork, poultry, milk, cheese, yogurt, ice cream, and eggs. Grain use tracks income upward until it reaches this level, producing a grain consumption ratio between those living in the world's wealthiest and poorest countries of roughly four to one.

The 800 kilograms of grain consumed per person each year in the United States translates into a diet rich in livestock products: as meat, it includes 42 kilograms of beef, 28 kilograms of pork, and 44 kilograms of poultry. (See Table 10-5). From dairy cows, it includes 271 kilograms of milk, part of it consumed directly and part as cheese (12 kilograms), yogurt (2 kilograms), and ice cream (8 kilograms). Rounding out this protein-rich fare are more than 200 eggs a year.

Table 10.5 Per capita grain use and consumption of livestock products in selected countries, 1990

		consumption						
country	***grain use***	***beef***	***pork***	***poultry***	***mutton***	***milk***	***cheese***	***eggs***
	(kilograms)				(kilograms)			
United States	800	42	28	44	1	271	12	16
Italy	400	16	20	19	1	182	12	12
China	300	1	21	3	1	4	-	7
India	200	-	0.4	0.4	0.2	31	-	13

Data rounded to nearest 100 kilograms, as the purpose here is to contrast the wide variation in consumption of livestock products associated with different levels of grain use. Total consumption, including that used to produce cheese, yogurt, and ice cream. SOURCE:
UN Food and Agriculture Organisation, *FAO Production Yearbook 1990* (Rome 1991).

Families in the western United States, for instance, often use as much as 3,000 litres of water a day-enough to fill a bathtub 20 times. Over development of water there has contributed to the depletion of rivers and aquifers, destroyed wetland and fisheries, and, by creating an illusion of abundance, led to excessive consumption. Meanwhile, nearly one out of every three people in the developing world-some 1.2 billion people in all-lack access to a safe supply of drinking water. This contributes to the spread of debilitating disease and death, and forces women and children to trek many hours a day to collect a few jugs of water to meet their family's most basic needs.

Disparities in food consumption are revealing as well. (See Table 1-2.) As many as 700 million people do not eat enough to live and work at their full potential. The average African, for instance, consumes only 87 percent of the calories needed for a healthy and productive life. Meanwhile, diets in many rich countries are so laden with animal fat as to cause increased rates of heart disease and cancers. Moreover, the meat-intensive diets of the wealth usurp a disproportionately large share of the earth's agricultural carrying capacity since producing one kilogram of meat takes several kilograms of grain. If everyone in the world required as much grain for their diet as the average American does, the global harvest would need to be 2.6 times greater than it is today-a highly improbable scenario.

Economic growth-the second driving force-has been fuelled in part by the introduction of oil onto the energy scene. Since mid-century, the global economy has expanded fivefold. As much was produced in two-and-a-half months of 1990 as in the entire year of 1950. World trade, moreover, grew even faster: exports of primary commodities and manufactured products rose elevenfold.

Table 1.2 Grain consumption per person in selected countries, 1990

country	*grain consumption per person*
	(kilograms)
Canada	974
United States	860
Soviet Union	843
Australia	503
France	465
Turkey	419
Mexico	309
Japan	297
China	292
Brazil	277
India	186
Bangladesh	176
Kenya	145
Tanzania	145
Haiti	100
World Average	323

SOURCES: World Watch Institute estimate, based on US Department of Agriculture, *World Grain Database* (unpublished printout) (Washington, DC 1992) Population Reference Bureau, *1990 World Population Data Sheet* (Washington, DC 1990)

The extent to which the overall scale of economic activity damages the earth depends largely on the technologies used and the amount of resources consumed in the process. Electricity generated by burning coal may contribute as much to economic output as an equal amount generated by wind turbines … but burning coal causes far more environmental harm. A similar comparison holds for a ton of paper made from newly cut trees and a ton produced from recycled paper.

Estimation as strategic guessing: three tasks

✣ AS4

How might you go about the each of the following tasks?

- You have planned to bake a cake, using a recipe where all quantities are given by weight. Your kitchen scales are broken.
- You need to budget for your utility (power, phone, etc.) bills over the next 12 months.
- You need to convey the size of the room you are in to someone on the phone who wants to know if the room is big enough to hold a party for 30 people.

As you work through each situation, try to identify:

1 any material resources which could be useful in carrying out the estimation

2 any prior personal knowledge and experience which could be useful

3 what collective reasoning and logic might be useful

4 what calculations may be necessary

5 what calculation tools could be useful

6 what further information would be required, and where one would get it

and

7 what assumptions may have to be made.

The average house price ✣ AS5

Which average would be most useful for each of these people's purposes:

- the mean?
- the median? or
- the mode?

- A real estate agent discussing house prices with someone who is trying to sell
- A real estate agent discussing house prices with someone who is trying to buy
- A property developer who is writing an ad for the homes being built in a new estate in that suburb
- An individual who is thinking of buying a home in that suburb

Some critical questions

HO4

page 1

From Keiko Yasukawa (1994)

Precision: in whose interest?

Are astronomically big and microscopically small numbers always meaningful?

Once you define a centimetre, why not then a millimetre? Why not a nanometre? And a thousandth of that? If you thought a million was big, what about a billion? A trillion? If we have the names of these numbers, and a way of representing them, we can be as 'accurate' and 'precise' as we like. But is that what we 'like'? Does it matter if a country is ten billion dollars or a hundred billion dollars in debt if they do not have a million dollars to pay back anyway? Does it matter if 1287 people died or 1286 people died in a battle if one of them was definitely your friend? Does it matter if the average number of children in an Australian household is 2.3 or 2.35 if you yourself are unable to have any? What does 35% of a child mean anyway? Does it really matter if you put three or four tablespoons of chopped nuts in your cake? But is it wise to say to your anaesthetist, 'I weigh something between 45 and 50 kilograms'? What accuracy is accurate enough? When are we allowed to approximate and when do we require precision? How is precision used as a tool of control in a capitalist society?

Who benefits when prices in shops are 'rounded' to the nearest 5 or 10 cents? Does it matter if you round up or down? If you wanted 4 lots of things for $1.46 each, does it matter if you buy them together, or if you pay for each of them separately? Why don't they price things in lots of 5 cents?

If a picture can convey a thousand words

- can a graph tell a thousand lies?

Graphs can be used both to inform and deceive. Scaling, choices of axes, high-tech production, and surrounding (or absent) text are all tools which can be used effectively to convey information or to distort information.

What do people actually see when they see graphs in the mass media? What do they look for, and what don't they see? How do technology and 'artistic' presentation help to distract the reader?

And how are graphs used appropriately to convey mathematical relationships? How do graphs tell us about patterns of growth? How do we graph phenomena which grow exponentially? How does that 'look' differently to those which grow 'linearly?'

Does a larger sample mean greater representation?

Averages are manufactured from samples so if you want to use statistics to argue that your view represents the 'average', take a sample of one—YOU!

An average, be it mean, median, or mode, is a function of what we choose to include or exclude in our calculation. Data for averages are selected, and estimates based on averages are no less subjective than the subjectivity exercised by the person who selects the 'samples' and excludes other possible samples. Manufacturing averages is a very creative activity indeed. We associate some sense of 'reliability' and 'likelihood' with this 'averages' compared with the individual guesses. Are averages of guesses more reliable than any one guess alone?

Why convert?

—using standards which are imposed for control and conformity

The mean calculated over a period of time is sometimes called the *expectation*. This has the connotation that the mean is what is 'expected'. How does a society which values conformity and 'mediocrity' in this sense, shape and enforce a culture which can expect 'the average'? 'Universal' standards and references are established so that those who control and enforce the desired mediocrity can measure and compare the individual constituents against the 'expectation' to rank them relative to one another. In order to make this industry readily transferable to all the target areas in this crusade towards mediocrity and homogeneity , a single measuring reference is needed. A set of standard units is chosen, metre, litre, the IQ, and so on.

- and how are units of measure needed as a calculating tool in the astronomical and microscopic world?

If you are working out what your employer owes you for *n* hours of work at $x an hour and you can't decide whether to add, subtract, multiply, or divide, you can work out what happens to the units when you perform each of those operations with the two quantities you've got, time and hourly rate, to come to the conclusion that only if you multiply the two will you get something in dollars. Numbers by themselves are dangerously ambiguous; without the units of measure (whether standard or not) they do not tell us what quantity it is we are talking about. In this way, units can guide us in how we combine the numbers in order to derive the desired outcome.

But sometimes a unit of measurement can add to the confusion because it does not reveal the connection between it and the quantity or any other related measure. How is an *Ohm* related to a *Coulomb* except that they are both names of male electrical engineers?

Domains of learning

P1
page 1

From Newman 1993, *The third contract: theory and practice in trade union training*. pp. 175-180

Mezirow describes three 'domains of learning'...

Mezirow takes as his starting point certain ideas drawn from the German social theorist, Jurgen Habermas, and adapts them to the context of adult education. As I read him, Mezirow does not claim to represent Habermas's ideas in detail but simply draws upon them in a search for insights into adult learning. What follows, therefore, is a discussion of Mezirow's interpretations … and Mezirow's analogies.

Habermas … describes three areas in which we generate knowledge. The first is the *technical* area. This is where we create knowledge in order to control and manipulate our environment. This area of knowledge is to do with academic terms, this kind of knowledge and learning is to be found in the empirical-analytical sciences like physics and geology.

Mezirow relates this first area to adult learning and describes it as the kind of *instrumental learning* we engage in to achieve task-related competencies. In this domain of learning we are concerned with understanding relationships between cause and effect, with developing knowledge by testing hypotheses, with the gathering of observable evidence:

> *Instrumental learning always involves a prediction about observable things and events.*

This domain of adult learning relates closely to the mechanistic tradition, to the world of training people to achieve behavioural objectives, to the business of designing and providing learning through needs assessment, task analysis and the rest of it, to the business of training people to survive in the world and perform their jobs. It is concerned with solving problems through weighing up the likely outcomes of a number of options and then testing the validity of the most plausible option.

If Mezirow has taken his line from Habermas, then I would like to take my own line from Mezirow and try to sum this first domain up by saying that it is about *learning to perform a role better* -about gaining the knowledge and skills necessary to be a more efficient waiter or archivist or machinist or, in the union training context, to be a more effective member or organiser or workplace rep or advocate or occupational health and safety rep.

The second area in which we generate knowledge Habermas describes as the '*practical*' area. This is where we create knowledge in order to understand our condition, not so much in terms of our interaction with the material world but in terms of ourselves and our interrelationship with others. In academic terms, this kind of knowledge and learning is to be found in the historical-interpretive sciences such as history, theology and the descriptive social sciences like anthropology and sociology.

Mezirow describes the second domain as *learning for interpersonal understanding*. In contrast to the instrumental learning of the first domain, this domain is concerned through communication and interaction.

> *Most significantly learning in adulthood falls into this category because it involves understanding, describing and explaining intentions; values; ideals; moral issues; social, political, philosophical, psychological or educational concepts; feelings and reasons.*

In this kind of learning we arrive at generalisations and solve problems, not through objectively testing a hypothesis but by reaching a consensus through 'rational discourse', that is through careful and considered communication and consultation with others.

All the adult learning … to do with communication skill, assertion and empathy training, conflict resolution, listening, group work, self-expression, confidence building … would fall into this domain of learning, as would most adult education in the liberal tradition. So that, where I summarised the first domain by saying it was about learning to perform a role better, this domain might be described as being about *learning to be a better person* -that is, better at being a person, better at relating and communicating, more sensitive, more knowledgeable about culture and history, and so better at understanding one's own and others' condition.

I would agree that most if not all union training operates fairly and squarely in these first two domains. But Mezirow argues that there is a third domain, and that this third domain of learning is a peculiarly adult one.

Habermas describes the third area in which we generate knowledge as the *'emancipatory'* area. In this area we are concerned with self-knowledge, with knowing who we are, how we came to be who we are , and the factors that continue to constrain and shape the way we see ourselves. In academic terms this kind of knowledge and learning is to be found in the critical social sciences, such as psychoanalysis and the critique of ideology.

Mezirow describes this third domain as *learning for perspective transformation.* In this kind of learning we do not just learn to look at the world more clearly. We learn *to look at the world.* We learn how to perceive our perceptions. We become aware of our awareness, and of how our awareness is constructed.

Perspective transformation involves not only involves becoming critically aware of habits of perception thought and action but of the cultural assumptions governing the rules, roles, conventions and social expectations which dictate the way we see, think, feel and act.

In this kind of learning, then, we address problems through critical reflection: that is, through examining awareness, through identifying and examining meaning perspectives in order to understand how they influence the way we see the world and the kinds of position we adopt. In this process we may recognise that our perceptions are distorted by 'institutionalised ideologies', reified power relationships' and 'internalised cultural myths' (Mezirow, 1981); and we may set about transforming our perspectives so that we are freed from the kinds of influences that limit our visions and cramp our ability to act.

A union trainer operating in this domain, then, would help members examine their beliefs and how they come to have them. He or she would create situations in which union members could debate policy and question given wisdoms in order to understand how these policies and given wisdoms came to be accepted, and to establish whether they are vital, valid and in the interests of the union and its members, or simply 'institutionalised ideologies'.

The union trainer would help members examine their own ways of relating to others and to 'authority'. She or he would create situations in which participants could examine the 'culturally induced dependency roles' some workers adopt or the 'rectified power relationships' that exist in the structures in workplaces. Participants might, for example, examine how it is that some people -both bosses and workers - can continue accepting the 'right' of a multinational company to make decisions- to close down a factory, say - that will affect the culture and economy of a locality far removed from where the decisions were made.

And the union trainer would help members examine the stories, the dogma, the unquestioned versions of events—theirs and their unions' 'internalised cultural myths'—that lead them to accept particular ways of acting, that prompt statements such as 'it's always been done this way' or 'that's been tried before and didn't work' or 'he was paid to take risks'. The trainer would set up situations in which participants could examine the validities and the falsehoods enshrined in the stories of past struggles, and help them establish a feeling for people and movements and events based on sound information and critical reflection. She or he would help participants identify where set ways of thinking and behaving might lead them to needlessly repeat mistakes.

Mezirow, then, has a lot to offer the union trainer. After all, if we can help our members become critical thinkers, people who perceive their perceptions people who understand how values and assumptions, ideologies and beliefs come to be constructed, then they will be able to subject others- including the boss and anyone else in a position to influence and exploit them- to a similar, critical analysis. If they can see through themselves, they will be able to see through others. At the very least, our members will become people who are extremely difficult to fool.

Someone who successfully engages in learning for perspective transformation cannot stay the same, so if I try to encapsulate this third domain in the same way as I have tried for the other two, I would say the first domain is about learning to perform a role better, the second domain is about learning to be a better person, and the third domain is about *learning not only to be better but different.*

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Dealing with difference

P2

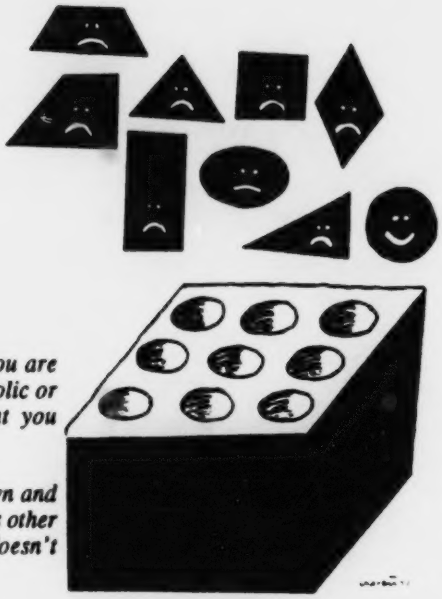
page 1

From Johnston, K. 1990, 'Dealing with difference', *Education Links*, Vol. 38, pp. 26–29

We have recognised over the last decade that social differences between learners profoundly influence educational outcomes. How do we understand, then, recent calls to turn our backs on social distinctions and concentrate on the technical tasks of teaching and learning? KEN JOHNSTON believes that to adopt this stance is to throw the baby out with the bath water.

Dealing with difference

TO YOUR PLACES, PLEASE!



Don't tell me about your social background. I don't care whether you are black or female. I'm not interested in whether your father is an alcoholic or whether your family is poor. All that I am interested in is that you understand calculus and pass the examination.

All kids have got the same needs. We might live in a poor area of town and my school might be disadvantaged but basically we are just the same as other kids. Just because we're not as well off as kids from other schools doesn't mean we are dumb. We can succeed just like any other kid.

Reasonable statements? The spirit of the age? The first epitomises the attitude of the teacher-hero in the popular recent film, *Stand and Deliver*. The second is the voice of a high school student talking at a forum for teachers from the Disadvantaged Schools Program that I attended recently.

They seem to sum up the emerging conventional wisdom of the 1990s. Teachers are not in the job of making social distinctions. At best they distract our attention from what we are good at - teaching and learning - while at worst they stigmatise individuals and communities, and lower expectations of success. If in the seventies we were preoccupied with how social class, gender and race contributed to school failure, our challenge in the nineties is to look at how teaching itself can contribute to success. It's winning, not losing, that matters and sparking the will to win is what the teaching game is all about.

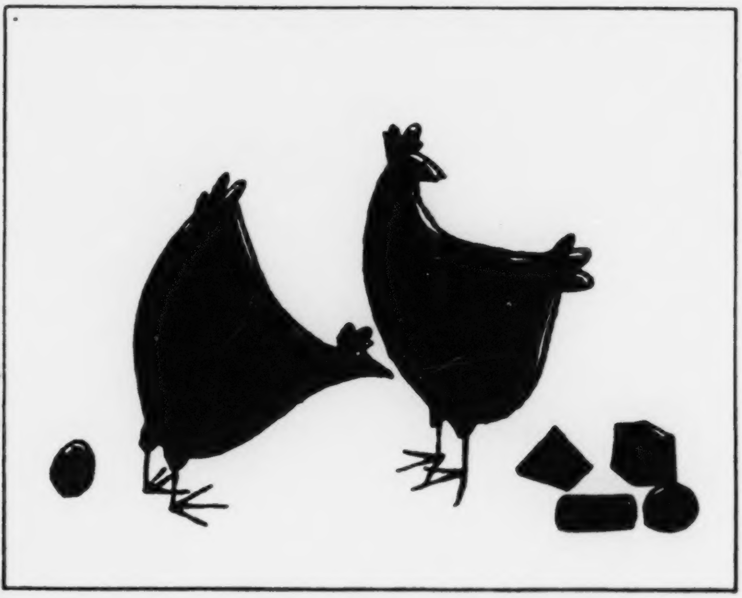
Before we all embrace this rather heroic sense of our vocation and turn our back on the social contexts of learning, it might be useful to clarify how social distinctions enter into the work of teachers. I have recently worked on a research project to examine assessment and evaluation practices in schools that are part of the Disadvantaged Schools Program. We found in talking to teachers about social and educational disadvantage that there is considerable confusion about whether they should take into account the social contexts of their students and if so, how it should influence the work they do. Do disadvantaged students have the same needs as other students? Is there essential knowledge that pertains to all young children regardless of their class, gender or race? Do social distinctions, in short, provide a legitimate or illegitimate basis for making differences in classrooms?

. . . .

I have argued that we need to recognise the way in which social distinctions enter into the cognitive and moral judgements that we use in our day to day practice as teachers. Recognition can help identify the dangers to be avoided. One danger is that by dwelling on difference we may breath new life into old and limiting gender, class and racial stereotypes that teachers bring with them into their work. Another is that difference thinking may construct alternative pathways through the curriculum jungle that lead to less powerful knowledge or less lucrative and satisfying careers. Faced with these obvious dangers, one can understand the tendency to turn one's back on social difference itself and concentrate instead on skills and learning outcomes that are in some sense universal for all learners.

The problem with **difference thinking** is that it may restrict the learners capacity for action and exclude them from powerful forms of knowledge and understanding. The solution, however, is not to throw out the baby with the bathwater. To ignore social difference, on the grounds of its misuse, makes the social nature of our judgements even more invisible and insidious. At present, the social bias built into our moral and cognitive categories works in a relentless, hegemonic way to the advantage of those who have social and cultural power. The solution is not to turn our backs on the social distinction we inevitably use in our teaching but to make them work for those who are most disadvantaged in our classrooms. It implies that we should consistently look at our practice as teachers from the point of view of those who are most disadvantaged in our schools and develop moral and cognitive distinctions that will suit their social purposes.

In order to use this counter-hegemonic strategy in our day to day teaching we need to become much more skilled in **contextual thinking.** here it is useful to distinguish between the principles, skills and knowledge to be learnt (universally applicable for all learners regardless of social background) and the social contexts in which the learning takes place. One of the key characteristics of human learning is not only that we learn more than one think at a time, but that the more enduring learnings are those which we soak up, often unconsciously, from the context itself. All knowledge, even the most abstract mathematical relationship, is context-dependent; it is mediated through language in social situations that reflect class, gender and ethnic inequalities. The implication is that it is not the 'universal' learning (rigorous thought, logical analysis, abstraction etc) that alienate learners but the social and moral contexts in which they are inextricably bound. An essential requirement for counter hegemonic teaching, therefore, is the ability to decode the social and cultural contexts of learning.



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Meeting individual needs HO5

Adapted from *Breaking the Maths Barrier*, Marr & Helme 1991, DEET, Canberra, pages 275–6.

Differences in Ethnic, Cultural or Political Background and Values

- Avoid culturally biased materials. If such material is unavoidable, or comes up as a natural outcome of class discussion, make sure that it is explained.
- Acknowledge cultural differences in the class and make use of them, for example by discussing different techniques of calculation, or sharing favourite recipes, or using a map to locate students' birthplaces.

Differences in Skills Levels and Learning Styles

- Use a variety of teaching strategies and tasks to suit the needs of individual learners. Include whole group, small group and individual activities, hands on materials and visual aids.
- Divide the class into small groups of students at a similar level working together or students with a similar goal, such as passing a selection test.
- When working on a particular topic have available a range of worksheets and activities which treat the topic at different levels. Students can then work at a pace and level that suits them.
- Use self paced materials or individualised worksheets, supplemented with group activities. For example, start the session with a group activity, game, or discussion which involves everyone before breaking into small groups or individual programs. In this way students work at their own pace but also have opportunities for interaction with others.
- Use 'core' and 'extension' questions on teaching materials and worksheets.
- Have worksheets or problem solving activities to challenge or extend students who have completed set work. These can be on worksheets, cards, or posted on the walls.
- If students have finished a task, encourage them to review previous work or return to an ongoing task such as learning multiplication and addition facts. Have a range of activities, games and resources to use, such as dice or cards for generating numbers to practise basic operations.
- Encourage students who have completed set tasks to make up their own problems or quizzes and swap them with each other.
- In a group where some students have reading difficulties, group students so that readers can assist with reading instructions and information.
- Use cross tutoring. More advanced students may enjoy tutoring other students. Take care that this strategy is not overused or that 'tutors' do not confuse or intimidate their 'students'.

Differences in Students' Expectations and Reasons for Attending

- Discuss students' expectations of classes openly and if necessary, negotiate individual goals and work programs.
- To avoid a totally individualised approach … [also use] group activities.

Irregular Attendance and Changing Class Numbers

- Make sessions self contained, with activities which can be completed within the time of the session. Students who miss classes can then rejoin the group without being disadvantaged.
- Encourage students to welcome latecomers and those returning to class after an absence, so that they still feel accepted by the group.

The practice of mathematics OHT3

For any discussion of
the practice of mathematics
we need to confront the idea of context.
This implies a study of everyday life *in situ*
and an emphasis on the 'lived experience'
of people's lives,
rather than focusing exclusively
on people's behaviour
in relatively 'unusual' or artificial settings
such as experimental laboratories
or test situations in schools.

From Evans & Harris 1991, 'Theories of practice'.
in M. Harris (ed.) *Schools, Mathematics and Work*, pp. 204–206.

Work contexts

HO6

page 1

From Evans & Harris 1991, 'Theories of practice' in Harris M. (ed.) *Schools, Mathematics and Work*,
© The Falmer Press, London, pp. 204–206.

The factory chosen for Scribner's research was a milk-processing plant employing about three hundred people. After an ethnographic study of the whole plant, four common blue-collar tasks were chosen for cognitive analysis; all of these were essential for the performance of the job and all of them involved operations with written symbols including numerals. The tasks were product assembly, counting product arrays, pricing delivery tickets and using a computer form to represent quantities of the various products. The objective was to describe the skilled performance and identify the systematic characteristics of each task. Only one of these tasks, product assembly, will be discussed here, very briefly.

Product assembly is carried out by 'pre-loaders' whose task is to make up orders for delivery drivers. It is classed as an unskilled job, is low-paid and is carried out in the low temperatures required for the storage of dairy products. The pre-loaders' work is to collect the various dairy products required by the route drivers who deliver wholesale orders consisting of a range of products specified in gallons, quarts, pints or half-pints. In the plant however, all products are packed in standard-sized cases which contain different quantities of the different products depending on how the individual product items are packed. Although the drivers specify their orders in units, the firm's computer that handles all the paper work, specifies them in case lots. If the number required for an order is not a round case load, then the computer expresses the number as ' case + number of units' for numbers up to and including half a case or as 'case – number of units' for numbers of more than half case. On the print-out, such numbers would appear as '1 + 3' or '2 – 4' for 'one case and three units' or 'two cases less four units' respectively. Detailed observation and carefully controlled intervention with the actual numbers revealed that, in the unpleasant working conditions, the pre-loaders mostly did not work literally from the computer instructions by adding or subtracting items exactly as instructed, but had a large repertoire of strategies involving the saving of physical effort for dealing with what appears to be on the surface such unskilled and repetitive work.

> For example, if an order is 1 – 6 (10) quarts and a preloader has the option of using a full case and removing 6 quarts (the literal strategy) or using a case with 2 quarts already in it and adding 8, the literal strategy is optimal from the point of view of physical effort: it saves 2 moves. If the partial case, however, has 8 quarts and only 2 quarts must be added, filling the order 8 + 2 is the least-physical-effort solution (the saving is 4 quarts) …

The numerical features were triggered by visual arrays in the incomplete cases (not all of whose contents would have been visible). As a result of the observations, the researchers proposed an explanatory 'law of mental effort' by which the workers expended mental work to save physical work. The amount of physical work involved could be checked immediately from observational records. It was clear that in the majority of cases where the pre-loaders did use literacy strategies, the strategies were also the ones that required least physical effort. On every occasion where non-literal strategies were used, they were also least-physical-effort strategies.

The researchers then went on to determine by means of laboratory studies of simulations in which 'pre-loader' performance was compared with 'novice' performance of clerks and students, whether such solutions strategies would transfer to different situation. The inclusion of office clerks provided the most marked novice–expert contrast … for although they shared a common 'cultural knowledge' of dairy products with the pre-loaders they did not actually label the products themselves. Laboratory-based simulations were devised, the outcome of which provided evidence that transfer of solution strategies from the natural to laboratory context did occur and that even when pre-loaders did fail to use optimal strategies, they were never as inefficient as those of the comparison groups.

> Clerks showed little tendency to adapt strategies to the properties of the problem at hand, using non-literal solutions in less than half of the instances in which there were strategies of choice. [School] students by and large were single algorithm problem-solvers; they were over-whelmingly literal ... their use of optimal strategies continued to [depend] on the nature of the problem and the type of conversion required …

In short the performance of the 'unskilled' workers was revealed to be more efficient than that of the 'skilled' workers and it was the latter groups who provided all the examples of the most inefficient solutions …

In product assembly, we have a suggestion as to one possible defining characteristic of practical thinking which might warrant the use of the qualifier 'intelligent', that is, the extent to which thinking serves to organise and make more economical the operational components of tasks ... The issue is not accuracy or error but rather modes of solution. Strategy analysis demonstrated that experience makes for different ways of solving problems, or to put it another way, that the problem-solving process is restructured by the knowledge and strategy repertoire available to the expert in comparison to the novice …

What is called the 'same operation' is done now in one way and now in another, but each way is, as we say 'fitted to the occasion' … Practical thinking is goal-directed and varies adaptively with the changing properties of problems and changing conditions in the task environment. 'In this respect, practical thinking contrasts with the kind of academic thinking exemplified in the use of a single algorithms in this way that is held up by formal mathematics to be an example of the highly valued property of generalisability.

Everyday contexts

HO7

From Evans & Harris 1991, 'Theories of practice', in Harris M. (ed.) *Schools, Mathematics and Work*, © The Falmer Press, London, pp. 206–207.

The suggestion that academic thought may not be a proper yardstick with which to measure, diagnose and prescribe remedies for 'everyday thought' is one of the points taken up by Lave (1988) in her work with the *Adult Mathematics Project* (AMP) in which arithmetic practices in various settings were investigated with the aim of developing a different perspective on problem solving from that of school or laboratory. Lave's investigations arose from concern about the ecological validity of experimental cognitive research in the light of 'speculation that the circumstances that govern problem solving in situations which are not prefabricated and minimally negotiable differ from those that can be examined in experimental situations' (Lave, Murtaugh and de la Rocha, 1984). One such investigation was of arithmetic decision-making processes during grocery shopping. A supermarket appears to be like a factory, in that it is a highly structured environment geared to support a specific task.

The study began with simple questions about the place of arithmetic in such settings, how it is used, whether it is a major or minor part of the activity and what are the procedural differences between arithmetic in such situations and in school. It chose deliberately to conduct its investigations in places other than the 'academic hinterland' and with ' ordinary people' in 'everyday activity'. The research involved 'extensive interviewing, observation, and experimental work with twenty-five adult, expert grocery shoppers' from a wide range of ages and incomes and from both sexes. Field data was obtained by participant observation. Shoppers wore a tape recorder and talked with an accompanying research worker, a procedure that was found less uncomfortable than talking in monologue; the researchers took care not to interpret the situation, but simply to clarify the shoppers' behaviour for the record.

The shoppers' arithmetic in the supermarket was then compared with their performances on pencil and paper tests of arithmetic.

Arithmetic problem solving in the test and shopping situations was quite different. The shoppers' scores averaged 59% on the arithmetic test 'compared with a startling 98%—virtually error free—arithmetic in the supermarket' … Not only was the 'practical workplace arithmetic' almost error—colleagues that the shoppers had already assigned 'rich content and shape to a problem solution by the time arithmetic becomes an obvious next step'. Far from being applied to the problem from the outside, the form and content of the arithmetic used so accurately grew out to it. The arithmetic practice was quite specific to the situation and appeared as … a 'gap-closing process' that draws the problem and the already anticipated form of problem solution closer together. It was the interactive use and monitoring of these processes by the shoppers that accounts for the extraordinarily high level of successful problem solving observed.

Street contexts

HO8
page 1

From Carraher, D. 1991, 'Mathematics learned in and out of school', in Harris M. (ed.) *Schools, Mathematics and Work*, © The Falmer Press, London, pp. 169–170 & 194–196.

On a shady street bordering a city square in Recife, Brazil, residents from the neighbourhood are doing their weekly grocery shopping among rows of fruit and vegetable stands. The following exchange occurs between a customer and a 10 year old vendor.

Customer: I'll take two coconuts (at Cr $40.00 each. Pays with a Cr $500.00 bill). What do I get back?
Child: (Before reaching for customer's change) 80, 90, 100, 420.

In a subsequent interview given at his own home the child, who has had three years of schooling, is asked to determine the sum of four hundred and twenty plus eighty. He picks up a pencil and writes the number 42 with 8 underneath and claims that the answer is 130. (cf. Carraher, Carraher, and Schliemann, 1985, originally in Portuguese)

The issues

The existence of mathematics out of school

There can be no doubt about whether people learn mathematics out of school. Firstly, it has been clearly established that children's understanding of number begins before they have been to school (Piaget, 1952; Gelman and Gallistel, 1978; Hughes, 1986). Secondly, unschooled adolescents and adults routinely perform calculations at work, in naturally occurring situations (Scribner, 1984; Carraher and Schliemann, 1983, 1985; Carraher, Schliemann and Carraher 1988 and in press 1990; Lave, Murtaugh and de la Rocha, 1984). Lastly, it can be shown that people who have been to school occasionally represent, solve or think about mathematical problems in daily life so differently from how they are taught in school that it is difficult to deny that something important has been learned or discovered outside of school. For example, sugar farmers in Brazil's north-east (Abreu and Carraher, 1989) employ (1) a system of measures different from the metric system introduced in schools and (2) procedure for determining land area which actually go against what they are taught in school. We believe it can be reasonably argued that schooled people develop a substantial part of their mathematical knowledge out of school. The present analysis, together with previous analyses by others … bears upon this issue.

Key issues

The main question, however, is not whether people can or do learn mathematics out of school, but rather what is the nature of mathematical knowledge learned out of school and how is it similar to and different from mathematics learned in school. What are the principal characteristics of this knowledge? How is it organised? How is it acquired? To what extent is it taught by others? To what extent is it discovered or constructed by the individual? How is it deployed in actual settings or situations? What sort of problems is it particularly suited for? How extensive, flexible, unified, general, explicit, precise, abstract how powerful is it? How is it related to cognitive development in general? How is it looked upon by the people who use it? What sense do they make of it? What symbolic representations, internal or external, are associated with the knowledge and what role do these representations play in understanding or solving mathematical problems? What modifications are introduced in representations and procedures as people confront new situations?

Overview and Conclusion

In reviewing the above studies of mathematics at work it has become clear that there is no unitary 'work mathematics' nor a unitary school mathematics. Some professions place varied mathematical demands upon workers. Foremen for example, must understand concepts such as volume and area and be able to work with proportion relationships which are challenging even to students with several years of formal instruction. Other jobs, such as that of bookies, may require constant use of mathematics, but in what could be described as routine tasks. As we noted with the bookies, props such as tables may be devised to allow a worker to solve a problem without his or her having to develop an extensive knowledge of all of the invariants relevant to the solution.

Work provides challenges and opportunities for people to develop mathematical knowledge which, as Carraher (1988) has argued, is meaningful. Mathematical knowledge developed at work appears to be meaningful in the sense that the representations used have clear referents. As a result, people are able to monitor their reasoning, checking the appropriateness of their conclusions. Self-invented methods may be necessarily meaningful: perhaps only if people truly understand a problem can they discover on their own a method which will work. But even where a community passes down procedures for solving a problem, as in the case of the farmers, there appears to be an attempt, successful in this case, at understanding the approach. So the major virtue of self-invented or intuitive mathematics is meaningfulness; its major liability consists in the limited conditions to which this knowledge may be useful …

The role of schooling in the development of mathematical knowledge is not a simple one. If the 'street maths' and 'written and oral maths' studies were to be taken in isolation, it might appear that the mathematics taught in schools is irrelevant or even detrimental to solving maths problems in real life (even though this was not concluded by the authors). However, several observations from other studies suggest that this would be an unwarranted conclusion. The error rates associated with written maths approaches seem to drop considerably when one looks across a wide range of schooling. It could be argued that the 'stronger' students are more inclined to continue in schools in Brazil. However, it seems more reasonable to suppose that students take several years to become comfortable with place value notation and with column-oriented algorithms for calculating, in which one operates not directly upon quantities but rather on algorithms, the relations of which to the original quantities are not direct. One could thus argue that the mathematics taught in schools constitutes a long-term investment, the payoff of which can only be felt in the long run. But what sort of payoff?

The studies reviewed offer some hints. The investigation of bookies suggests that through schooling, they begin to note similarities and treat as similar situation which, on the basis of contents involved (colour, number, or letters), would appear to be different. It has been suggested that the candy sellers with relatively high levels of schooling begin to distance themselves from the characteristics of the sales situations in which they work and begin to analyse their sales in terms of more abstract notions, such as 'profit per unit purchased', a derived quantity never directly encountered in their work. The work with the farmers from the south of Brazil suggests that schools may promote the generation of multiple solutions which are removed from the immediate situation. We also saw in this case that the cost for leaving the immediate situation can be high; students showed many inappropriate strategies for representing problems. Are these 'teething problems' that will go away or do they reflect a failure of schools?

Resnick (1986) characterises well the paradox which schools find themselves up against in the emphasis upon formalisation in maths:

> On the one hand, the expression, A + B, takes its meaning from the situations to which it refers. On the other hand, it derives its mathematical power from divorcing itself from those situations. (p. 30)

She then elaborates on this paradox in discussing the 'dual role of formal mathematical language as both signifier and signified':

> To become truly proficient at mathematics one must be able, eventually, to reason with and on the formal symbols themselves. Part of the power of algebra, for example, is that once an appropriate set of equations is written to express the quantities and relationships in a situation, it is possible to work through extensive transformations on the equations without having to think about the reference situations for the intermediate expressions that are generated. In this sense the 'meaning' of algebra is encompassed within the formal system; a meaningful expression is one that is legal within the formal system, and the application of the correct transformation rules insures that all expressions that are generated will be legal. ...

In science, power refers to a relationship between work and time. Powerful tools and methods are those which accomplish a lot of work in relatively little time. The power of formalisation lies in their helping us accomplish mathematical tasks such as determining answers and expressing relations succinctly and efficiently.

Language contexts HO9

From Walkerdine V. & The Girls and Mathematics Unit 1989, *Counting Girls Out*., © Virago in association with the University of London Institute of Education, pp. 52–53.

Mathematical meanings … cannot be separated from the practices in which the girls grow up. The mother is positioned as regulative in these practices, in which desires, fears and fantasies are deeply involved. So 'mathematical meanings' are not simply intellectual, nor are they comprehensible outside the practices of their production. Yet in school this is precisely what happens to them. Children have to learn that there are special meanings to these terms, which are not necessarily those used at home. These meanings lead to the generation of mathematical statements of enormous power, because they can relate to anything. All the meanings at home are produced in aspects of domestic regulation. For example, taking the pair *more/less*, all instances of *more* in these transcripts come from the mother's regulation of the child's consumption of commodities and are therefore part of her regulation of the domestic economy. We analysed the transcripts of recordings of thirty mother–daughter pairs by Professor Barbara Tizard (see also Walkerdine, 1988; Walkerdine and Lucey, 1989) to draw out all occurrences of *more* and *less*. While there were many examples of *more, less* did not occur once. At first one might think that this is because *more* is a semantically easier term than *less* and therefore acquired first. However, this interpretation is not so easily supported when it is noted that all instances of *more* come from mother–daughter exchanges where the daughter's consumption of scarce or expensive resources and food is regulated by the mother.

C. I want some more.
M. No, you can't have any more, Em.
C. Yes! Only one biscuit.
M. No.
C. Half a biscuit?
M. No.
C. A little of a biscuit?
M. No.
C. A whole biscuit?
M. No.

C. Who gave you that ? (Cleaning fluid for sink.)
M. Granny gave me that little bit 'cause I ran out.
C. Has you still got some more?
M. Hm?
C. Have you still got some more?
M. Just enough for today and get some more tomorrow.

The dimension of *less* is simply not relevant. The opposite of *more* in food regulative practices is something like *no more, not as much*, and so on. *More* and *less* form a contrastive oppositional pair only with respect to certain practices, and these practices are pedagogic.

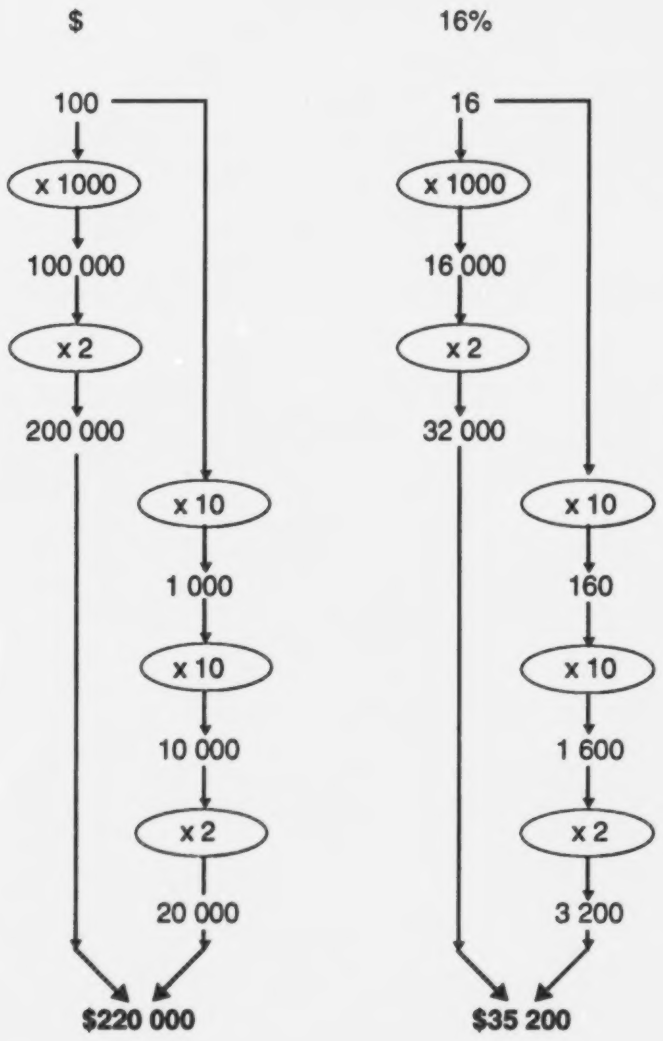
A farming context

HO10
page 1

From a paper delivered by Isabel Soto, of Chile, at the UNESCO
Adult Numeracy Seminar at Marly-le-Roi, France, 1993

There is a gap between the practice and the theory of teaching maths to adults. In management of small land properties we detected a praxis of informal maths that we hadn't known about, and didn't understand. The peasants in this study were interviewed altogether for between 3 and 5 hours, and were also observed. I would like to make one initial remark: these problems on proportion are actual, real problems.

E.g. 16% of $220,000 (to follow the reasoning, we developed a schema of arrows)


$
16%
100
x 1000
100 000
x 2
200 000
x 10
1 000
x 10
10 000
x 2
20 000
$220 000
16
x 1000
16 000
x 2
32 000
x 10
160
x 10
1 600
x 2
3 200
$35 200

The methods used [here, and in other similar problems] were not those that are taught in school. This one uses an isomorphism, duplicating both columns, and using simpler figures to generate more difficult ones.

Results

1 Methods were essentially oral and practical, with numbers retaining their sense throughout.

2 There was an adequate use of the decimal system and complicated structures.

3 The working was always from left to right.

4 There was an appropriate use of the fundamental properties of operations.

The research puts into question traditional school methods. Many structures considered complex mathematically gave the peasants little trouble. Perhaps these can be used to develop algorithms that are more relevant to adults.

Why I like numbers.... P3

From P. Hoeg 1992, *Miss Smilla's Feeling for Snow*, © Flamingo, pp. 101–102.

'Do you know what the foundation of mathematics is?' I ask. 'The foundation of mathematics is numbers. If anyone asked me what makes me truly happy, I would say: numbers. Snow and ice and numbers. And do you know why?'

He splits the claws with a nutcracker and pulls out the meat with curved tweezers.

'Because the number system is like human life. First you have the natural numbers. The ones that are whole and positive. The numbers of the small child. But human consciousness expands. The child discovers longing, and do you know what the mathematical expression is for long?'

He adds cream and some drops of orange juice to the soup.

'The negative numbers. The formalization of the feeling that you are missing something. And human consciousness expands and grows even more, and the child discovers the in-between spaces. Between stones, between pieces of moss on the stones, between people. And between numbers. And do you know what that leads to? It leads to fractions. Whole numbers plus fractions produce the rational numbers. And human consciousness doesn't stop there. It wants to go beyond reason. It adds an operation as absurd as the extraction of roots. And produces irrational numbers.'

He warms French bread in the oven and fills the pepper mill.

'It's a form of madness. Because the irrational numbers are infinite. They can't be written down. They force human consciousness out beyond the limits. And by adding irrational numbers to rational numbers, you get real numbers.'

I've stepped into the middle of the room to have more space. It's rare that you have a chance to explain yourself to a fellow human being. Usually you have to fight for the floor. And this is important to me.

'It doesn't stop there. It never stops. Because now, on the spot, we expand the real numbers with the imaginary ones, square roots of negative numbers. These are numbers we can't picture, numbers that normal human consciousness cannot comprehend. And when we add the imaginary numbers to the real numbers, we have the complex number system. The first number system in which it's possible to explain satisfactorily the crystal formation of ice. It's like a vast, open landscape. The horizons. You head towards them and they keep receding ...'

Unemployment rates

OHT4

Table 1

Males in the labour force: unemployment rates, Australia, 1978 to 1984

year	***1978***	***1979***	***1980***	***1981***	***1982***	***1983***	***1984***
%	5.2	5.1	5	4.8	4.8	8.9	9.3

Approximated from Ian Castles 1992, *Surviving statistics*, Australian Bureau of Statistics, Sydney, p. 32.

Your metric conversion chart ✋ HO11

Draw up for yourselves a metric conversion chart by listing familiar quantities which are examples of things measured by metric units.

e.g. normal body temperature = 37° Celsius

Then try to think of other familiar units which approximate the initial quantity, possibly using imperial system units, and/or non-standardised but common references such as coffee cups, arm lengths, baby's body weight etc.

e.g. normal body temperature = 37° Celsius
= temperature on a very hot day
= almost 100° (on the 'old' scale)
= 98.6° Fahrenheit

Include a summary of the basic metric units for
- linear measurements
- liquid volume
- solid volume
- area
- mass
- temperature

and a range of units with prefixes, e.g. kg, cm and mm^2.

Using your previous activities on place value with concrete materials illustrate, for example, the relationship between
- a metre and a centimetre
- a square millimetre and a square metre
- a cubic metre and a cubic centimetre.

Percentage scenarios

✋ HO12

Can you make sense of the following four scenarios?
They are all adapted from actual situations.

1 A quote in a newspaper:

'Ten years on, the members of the class of '83 have gone their different ways. Seventy-one percent of the class are males in full-time employment, while sixty-two percent are females in full-time employment.'

2 Another quote in a newspaper:

'Metropolitan rail fares are likely to increase by 100% (e.g. from $3.20 to $6.40) while some country fares will increase by as much as 300% (e.g. from $12 to $36).'

3 'I won't make anything, but I won't lose.'

A bookshop usually marks its books up 25% on the original cost price. After a while, it is clear that some detective novels are not selling, so the bookseller decides to mark down their present selling price—by 25%. 'I won't make anything, but I won't lose,' she says.

4 In 1989 a report argued:

'A policy of positive discrimination has allowed the number of women in university professorial positions to rise by 60%. The number of men in similar positions over the same period of time has risen by only 6%. It is clearly time to revert to a fairer policy.'

Loans: whose interest?

HO13

You are borrowing $10 000 at 10% reducible interest, and you have agreed to repay it at the rate of $100 a month. You will pay interest annually, and your initial interest payment of 10% will be made at the time you take out the loan.

Estimate

- how long you think it will take you to repay the loan
- how much money you think you will repay in total.

1 Working in pairs or small groups, do enough calculations to draw a fairly detailed graph showing how much money you owe the bank over the period from now until the loan is paid off.

How long will it take? How much will you repay altogether?

2 Next, try and work out what conditions might give you a better deal on this loan. Choose one of the following and see what happens in each case.

- You pay interest twice a year, and your initial interest payment of 5% is made at the time you take out the loan.
- You pay interest monthly, and your initial interest payment of 0.83% is made at the time you take out the loan.
- You pay back $80 a month instead of $100.
- You pay back $120 a month instead of $100.
- You pay in an extra $500 at the end of the fifth year.
- You pay in an extra $500 at the end of the first year.

You may not need to work out the whole sequence again (as in Q. 1), but do enough to be clear about the trend of what is happening.

Social practice of mathematics P4

From Christine Keitel 1989, 'Mathematics education and technology',
For the Learning of Mathematics, Vol. 9, No. 1, pp. 9–10.

We easily accept the following statement as a commonplace: 'The ultimate reason for teaching mathematics to students at all educational levels, is that mathematics is useful in practical and scientific enterprises in society'. (Carss et al., p. 19)

As often with trivial wisdom, we do not really reflect on it. Otherwise we might note that it is more of a conjuration than a justification. For it does not help to explain a contradiction which has been with us for a very long time, namely

> no modern society can exist without mathematics, but
> the overwhelming majority of people in a modern society can and do live quite well *while doing hardly any mathematics.*

In fact, the hand-calculator is the culmination so far of a development by which, while reality is being structured more and more by mathematics, the average individual is more and more relieved of the need to use mathematics—again and again nourished by common practice and proved by empirical research—that in general one does not really need the mathematics learned at school- seems more justified today than ever.

It is true, of course, that an immeasurable amount of mathematical knowledge is available today and is rapidly expanding, and that there exist people who professionally use specific sections of this knowledge. However, the very scope of it and of its eventual specialisation puts it beyond general education, and, as R. Fischer [Fisher, 1984] points out, numerous agencies outside school fill in this gap, and they are much more prepared to provide special knowledge purposefully and effectively to those who need it. In fact, the need for high achievement in mathematics for all cannot be justified by this kind of argumentation. So what do we do about the contradiction between the increasing *mathematisation* of modern society and the potential *demathematisation* of its members [Chevallard, 1988]?

Demathematisation is brought about by the very existence of the products of our technologically-structured environment: demathematisation is *inherent* in these products as it is in technology.

If we turn back to an early technological achievement as represented by the clock, we immediately see that the original mathematical considerations which resulted in the conceptualisation of the clock and its eventual construction may be extremely fare from the thoughts of an actual user of a watch who does not wish to miss his train. And so may be all the subsequent additions of mathematical and technical ingenuity up to the quartz-crystal clock and digital equipment. They are all incorporated in the actual instrument- and yet, to use it appropriately, we need not have the slightest idea of them.

Thus it is an effect of technology to be a substitute for our own imagination, mathematical knowledge, and technical skill; and even more; it summarizes the best talents of generations of specialists before us. It is conceived to *replace* them.

Thus the quintessential product which is in our hands, make the enclosed mathematics *implicit* mathematics. Mathematics continues to be effective, but without requiring respondent capabilities on the user's side. That is how demathematisation takes place. Whereas *explicit mathematics* vanishes beyond the clouds in the summits of research and extreme specialisation, *implicit mathematics* makes mathematics disappear from ordinary social practice.

From time to time flashes of lightning from the lights of explicit mathematics illuminate society. Scientific mathematics is the more met with respect the more it is wrapped in mystery and the more astonishing technological achievements it apparently inspires. On the other hand less attention is given to implicit mathematics—which is also how school mathematics reacts to these phenomena.

Planning a program

HO14
page 1

In your pairs or small groups, visualising a theoretical student group that you profiled in section C2.1, work through this activity. The aim of the activity is to undertake a small curriculum planning exercise mapped against a relevant state, territory or national adult basic education curriculum framework or accredited course.

1 Decide where your group of students are located against the state or national curriculum frameworks you are expected to teach towards. Decide on the relevant levels and/or streams. If you do not have to work to any such curriculum undertake the activity as a curriculum/program planning activity.

2 Decide upon a theme or topic that you would like to work on with this group of students. This could be something linked to the various investigations and excavations that you have met in this section of the course, but feel free to choose something that you all agree is relevant and important to your chosen group of students.

3 Decide upon the objectives that you want to achieve in this theme topic.

4 Identify the competencies—units and elements etc.—against which you will be teaching this topic, and start to plan a number of teaching activities and lessons that could achieve these objectives as well as meeting the given curriculum aims.

5 Include, where appropriate, the necessary materials and resources you would use and any references, books or materials you could use. Feel free to use any of the resources made available during this Mathematical Journey or any other resources you know of.

You can use the attached Planning Grid on the next page to help you write up your plan, or else develop your own.

HO14 Planning grid

Theme/topic: ______________________________

Curriculum framework level/stream: ______________________________

Educational goals	Curriculum framework competencies/elements	Numeracy skills	Teaching activities	Resources

Gender and maths: some issues ✣ AS6

M1

An example of mystification, showing *misinterpretation of evidence*, comes from the relationship between class size and pupil attainment. In a number of educational studies there appears to be a 'positive correlation' between the two, i.e. as class size increases (across different classrooms) the level of attainment also tends to increase ... Thus the statistics seem to challenge what teachers know by 'common sense'.... Yet they may not be confident enough to challenge the interpretation of the statistics which ignores that 'correlation is not causation' and which fails to investigate alternative explanations for the correlation observed.

Evans J. 1989, 'The politics of numeracy', in P. Ernest (ed.) *Mathematics Teaching: the state of the art*, Falmer Press, London.

M2

[Whitley et al. have argued that] publication bias towards studies in which significant findings are reported has contributed towards an exaggeration of gender differences ...

Gilah Leder 1992, 'Mathematics and gender: changing perspectives', in Grouws D.A. (ed). *Handbook of Research on Mathematics Teaching and Learning: a project of the National Council of Teachers of Mathematics.*

M3

Academic staff numbers by gender Macquarie University 1992

	men	women
Behavioural Sciences	25	25
Biology	30	8
Chemistry	15	2
Computing & Electronics	21	1
Early Childhood Inst	4	31
Earth Sciences	35	5
Econ. & Financial Studies	58	15
Education	17	14
English and Linguistics	27	14
History, Philosophy, Politics	48	9
Law	17	8
Mathematics	17	1
Modern Languages	19	8
Physics	13	2

Macquarie University Calendar, 1992

M4

> ... it appears that in many countries and cultures some differences in mathematics achievement do exist, but typically they are small and do not always favour boys, do not appear at all age and ability levels, and are not consistent for all types of mathematics learning or all mathematics topics. In Australia, when participation is equal, achievement levels are likely to be as high for girls as they are for boys except in the top 1% of the mathematics achievement range. Regardless of the extent or cause of differences in ' mathematical giftedness', most girls achieve at least as well in mathematics as most boys.
>
> Sue Willis 1989, *'Real girls don't do maths': gender and the construction of privilege*, Deakin University Press, Geelong.

M5

Inquiry tips single sex classes to help boys

By MICHAEL WILKINS

More single-sex classes for boys are being considered in a drive to improve their performance in school.

A parliamentary committee examining a surge in girls' academic standards in NSW schools will also consider encouraging a new 'men's revolution', its chairman says.

Because males tend to show off in front of their female classmates, separating them into different classrooms has to be considered, committee chairman Stephen O'Doherty says.

'If boys are just showing off in front of girls, you have to ask whether it is better to have single-sex classes,' Mr O'Doherty says.

Many students in co-educational schools could be divided by sex to create better discipline and a more peaceful learning environment.

The committee's inquiry has come in the wake of figures which show girls are outperforming boys in high schools across NSW.

Girls are now topping more than 60 per cent of the HSC subjects and scoring significantly higher tertiary entrance marks than boys.

A strategy launched five years ago to improve the standard of girls' performance has worked too well and left boys behind.

Mr O'Doherty says society is recognising the need to raise male education standards.

'We are just picking up on something that is akin to a women's revolution-type shift', he says.

'I don't think it would be anywhere near as powerful as the women's revolution because men are not a subjugated class'.

Mr O'Doherty says single-sex classes might give teachers the chance to concentrate on teaching rather than constantly disciplining them.

© *The Sunday Telegraph*, Sydney, May 1, 1994, p.3

M6

Proportion of female maths teachers

from G. Hanna, E. Kundiger and C. Larouche 1990, 'Mathematical achievement of Grade 12 girls in fifteen countries', in L. Burton (ed.) *Gender and Mathematics: an international perspective*, pp. 93–94, © Cassell, London.

Many researchers have suggested that a female's perception of maths as a male domain may negatively affect her motivation to do well in the subject and hence affect her achievement (Fennema and Sherman, 1977; Sherman and Fennema, 1977; Leder, 1982). In fact, it is thought that girls often fear success in mathematics, believing that their social relationships with their male peers will suffer if they are perceived as superior in a sphere that they imagine to be forbidden to women (Moss, 1982). Finn *et al.* (1979) have suggested that girls' performance improves most significantly in programs that rely on older girls to counsel, encourage and tutor younger girls.

In this light, it could seem reasonable to suggest that the ratio of female to male maths teachers may be an important factor in explaining sex differences in mathematics achievement, since it most likely affects the degree to which girls subscribe to the notions that maths is the preserve of men. Thus, according to this reasoning, countries with negligible sex differences in achievement would be expected to have higher proportions of female maths teachers than their highly sex-differentiated counterparts. Although this notion seems intuitively sound, as Table 9.4 shows, our data do not support it. For instance, in British Columbia, very small sex differences were observed even though only 3 per cent of the maths teachers were female, while in Hungary very large sex differences emerged despite the fact that a majority of the teachers (60 per cent) were female.

Table 9.4 *Contextual variables by country: female maths teachers, school organisation, years of anticipated post-secondary education and home support.*

	Thailand	*British Columbia*	*England*	*Israel*	*Japan*	*Hungary*
Female maths teachers (%)	52	03	30	38	05	60
Students planning 2 or more years of post-secondary education (%)	78	73	93	63	80	92
School organisation	mixed	mixed	mixed	mixed	mixed	mixed
Home support						
Parents encourage maths—much or very much (%)	79	77	81	69	44	49
Parents want me to do well—much or very much (%)	90	94	95	83	81	80

M7

Our data do not support ...'

The effect of female role models on girls' performance is just one of the many concerns examined by Gila Hanna, Erika Kundiger and Christine Larouche in 'Mathematical achievement of grade 12 girls in fifteen countries', an analysis of gender issues based on the results of the Second International Mathematics Study. They hypothesised that 'the ratio of female to male math teachers may be an important factor in explaining sex differences in mathematical achievement', or more precisely that 'countries with negligible sex differences in achievement' would have 'higher proportions of female maths teachers'. It is not clear how the ' proportions of female maths teachers' were measured. For instance, when the authors say that in British Colombia only 3% of those students who wrote the test had female teachers at the time. Only the latter interpretation tells us anything about the sex of the girls' teachers at the time of the test; nothing seems to be known about the sex of their mathematics teachers throughout the rest of their schooling.

In this study the measure of mathematical achievement was taken uncritically to correspond the global mark students to the SIMS test. Comparing the three countries with the smallest sex differences in marks (i.e. 'achievement') with the three having the greatest sex difference, the authors conclude that 'our data do not support' the hypothesis. They cite the example of British Columbia where sex differences were minimal and the percentage of female maths teachers low (5%). What is important here is that the authors concluded that differences in achievement could not be attributed to the variable 'sex of the teacher' nor, perhaps, to the proportions of female mathematics teachers. They were careful to explain in their conclusion that 'This does not mean that these variables have no influence on math learning, but rather that their influence may be exerted in interaction with other societal factors'.

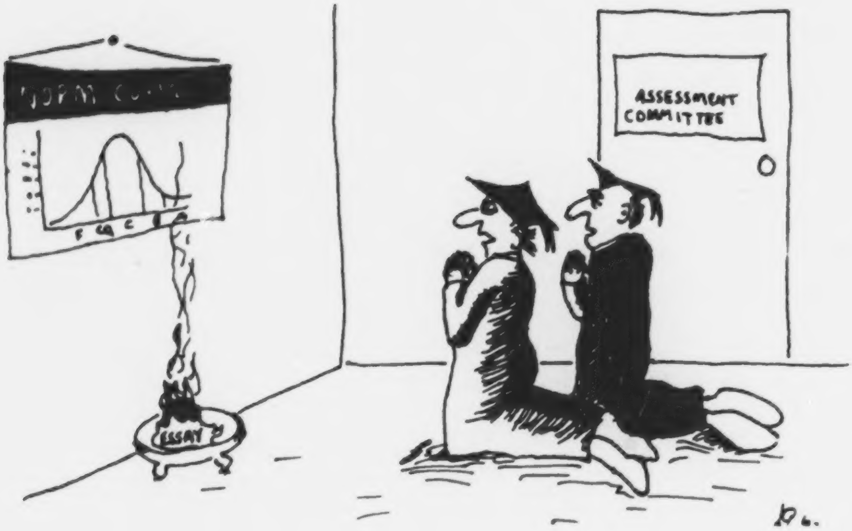
From L. Lee 1992, 'Gender fictions', *For the Learning of Mathematics*,
Vol. 12, No. 1, pp. 30.

M8

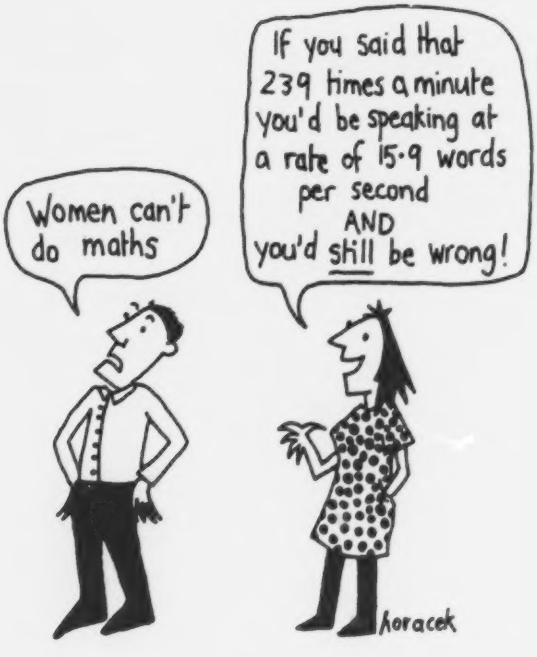
An advantage of large-scale sample surveys is that they allow a fairly precise description of the population in question. Unfortunately, routine application of the significance-testing procedures developed for small-scale experiments can be misleading. There is a close link between sample size and statistical power: the ability of a statistical test to detect a difference. Consequently in large surveys trivially small differences may be highly significant statistically, and this significance may be deceptive... Part of the problem is the seductive word 'significant': surely something that is significant (with or without the adjective statistically) must be important. This concept of 'significance' is part of the pidgin statistics of social science. Yet its meaning is not sufficiently appreciated....

Walkerdine 1989, *Counting Girls Out*, London, Virago Press.

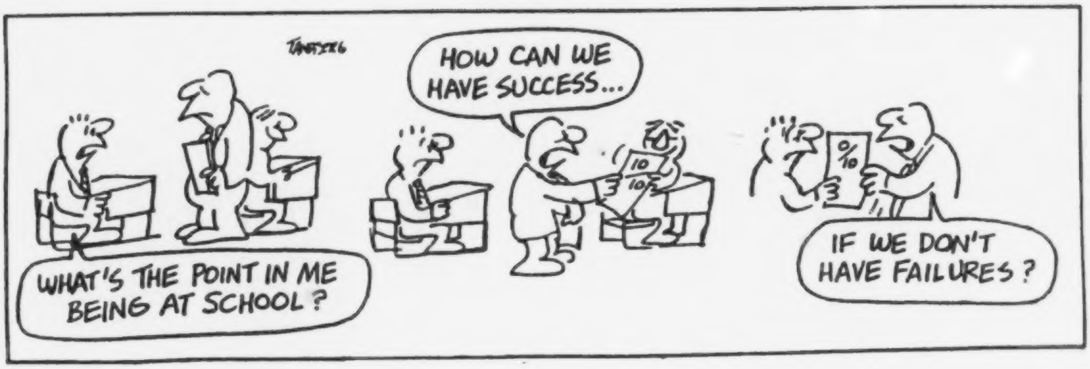
M9


NORM CURVE
ASSESSMENT COMMITTEE
ESSAY

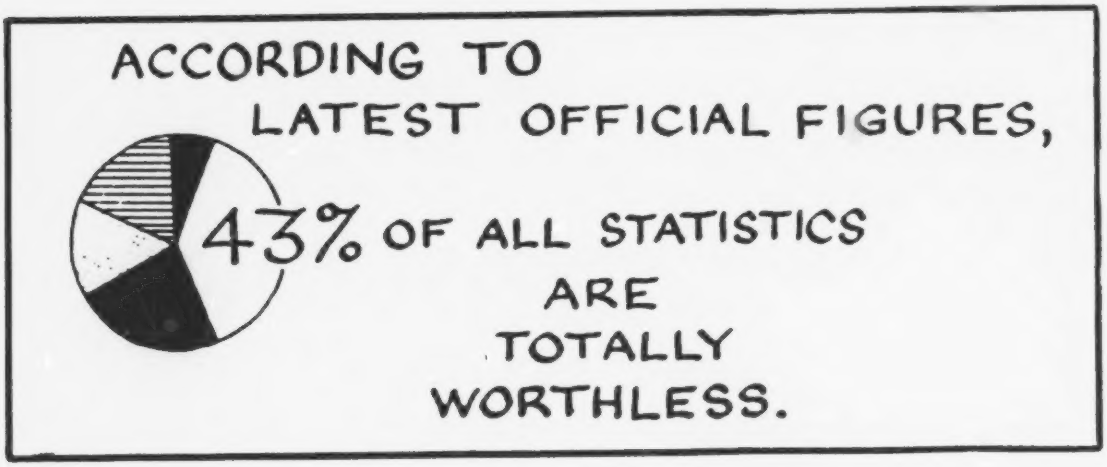
M10


Women can't do maths
If you said that 239 times a minute you'd be speaking at a rate of 15·9 words per second AND you'd still be wrong!
horacek

M11


WHAT'S THE POINT IN ME BEING AT SCHOOL?
HOW CAN WE HAVE SUCCESS...
10/10
%/10
IF WE DON'T HAVE FAILURES?

M12


ACCORDING TO LATEST OFFICIAL FIGURES, 43% OF ALL STATISTICS ARE TOTALLY WORTHLESS.

The best laid plans... ✋ HO15

From J. McGuirk 1994, 'Curriculum theory and practice', Course notes for Graduate Diploma of Adult Basic Education, University of Technology, Sydney, pp. 6–7.

best laid plans ...

However, as with the *best laid plans and programs*, they are bound to go astray. A program can only, at best, have an indirect influence on actual learning. It is mediated by teaching and the classroom context within which instruction is only one element. It is further mediated by learners' participation in classroom work and by their own interpretation of appropriate objectives and content for learning. Allwright (in Breen 1987) states that learners don't learn what teachers teach. It is not the content of a lesson that is the basis for learning but the process of classroom interaction which generates opportunities for learning.

The variables that intervene between the planning of a program and the actual learning which the plan intended are such that the original plan may be virtually irrelevant. (!) The critical content of any learning experience is the method or process through which the learning occurs.... 'it is not what you say to people that counts; it is what you have them do' (Postman & Weingartner in Breen 1987).

A crucial function of a program is to facilitate the learning of new knowledge and capabilities through such organisation. If it's true that the teaching-learning process actually redefines and further reorganises content, then how can a program serve this facilitative purpose.

process syllabus

Breen proposes the *process syllabus* where students and teacher agree about *participation*—who works with whom? *procedure*—which activity or task will be undertaken; how will it be worked on; what resources should be used; when should it be undertaken and for how long; how shall outcomes be evaluated? and *subject-matter*—what shall be the focus of the work? for what learning purposes?

So you get a cycle of decision-making about preferred ways of working, content and activities & tasks and evaluation.

negotiated curriculum

This is very similar to the *negotiated curriculum* as put forward by Cook (1992) and Boomer (1992). Questions remain of course about who should do the planning. When will the planning take place? What is appropriate subject-matter? What are the preferred procedures? What organising principles are helpful or unhelpful?

A mathematical excavation ✋ HO16

Rationale and aims

Teaching maths in context and looking at embedded maths is all very well. But standing back and admiring the embedded maths doesn't add much to extending our understanding of either the maths or the context. This exercise is aimed at getting you to excavate the maths embedded in a particular context, and to mess around and grapple with the maths so that you:

1. extend your understanding of the mathematical concepts you have excavated;
2. examine the role that maths plays in conveying information, and the interplay between maths, language and context, including political context;
3. critically assess the appropriate and inappropriate use of maths in the context that you have examined;
4. experience and become more conscious of the messiness that an authentic practice of maths entails;
5. identify through experience some strategies and resources which are useful in learning maths.

Conducting the excavation

You will have about an hour to spend on this activity in which you excavate materials on the topic of gender and mathematics and prepare a short presentation.

1. With the other members in your group, read the materials you have been given and briefly discuss the following questions.
    - Is there maths involved?
    - How is it involved? Is it used appropriately?
    - What other maths might be relevant?
    - Who is using the maths? In whose interests does it work?
2. Decide amongst the group:
    - on an area of maths for each member to activate and grapple with;
    - how you will work, e.g. in pairs or individually;
    - when you will reassemble to pool findings, e.g. report back after 20 min ...
3. Use all available resources, including the resources you have brought in with you, to learn and explore the maths. Don't worry initially whether or not this maths plugs directly back into the context of maths and gender.
4. When you reassemble, discuss your findings with your group. Together discuss
    - whether and how the maths adds meaning to the context
    - whether further grappling with the maths might help
    - whether the maths has turned out to be a dead end.
5. What knowledge other than maths would help uncover the context further? Where could you find that knowledge?
6. Revisit the questions you looked at in Question 1, and see if you would add to what you discussed then.
7. Prepare a short presentation for the rest of the large Group.

Criteria for good Adult Basic Education resources

HO17

Adapted from Kindler, J. and Tout, D. 1991, 'Criteria for good Adult Basic Education resources', *ARIS Bulletin*, Vol. 2, No. 4, pp. 12–13.

Methodology

- Materials should support the methodology and teaching approaches used.
- Materials should support the curriculum or content you are teaching.
- Materials should combine theory and practice. The methodological base of the resource should be linked to the student material.
- Materials should act as a catalyst for student work and investigation.

Content

- The content of materials should be holistic and integrated (eg maths/literacy/science).
- Materials should be meaningful to adult students (based on prior knowledge or ideas and language discussed before students are presented with the material).
- Materials should be relevant to adult students in terms of:
  - educational needs;
  - functional needs;
  - interest; or
  - personal development.
- Materials should be culturally appropriate or sensitive.
- Materials should be capable of challenging and extending students.
- Materials should use language that is not artificially contrived.
- Reading materials should be complete (not fragments).
- Materials should be flexible and be able to be adapted to different situations.
- Materials should be current and up to date.

Accessibility

- Materials should be readily available.
- Materials should be within budget.

Presentation

- Presentation and layout of materials should be appropriate to the type of material.
- Materials should be teacher friendly – it should be easy for teachers to identify which are notes to teachers, discussions of the theoretical base or student worksheets and exercises.
- Materials should be adult and non-patronising in print, appearance, format, readability and graphics.

Collections of resources

- Materials should be varied to cover a range of purposes (taking into account the range of information that must be computed and read in a day).
- Materials should promote the development of listening, speaking, and critical thinking as well as maths, reading and writing.
- The format of a collection of materials should cover a range of formats – print, video, audio, computer and concrete.
- Within a collection materials should offer a range of structures, eg modules, single lesson, or capable of being used over a number of lessons.

Literacy AND numeracy? HO18

From Lee A., Chapman A. & Roe P. 1993, *Report on the Pedagogical Relationships between Adult Literacy and Numeracy*, DEET, Canberra, p. 35.

It is useful to reflect on what a pedagogy for numeracy might look like, seen through this framework.

In terms of the first approach, the 'decoder' will recognise the basic notational and symbolic conventions and representational modes of written mathematics. They will be able to perform operations upon those symbols and representations, according to formulae and learned methods.

In terms of the second approach, the 'participant' will understand underlying mathematical concepts, whether coded in formal mathematical modes or embedded in print, and be able to make sense of a mathematical statement or representation.

In terms of the third approach, the 'user' will understand the social functions and purposes of mathematics, will understand contexts in which it is appropriate to use mathematics, will know what mathematics to use and will be able to understand how to use the results provided by the mathematics.

In terms of the fourth approach, the 'analyst' will understand the power of mathematics, its capacity to be used to mystify, to misrepresent and to exclude as well as to codify information concisely and precisely and to reach solutions.

In a similar manner to the use of the framework for discussing literacy pedagogy the different approaches focus to differing extents on the active 'doing' of mathematics as distinct from the more receptive reading and comprehending of mathematics in whatever context it is imbricated. The parallels between the pedagogical issues for literacy and numeracy can be mapped onto the framework as follows:

text decoder	codes and conventions
text participant	comprehension and understanding
text user	functions and purposes
text analyst	critique and resist

Critical literacy awareness HO19

Adapted from Wallace C. 1992, 'Critical language awareness in the EFL classroom,'
in Fairclough N. (ed.)*Critical Language Awareness*, Longman, London.

Text level strategies

1. What is the topic?
2. Why is the topic being written about?
3. How is the topic being written about?
4. What other ways are there to write about the topic?
5. Who is writing to whom?

Strategies at the broad level of reading practices in social context

1. Which reading practices are characteristic of particular social groups, e.g. what kind of reading behaviour typifies a particular family or community group?
2. How is reading material produced in a particular society, i.e. how do advertisements, leaflets and public information material come to us in the form they do, who produces them and how do they come to have the salience they do?
3. What influences the processes of interpreting texts in particular contexts (i.e. intertextuality)?

Teaching critical maths

HO20

From Barnes M., Johnston B. & Yasukawa K. 1995, 'Critical Numeracy', a poster presented at the Regional ICMI conference, Monash University, April.

A critical approach to mathematics teaching would encourage students to pose their own questions, arising either from stimulus material provided by the teacher or from the students' own interests and concerns. Students would be encouraged to ask questions such as:

- What mathematical questions arise out of this situation?
- What mathematics is being used, or could be used, in this context?
- Which groups in the community are affected by the circumstances described?
- Which groups are likely to benefit from the use of mathematics in this context?
- Could you look at the questions in a different way? Would this produce different answers?
- Are there important factors which have been ignored?
- Is there any information not given here which might help you answer your questions?

Starting points for critical mathematics explorations of important social issues could include cuttings from newspapers, magazines or journals, or real data from sources such as publications of the Australian Bureau of Statistics.

Questions for critical numeracy

HO21

From Keiko Yasukawa 1995, Notes on Critical Numeracy
for the Graduate Diploma of Adult Basic Education, UTS.

- Who is the potential audience?
  Who are the writers?
  What are some of the features which effective and responsible communication ought to entail?

- What sorts of information are of interest?
  For whom? From whom?
  What are your materials trying to convey?
  What issues of power, equity and social justice might be involved?

- What sorts of language, maths, texts and other means are employed to convey the information?
  Does the use of these limit the audience?
  Do they distort the information for some and not for others?
  Who gains and loses out of the distortions?

- What maths is being used explicitly or implicitly?
  Is there any other maths that could be relevant?

Mathematical excavations ✋ HO22

Rationale and aims

Teaching maths in context and looking at embedded maths is all very well. But standing back and admiring the embedded maths doesn't add much to extending our understanding of either the maths or the context. This exercise is aimed at getting you to excavate the maths embedded in the contexts you have chosen, and to mess around and grapple with the maths so that you:

1 extend your understanding of the mathematical concepts you have excavated;

2 examine the role that maths plays in conveying information, and the interplay between maths, language and context, including political context;

3 critically assess the appropriate and inappropriate use of maths in the contexts that you have examined;

4 experience and become more conscious of the messiness that an authentic practice of maths entails;

5 identify through experience some strategies and resources which are useful in learning maths.

Conducting the excavation

You will have about an hour and a half to spend on this activity in which you excavate the materials you have brought and prepare a short presentation.

1 Share the materials that you have brought in with the other group members. Spend 5 or 10 minutes deciding what context you are most interested in excavating mathematically. See if there are others who would like to work on the same topic with you. Divide into groups, according to interest areas.

2 With the other members in your interest group, read all your materials. Use the questions you developed in Session C6.1 to focus on a brief initial discussion of the materials.

3 Decide amongst the group:
   - on an area of maths for each member of the group to activate and grapple with
   - how you will work, e.g. in pairs or individually
   - when you will reassemble to pool findings, e.g. report back after 20 min ...

4 Use all available resources, including the resource audit checklist developed in Session C5.2, to learn and explore the maths. Don't worry initially whether or not this maths plugs directly back into the context you are excavating.

5 When you reassemble, discuss your findings with the group. Together discuss whether and how the maths adds meaning to the context, whether further grappling with the maths might help, whether the maths has turned out to be a dead end.

6 What knowledge other than maths would help uncover the context further? Where could you find that knowledge?

7 Revisit the questions you looked at in Question 2, and see if you would add to what you discussed then.

8 Prepare a short presentation for the rest of the Group.

References for Module C

HO23

page 1

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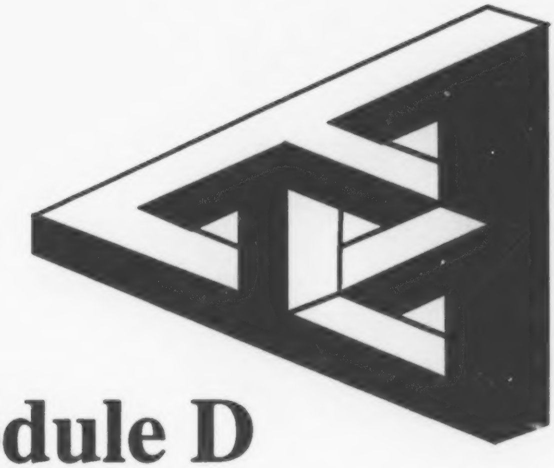

Module D
Naming theories: implications for practice

Presenter's notes

Module D: Naming theories: implications for practice

Nominal time: 15 hours

Brief description

Participants start by naming theories of teaching maths developed throughout the course and look at a number of issues/areas in the light of these theories:
- technology and calculators
- measurement
- assessment, including Curriculum Project 4: Assessment tool
- theory into practice
- probability and chance.

The section and the course then finishes with a look at the question—so what is numeracy?

Rationale/aims

This last section is the culmination of the course and attempts to pull out from previous discussion the three theories of transmission, constructivism, and critical constructivism; apply these theories to teaching, planning, and assessment; and come to the conclusion that numeracy is not less than maths but more. The course designers therefore hope that participants will be able to help their students move from numeracy as doing basic maths to numeracy as being at-home with mathematics to numeracy as activating mathematics with a critical stance.

Learning outcomes

Participants should be able to:
- Identify and describe theories relevant to the teaching of adult numeracy
- Identify the implications of the theories to curriculum, teaching and assessment practices in adult numeracy.
- Identify and explain the concept of adult numeracy and it's relationship to mathematics
- Analyse and apply specific areas of mathematics, and identify teaching and learning strategies and resources for these specific areas.

Assessment requirements

1 Maintain a written journal reflecting on the content of the section and any teaching implications.
The journal is to be personal reflections on the course to be done out of classroom time, consisting of one brief informal entry for this section and one summative entry reflecting on the course and the question 'What is numeracy?'.

2 Curriculum Project 4, Assessment tool, CP4 is a small group or individual activity, involving course discussion and materials development. It will involve developing an assessment task for a learning/teaching activity.

References

A bibliographical list of references for this module is given at the end of Resources B [HO15].

Module D Outline

module	*time*	*development of issues/activities*	*maths involved*
D1 – And so to theory			
D1.1 Naming the theories	1 h	– *two (and a half) models* – *different theories, different questions*	
D1.2 For example: technology	2 h	– *past technologies* – *what are calculators good for?* – *back to our pedagogical frameworks* – *calculating—and computing—claims* – *the social shaping of technology*	decimals, graphs, exponential growth, estimation, multiplication
D2 – Implications for teaching			
D2.1 Case-studies: constructivist and critical	1.5 h	– *theories in practice: some case-studies* – *approaches to scale* – *implementation or critique?*	scale, measurement
D2.2 A lesson on measurement	1.5 h		time measurement
D3 – Implications for assessment			
D3.1 Assessment alternatives	1.5 h	– *current practices* – *our three frameworks* – *towards guidelines for good assessment*	
D3.2 CURRICULUM PROJECT 4	1.5 h	Curriculum Project 4: Assessment tool Part 1	Participant's choice
D4 – Theory and practice: closing the gap			
D4.1 CURRICULUM PROJECT 4	1.5 h	Curriculum Project 4: Assessment tool Part 2	Participants' choice
D4.2 Theory into practice	1.5 h	– *beliefs, theories and implications* – *ANT: what has it been for you?* – *closing the gap between theory and practice*	
D5 – And so what is numeracy?			
D5.1 The chance of numeracy	1.5 h	– *probability fair* – *maths and Lotto*	probability, statistics
D5.2 Towards numeracy	1.5 h	– *five strands of meaning* – *so what is numeracy?*	

D1 And so to theory

D1.1 Naming the theories

Brief description

Participants are introduced more formally to three theories about knowledge, and their implications for learning and teaching.

Rationale/aims

This session makes explicit the theoretical foundations of the course. It discusses three approaches to numeracy teaching: transmission, constructivism and critical constructivism. Participants have already examined and reflected on their own assumptions about teaching throughout the course, and this session allows them to relate that analysis to the three approaches outlined.

Preparation

Presenter
Read
von Glaserfeld (1988) and Lerman (1993)

Participants
Read
Mathematics for empowerment:... [P1]
Constructivism in education [P2]

Time: 1 hour

Materials needed

Handouts/OHTs/paper
Mathematics for empowerment:...[P1]
Constructivism in education [P2]
Two (and a half) contrasting views of knowledge [HO1] and [OHT1]
Different theories, different questions [OHT2]

References

Johnston, B. 1993, 'Mathematics for empowerment: what is critical mathematics education?' in S. McConnell & A. Treloar (eds) *Voices of Experience: a professional development package for adult and workplace literacy*, Book 4, 'Reframing Mathematics', DEET, Canberra, pp. 10–15

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D1.1 Naming the theories

Detailed procedure

This section consists largely of discussion, in small or large groups:

–	**Two (and a half) models**	(45 min)
–	**Different theories/different questions**	(15 min)

Two (and a half) models

1 Remind the participants of the discussion in A3.1 arising from the quote, 'I taught them but they didn't learn.', about the relationship between the learner, the teacher, the knowledge. Get them to look again at the metaphors they developed then and later (A6.3). Do the metaphors still hold? Would they like to change them? How?

2 Working in small groups, get the participants to discuss *Mathematics for empowerment:* [P1]. Ask them to draw up a table, showing what they think are the assumptions about knowledge, learning and teaching behind the three ideologies: instrumental (positivism, transmission), interactive (constructivism), critical (critical constructivism).

3 Hand out *Two (and a half) contrasting views of knowledge* [HO1]. Ask participants to compare their results about instrumental and interactive approaches to the first two views.

	Positivism	**Constructivism**	**Critical constructivism**
Knowledge as	– external reality – objective – above human – true – God-given – unproblematic	– internal reality – subjective – human activity – viable – socially negotiated – problematic	as for constructivism + – situated – political
Learning as	– reception of information – absorption of facts – reproduction	– constructing – dealing with perturbation – reconstructing	as for constructivism + – purposeful – critically reflective – challenging power relations
Teaching as	– transmission – expert – concern for product	– questioning, provoking – collaborating, facilitating – concern for process	as for constructivism + – awareness of power relations – negotiation of power relations

Discuss Table 1 in OHT 1. Points that might arise:

- Some metaphors used earlier may be modelled on one of these ideologies, or use a combination of factors ... try to make connections. For example, metaphors of 'distilling' or 'transmitting' could refer to the instrumental ideology. A metaphor of 'building' or 'provoking' is based on the more interactive/constructivist approach.
- Theories and practices don't arise in a vacuum but emerge out of particular historical and social conditions—see *Mathematics for empowerment:...* [P1].

4 As a Group, using the experiences of Module C, discuss how the column 3 in OHT 1 might be filled in. Ask:

How could a constructivist approach be developed to include a critical perspective?

5 Recall the discussion in Section A3.1 about different ways of learning about area:

- for most people, as formulae ... A = l x b
- maybe, as in session A2.1 in *Looking at boundaries*, through confronting assumptions, developing conceptual understanding
- perhaps from use in real life contexts, e.g. building a hen-house

or learning about subtraction:

- by rote ... borrow and pay back
- by shuffling numbers ... decomposition
- by 'street maths' ... shopkeeper's subtraction
- by estimation.

Ask: *How do these ways of learning and teaching fit into our three models?*

Different theories, different questions ...

1 Earlier, when we were discussing maths anxiety (Section A3.1), we argued that it is important to recognise that different theories allow or disallow different questions and explanations, and lead to, or exclude different actions. Recall the points made then:

- we all have theories, explicit or implicit, whether we admit to them or not
- different theories allow or disallow different questions and explanations, and lead to or exclude different actions
- making our assumptions and theories explicit gives us a chance to choose which to hang on to, which to discard.

What questions, explanations, actions might arise from these three different theories?

2 After some discussion, show *Different theories, different questions* [OHT2] and get participants to develop the questions a little. Examples:

- In the transmission model, we might ask what went wrong with the transmission ... were my words not clear? was the student not paying attention?
- In the constructivist model we might ask how we can fit our understanding to the student's ... how can we know what the student's construction is?

D1.2 For example: technology

Brief description

Participants consider how technology can be used as a tool for learning in mathematics, and reflect on different pedagogical frameworks.

Rationale

Technology permeates our lives, at home or at work. In this section we consider some ways we can use it to enhance the process of learning to be numerate. Participants glimpse how technologies have always been used in maths and are socially constructed and historically situated. By involving participants in a variety of activities on which they can reflect, the session shows that calculators can be used to reinforce and demonstrate concepts as well as to do straightforward calculating. Participants have a chance to tease out how pedagogical frameworks apply to the use of calculators in the numeracy classroom, and those who are unfamiliar with calculators have an opportunity to strengthen their confidence and skills.

Preparation

Presenter
Read
Noss (1991)

Collect 'technologies' used in maths in the past
Cut one copy of *Calculating claims* [AS1] into strips

Participants
Read
The social shaping of technology [P5]

Time: 2 hours

Materials needed

Concrete materials
a set of simple 4-function calculators with memory—preferably 1 each
old technologies:
different kinds of abacuses, slide rule, logarithm tables, shop till, Napier's rods, picture of *quipu* (e.g. in Ascher, *Ethnomathematics*, useful pictures, information in Pappas 1986, 1991)

Handouts/OHTs
Get rich quick [OHT3]
How numbers grow [HO2]
Target [HO3]
Four-in-a-line [HO4]
Decimal patterns [HO5]
The case for calculators [P3]
The impact of technology [P4]
The social shaping of technology [P5]
Two (and a half) contrasting views of knowledge [HO1] and [OHT1] *from Section D1.1*

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Shuard, H. 1986, *Primary Mathematics Today and Tomorrow*, Longman, London.

The Shell Centre for Mathematical Education, *Calculators in the Mathematics Classroom: a day course for teachers*, Shell Centre for Mathematical Education, Nottingham.

Thiering, J. & Barbaro, R. 1992, *Numeracy and How We Learn*: a professional development program, TNSDC, Sydney, pp. 48–51.

D1.2 For example: technology

Detailed procedure

This section consists of three activities, mostly done in small groups, with some Group discussion to bring together the main issues.

– **Past technologies**	(15 min)
– **What are calculators good for?**	(45 min)
– **Back to our pedagogical frameworks**	(15 min)
– **Calculating—and computing—claims**	(30 min)
– **The social shaping of technology**	(15 min)

Past technologies

1 Set up a display of 'mathematical technologies' (for calculating) from the past, so that when participants arrive they can wander round and try out any they are interested in.

2 Discuss the use of technologies through time, talking about how they are historically and culturally based and considered appropriate or not. Talk about pencil and paper as a technology ... preceded by, e.g. slates. Point out calculators and computers as the dominant current technology.

What are calculators good for?

1 Brainstorm the question *What are calculators good for?*
and quickly list the answers without comment, judgment or discussion. The question will be revisited at the end of the session.

2 There are four calculator activities included to allow participants to experience a range of ways in which the calculator might be useful in numeracy classes. There may not be time to do all four in class. It is better to do three well—*How numbers grow* [HO2],*Target* [HO3] and *Four-in-a-line* [HO4]. Participants can try out *Decimal patterns* [HO5] in their own time.
There are also good calculator games and activities in Marr & Helme 1987, *Mathematics: A New Beginning*, e.g. *Target I* and *Decimal Dilemma*, which could be used here if participants want further ideas for calculator usage. In Marr, Anderson & Tout 1994, *Numeracy on the Line*, there is also a section on calculator use, *Calculators—a tool to use,* which focuses on using a calculator to recognise and use the language of numbers and the four operations.

3 Using *Get rich quick* [OHT3] ask the Group the following question:

You have a job working every day for four weeks. Your boss gives you a choice about your wages. You can be paid $1000 a day, or you can be paid 1c the first day, 2c the second, 4c the third, 8c the fourth and so on, doubling each day.
Which method of payment will you choose?
How much do you think you would earn the second way?...Guess!

$1 $10 $100 $1000 $10 000 $100 000 $1 000 000

4 Once everyone has committed themselves to an answer, not necessarily publicly, give participants a calculator each and get them to work out the answer. Those who know how to use the repeat function key could show others. There will probably be initial problems with:

- which number to put in first to get the desired repeat, and
- keying in 1c and getting a number too large for the calculator without rounding or scientific notation.

You could look at the patterns here in the sequence of numbers created, especially if some of the maths teachers have used a formula, or tried to remember one, for calculating the total. The pattern easily leads to seeing the powers of two, and then to the relation between the running total and the individual days figures. This can also be extended and used to see how $2^0 = 1$, and even to negative indices.
Plotting the graph of the earnings also clearly demonstrates the effect of exponential growth.

5 Hand out *How numbers grow* [HO2] and use it to discuss the idea of exponential growth, looking at the graphs and reading the David Suzuki example.

6 In pairs, get participants to play *Target* [HO3] and *Four-in-a-line* [HO4]. If there is time they can go on to do the activity *Decimal patterns* [HO5]. After about 25 min, ask:
What could students learn from these activities ?
Extract from the discussion three important uses of the calculator in teaching numeracy.

- *For calculating:*
  In *How numbers grow* the calculator allows a large number of awkward calculations to be done in a short time. Once they are proficient with the use of the repeat button, learners can see the overall picture without getting bogged down in the details. One of the results of this is that students can meet relevant concepts like exponential growth earlier than if they had to wait for algebraic statements.
- *For reinforcing concepts*:
  Four-in-a-line encourages students to estimate and to use partial knowledge to fill in the estimations.

- ***For demonstrating concepts*:**
  Target **makes students confront their assumption that multiplication always makes numbers bigger. Here the calculator is important in demonstrating and developing important relationships. This is true also for *Decimal patterns*.**

7 Point out to participants the useful material on the skills and knowledge students need to use calculators effectively in *Breaking the Maths Barrier*, pp. 90–95. If there is time, talk about some of this knowledge.

It may also be important to acknowledge that some students are still put into situations, usually in entrance exams for courses or occupations, where they are not allowed to use calculators. Discuss how you can support these students, especially those who feel confident and competent with a calculator, but who feel that they are not proficient at pen-and-paper methods any more.

Back to our two (and a half) pedagogical frameworks

1 Referring to handout HO1 or OHT1, *Two (and a half) contrasting views of knowledge*, from Section D1.1, get participants to discuss the question:

Can you see how a calculator might be used differently according to where a teacher's beliefs fell on a continuum from transmission to constructivist to critical constructivist models of teaching?

Points arising might include:

- a constructivist view of learning would be more likely to promote the use of the calculator to demonstrate concepts, to pose questions and problems (like *Target* or 'Using a variety of different calculators key in 2+3 x 5... do they all give the same answer? Why?') which provoked the student into questioning their understanding
- a purely transmission view would be likely to use the calculator simply for calculation.

2 Conclude this section by pointing out that these activities have involved:

- not only (at times) the following of rules (transmission model)
- and the making of conceptual mathematical links (a more constructivist approach)
- but also—in *How numbers grow*—the use of mathematics to critically understand an aspect of society.

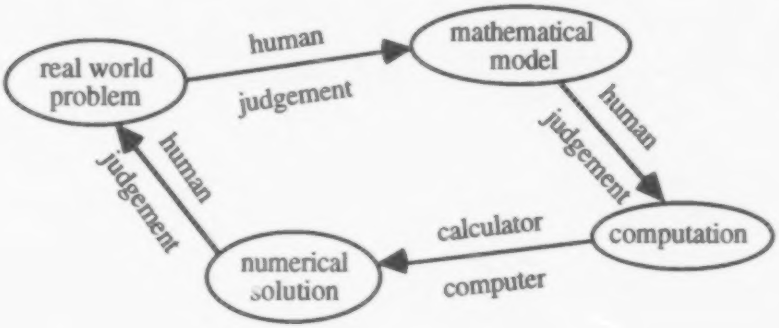
Calculating—and computing—claims

1 To tease out some of the wider issues raised in this session use *Calculating claims* [AS1]. To do this divide the participants into groups of two or three, give each group a slip of paper with one of the claims and get the group to respond to it. Get them to refer to *The case for calculators* [P3] and The *impact of technology* [P4].

Get the small groups to report back to the Group. Discuss some of the issues that have arisen, focusing on questions like:
What are calculators good for?
What skills are needed to use a calculator effectively?

2 Above all the discussion should bring out the ideas that:

- all technologies both extend and limit possibilities ... how? ... consider slates, abacus...;
- the calculator does more than calculate: it can also reinforce learning, and demonstrate concepts;
- there are skills and knowledge that students need if they are to use a calculator effectively, e.g. estimating, selecting the right operation, understanding the meaning of the decimal point, making sense of the answer;
- human judgment is also intrinsic to the appropriate use of technology, e.g. What is a human and useful interpretation of 'the average number of children is 2.3'? The following diagram can be used to help explain the role of human judgment and its relation to the use of technology.



3 We have been making assertions about the use of calculators in teaching numeracy. Can we parallel these assertions when we consider the use of computers in teaching numeracy, e.g. are computers also useful for calculating, reinforcing and demonstrating concepts?

4 List some of the parallel statements on the board and critique them. They might include:

- *Computers will make you lazy.*
- *Computers will allow you to think about the problem rather than getting bogged down in detail.*
- *Computers are good for practising skills.*

Participants might have some suggestions for how computers can be used in teaching numeracy.

The social shaping of technology

1 We considered briefly at the beginning of the session how different cultures and histories had produced different calculating/recording technologies. Finish with a discussion focusing on the point that how things are used and which things are used is not predetermined. Discuss The *social shaping of technology* [P6].

Ask: *What are the implications for how we use calculators (and computers) in teaching?*

Possible points

- There are a range of ways in which calculators—and computers—can be usefully used.
- Both rote learning types of computer programs and more informal, constructivist type programs are possible if not widely available. What about more critical, real-world numeracy programs?
- We said before that all technologies both extend and limit possibilities: how do calculators and computers extend possibilities, how do they limit them? Do you use a computer much? Has it changed how you write, for instance?
- How things are used and which things are used is not predetermined ... we have some say in it. We don't have to use calculators and computers in a way that will, for instance, stop people thinking.

You might like to ask if any participants are familiar with graphic calculators, spreadsheets, mathematical computer games, statistics, databases etc. and whether they would be willing to share their knowledge.

D2 Implications for teaching

D2.1 Case-studies: constructivist and critical

Brief description

Participants examine and analyse several teaching/learning episodes.

Rationale/aims

Participants have now met and explored three main frameworks for understanding teaching and learning: the transmission, the constructivist and the critical constructivist models. In this brief session, they have a chance to analyse and relate models of constructivism and critical constructivism to classroom episodes, both in the course and outside it.

Preparation

Presenter
Read
Mellin-Olsen (1987) pp. 30–57

Participants
Read
Which map shall I use? [P7]
or
Constructivism in a straitjacket [P8]
Cups of birdseed [P11]

Time: 1.5 hours

Materials needed

Handouts/OHTs
- *Two (and a half) contrasting views of knowledge* [HO1] and [OHT1] from D1.1
- *Which map shall I use?* [P7]
- *Constructivism in a straitjacket* [P8]
- *The School of Barbiana* [P9]
- *Activity theory* [P10]
- *Cups of birdseed* [P11]
- *Implementation or critique?* [P12]
- *Gorbachev and gender* [HO6]

References

Davis, R., Maher, C. & Noddings, N. (eds) 1990, *Constructivist views on the teaching and learning of mathematics*, National Council of Teachers of Mathematics, Virginia.
Johnston, B. 1995, 'Which map shall I use?', *Numeracy in Focus*, No. 1, pp. 33–37.
Fitzsimons, G. & Sullivan, P. 1993, 'Constructivism in a straitjacket', in J. Malone, & P. Taylor (eds),*Constructivist Interpretations of Teaching and Learning Mathematics*, National Key Centre for School Science and Mathematics, Curtin University of Technology, Perth.
Mousley, J. 1993, 'Constructing meaning in mathematics classrooms: text and context', in J. Malone & P. Taylor (eds),*Constructivist Interpretations of Teaching and Learning Mathematics*, National Key Centre for School Science and Mathematics, Curtin University of Technology, Perth.
Mellin-Olsen, S. 1987, *The Politics of Mathematics Education.*, Reidel, Dordrecht.
Noss, R. 1990, 'Political dimensions, political axes', in *The Proceedings of the Political Dimensions of Mathematics Education Conference*, London.
The School of Barbiana, 1969, *Letter to a Teacher*, Penguin, London.

D2.1 Case-studies: constructivist and critical

Detailed procedure

In this session participants read and discuss several articles;

– **Theories in practice: some case-studies**	(30 min)
– **Approaches to scale**	(45 min)
– **Implementation or critique?**	(15 min)

Theories in practice: some case-studies

1 Get participants to read either *The School of Barbiana* [P9] or *Activity theory* [P10].

2 Participants discuss in small groups the two readings, Two *(and a half) contrasting views of knowledge* (HO1 and OHT1 from D1.1), and those they will have read in preparation (*Which map shall I use?* [P7] *or Constructivism in a straitjacket* [P8] or *Cups of birdseed* [P11]), looking at the question:
To what extent are these teaching episodes/philosophies informed by transmission, constructivist or critical constructivist beliefs? and what are the effects?

3 They report their findings back to the Group so that all participants have some idea of the content of all the readings and the issues involved.

Approaches to scale

1 Next, participants read *Cups of birdseed* [P11] and compare the activities in the two situations with each other, and with the activities participants themselves have recently experienced in doing *Why do babies dehydrate...?* [HO16 in B4.2]. What are the similarities? differences? Again ask the question used in step 2 above.

2 As a postscript to the *Cups of birdseed* discussion on scale, give participants *Gorbachev and Gender* [HO6]. Ask: *What is your immediate impression from these graphs?*

3 Ask participants to re-draw the graphs as bar graphs and compare them with the originals. Ask: *Can you see how the information has been presented to give a particular impression?* Analyse, with participants, the effect which scale makes, e.g. one of the Russian dolls looks an eighth of the 'size' of the one that is twice the scale.

Implementation or critique?

Discuss *Implementation or critique?* [P12] and in conclusion, suggest that successful mathematical learning would involve:

- not only (at times) the remembering of facts (transmission model)
- and the making of conceptual mathematical links (a more constructivist approach)
- but also the use of mathematics to critique and perhaps transform aspects of society.

D2.2 A lesson on measurement

Brief description

Participants use constructivist and critical constructivist frameworks to plan a lesson.

Rationale/aims

Participants have now discussed the three main frameworks that this course explores and they have analysed a number of case-studies involving these frameworks. In this session, they begin from the need to teach a particular mathematical topic—measurement—and try to explore the implications of the frameworks as they develop appropriate teaching activities and strategies.

Time: 1.5 hours

Materials needed

Concrete materials
a wide range of books that address practical, social, historical and critical uses of measurement (including for example those included under References). Include a variety of school and other textbooks, with examples of all three teaching approaches if possible.

Handouts/OHTs
Planning sheet [HO7]
The transmission model [OHT4]

References

Goddard, R., Marr, B. & Martin, J. 1991, *Strength in Numbers*, Division of Further Education and Training, Melbourne,

Goddard, R. & Regan, M. 1995, *The Value of Time,* Council of Adult Education, Melbourne.

Hogben, L. 1936, *Mathematics for the Million*, Allen and Unwin, London.

Jacobs, H. 1982, *Mathematics: a human endeavor*, W.H. Freeman and Co, New York (or Australian Edition, 1977, published by Lloyd O'Neil, Victoria)

Joseph, George Gheverghese 1991, *The Crest of the Peacock: non-European roots of mathematics*., I.B. Tauris, London.

Lovitt, C. & Clarke, D. 1988/9, *MCTP Activity Bank. Volumes 1 and 2,* Curriculum Development Centre, Canberra.

Marr, B. & Helme, S. 1987, *Mathematics: a new beginning*, Teaching Maths to Women Project, State Training Board of Victoria, Melbourne.

Marr, B., Tout, D. & Anderson, C, 1994, *Numeracy on the Line: language -based numeracy activities for adults*, National Automotive Industry Training Board, Victoria.

Pappas, T. 1986, *The Joy of Mathematics: discovering mathematics all around you*., Wide World Publishing/Tetra, California.

Pappas, T. 1991, *More Joy of Mathematics: exploring mathematics all around you*., Wide World Publishing/Tetra, California.

Shan, S. & Bailey, P. 1991, *Multiple Factors: classroom mathematics for equality and justice*, Trentham Books, Stoke-on-Trent.

D2.2 A lesson on measurement

Detailed procedure

1 Begin the session with a general discussion by getting participants to focus on linear measurement. Ask them to use *Two (and a half) contrasting views of knowledge* [HO1 from D1.1] to tease out what kinds of knowledge, learning and teaching might be emphasised in a transmission approach to planning a lesson on the measurement of length. Some initial ideas are given in *The transmission model* [OHT4].

2 Now get participants to work in pairs to develop a fairly detailed outline of how they might teach about another aspect of measurement, e.g. time or volume or mass/weight, if they were using a constructivist and/or critical constructivist model of teaching. Remind them:

- to focus on a particular learner or learner group
- to use *Two (and a half) contrasting views of knowledge* [HO1 from D1.1] to tease out what kinds of knowledge, learning and teaching might be emphasised in such models
- to use a wide variety of resources to adapt and expand activities.

Some participants might find the *Planning sheet* [HO7] useful. The table below illustrates how a beginning might be made on planning a few lessons on the aspect of time measurement.

The measurement of time: some issues and activities

constructivist	*critical constructivist*
knowledge as internal reality, subjective, human activity, viable, socially negotiated, problematic – units of measurement evolved through physical and historical conditions, e.g. rotation of earth round sun – collecting a list of sayings about time and tracing their origins, e.g. saving time, wasting time....	***knowledge as also*** situated, political – the use of measurement itself changing over the centuries, increasing with capitalism ... time important for profit and production

learning as	***learning as also***
construction, perturbation, reconstruction – evolve units of measurement, starting with informal units, e.g. clapping, finding need for standard unit – examining other cultural ways of measuring time, e.g. Chinese water clock – developing understanding of imprecision of any measurement	purposeful, critically reflective – activity evolving from need of learner to control/understand time aspects of her/his life – questioning how different workplaces require different accountability in terms of time (look at student's specific workplace) – challenging power relations (discussion of purpose of learning)
teaching as	***teaching as also***
questioning, provocation, collaboration, concern for process – setting up situation where measurement of time is needed and no instruments are available – provoking need for standard units...	awareness of power relations – questioning use of measurement of time, unnecessary? precision of digital watches – negotiating power relations, investigation of students' needs and wants

3 Get participants to report back to the Group, collate suggestions on OHT or whiteboard, and later photocopy what is generated and distribute to participants.

4 Round off the session by pointing out that

- we saw earlier how different theories lead to different questions and
- we have now just seen how they can lead also to different practices and actions.

Preparation for the next session: Curriculum Project 4 – Assessment tool

You need to give out to the Group 4 or 5 different readings (from CP4–1 to CP4–8) from the *Curriculum Projects and Numeracy Journal* section so that each person has one reading to do for the following week, and at least two or three participants will have read the same reading. Ask them to read with this three-fold question in mind:

What does this article say about:

- *why we assess*
- *what makes a good assessment*
- *what alternative assessment procedures we could use?*

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D3 Implications for assessment

D3.1 Assessment alternatives

Brief description

Participants examine current assessment practices and relate them to the three theoretical frameworks.

Rationale/aims

The three theoretical frameworks that permeate this course have implications for how we assess as well as for how we understand learning and teaching. Different kinds of knowledge may well need different kinds of assessment. This session gives participants a chance both to share their current practices and match them with the different frameworks, and also to begin to develop assessment tasks that would derive from particular theoretical positions.

Preparation

Participants
Ask participants to bring two examples of assessment tasks that they or colleagues are currently using, preferably one that works well and a second that is problematic.

Time: 1.5 hours

Materials needed

Handouts/OHTs
Two (and a half) views of assessment [HO8]
Assessment issues [OHT5]

References

Leder, G. (ed.) *Assessment and Learning of Mathematics*, ACER, Melbourne.
Stenmark, J.K. (ed.) 1989, *Assessment Alternatives in Mathematics: an overview of assessment techniques that promote learning*, EQUALS, Lawrence Hall of Science, University of California.
Stenmark, J.K. (ed.) 1991, *Mathematics Assessment: myths, models, good questions and practical suggestions*, National Council of Teachers of Mathematics, Reston, Virginia.
Stephens, M. & Izard, J. (eds) 1991, *Reshaping Assessment Practices: assessment in the mathematical sciences under challenge*, ACER, Melbourne.
Webb, N. & Coxford, A. (ed.) 1993, *Assessment in the Mathematics Classroom*, 1993 Yearbook, National Council of Teachers of Mathematics, Reston, Virginia.

D3.1 Assessment alternatives

Detailed procedure

This section looks at participants' current practices in assessment, proceeds to tie in the discussion to our three theoretical frameworks, and begins to develop guidelines of good assessment.

– **Current practices**	(30 min)
– **Our three frameworks**	(45 min)
– **Towards guidelines for good assessment**	(15 min)

Current practices

1 Participants will have brought with them some examples of assessment tasks and methods they or colleagues are currently using. In small groups, get them to share the information and discuss what they have found useful or not about the particular tasks. Distinguish between initial, formative and summative assessment.

2 Display *Assessment issues* [OHT5] and get participants to discuss the questions in small groups before returning to the larger Group.

3 Continue with a Group discussion of the question
How do you assess? Are there other possibilities that you have either tried or heard about?
You could make a diagram on the board to record all the ways of assessment, noting in each case what each method was trying to assess.

Our three frameworks

1 Remind participants of the three theoretical frameworks we have been using. Hand out *Two (and a half) views of assessment* [HO8] and ask:
How does assessment fit into our theoretical frameworks for thinking about learning and teaching?

2 Get participants to tease out the implications of the different frameworks for assessment, filling in suggestions in the appropriate boxes if they like. Discuss conclusions.

3 In pairs, get participants to return to the plan for teaching measurement (developed in D2.2) and outline a method that would be appropriate for assessing one or more of those activities.

4 Report back to the Group.

Towards guidelines for good assessment

1 Conclude with a brainstorm to suggest guidelines for good assessment:

What makes a good assessment?

These might include statements like ... assessment should:

- improve learning
- focus on what students do know, rather than what they don't
- reflect all the goals of the curriculum
- be fair, valid and reliable
- allow students to learn at their own pace
- not label half the students as failures
- communicate to all involved that most real problems are messy and have more than one answer
- be reported in a clear and meaningful way ... etc.

See, for example, Stenmark (1989) and Clarke (1992) for more suggestions.

D3.2 Curriculum Project 4 – Assessment tool Part 1 – The readings

Brief description

This is the first part of Curriculum Project 4, Assessment tool, Part 1 – The readings. Participants explore a range of assessment alternatives, and develop a collection of relevant tasks

Rationale/aims

'Assessment tool' is the 4th of the four Curriculum Projects in the course. It is a short project, giving participants an opportunity to investigate a range of assessment alternatives for numeracy and to extend the criteria (developed in Session D3.1) for what makes a good assessment task. By the end of the project the participants should be aware of an extended repertoire of assessment methods and will have developed some tasks for assessment use in their class.

Preparation

Presenter
The previous week, the Presenter will give out to the Group 4 or 5 different readings (from CP4–2 to CP4–8).

Participants
Read
Assessment tool [CP4–1] and the given reading from CP4–2 to CP4–8.

Time: 1.5 hours

Materials needed

Handouts/OHTs/paper
Assessment tool [CP4–1]
Thinking about assessment [CP4–2]
Assessing students in numeracy programs [CP4–3]
Assessment alternatives [CP4–4]
Reshaping assessment practices: foreword [CP4–5]
Who assesses whom...? [CP4–6]
A socio-constructivist approach [CP4–7]
The role of assessment... [CP4–8]

References

Burton, L. 1991, 'Who assess whom and to what purpose?' in M. Stephens & J. Izard (eds) *Reshaping Assessment Practices: assessment in the mathematical sciences under challenge*, ACER, Melbourne.
Clarke, D. 1992, 'The role of assessment in determining mathematics performance', in G. Leder (ed.) *Assessment and Learning of Mathematics*, ACER, Melbourne.
Leonelli, E. & Schwendeman, R. 1994, *The Massachusetts Adult Basic Education Math Standards*, Massachusetts Department of Education, USA.
Marr, B. 1994, 'Thinking about assessment', *Numeracy in Focus*, No. 1, pp. 10–14.
McRae, A. 1994, 'Assessing students in numeracy programs', *Numeracy in Focus*, No. 1, pp. 15–19.
Stenmark, J.K. (ed.) 1989, *Assessment alternatives in mathematics: an overview of assessment techniques that promote learning* EQUALS: Lawrence Hall of Science, University of California.
Stephens, M. 1991, foreword, in M. Stephens & J. Izard (eds) *Reshaping Assessment Practices: assessment in the mathematical sciences under challenge*, ACER, Melbourne.
Yackel, E., Cobb, P. & Wood, T. 1992, 'Instructional development and assessment from a socioconstructivist perspective', in G. Leder (ed.) *Assessment and Learning of Mathematics*, ACER, Melbourne.

Procedure

This Project should be timed to start after Section D3.1 on assessment, and should extend over at least two weeks. There are two parts to the project:

Part 1	**– The readings**	1.5 h
Part 2	**– Comparing results: a range of alternatives**	1.5 h

Detailed instructions for this Curriculum Project will be found in the **Curriculum Projects and Numeracy Journal** section.

Preparation for Part 1 – The readings

At the previous session you should have given out to the Group 4 or 5 different readings (from CP4–1 to CP4–8), so that each person has one reading to do for the following week, and at least two or three participants will have read the same reading.

D4 Theory and practice: closing the gap

D4.1 Curriculum Project 4 – Assessment tool Part 2 – Comparing results: a range of alternatives

Time: 1.5 h

Brief description

This is the second part of Curriculum Project 4, Assessment tool, Part 2 – Comparing results: a range of alternatives. Participants explore a range of assessment alternatives, and develop a collection of relevant tasks

Detailed instructions for this Curriculum Project will be found in the **Curriculum Projects and Numeracy Journal** section.

D4.2 Theory into practice

Brief description

Participants review the theories they have met, and consider the gap between theory and practice.

Rationale/aims

Having reviewed theories they have met throughout the course, participants have a chance to focus on the vexed question of why practice does not always match theory, and to work out strategies for narrowing the gap.

Preparation

Presenter:
Read Ernest (1989)
Photocopy *Teaching: implied beliefs* [AS2], cut into sets of 12 pieces, and put each set in an envelope (1 set for every 2 students).

Time: 1.5 hours

Materials needed

Concrete materials
1 set/pair of the 12 pieces in *Teaching: implied beliefs* [AS2]
Handouts/OHTs/paper
Teaching: implied beliefs and practices [HO9]
The impact of beliefs on teaching [P13]

References

Ernest, P. 1989, 'The impact of beliefs on the teaching of mathematics', in P. Ernest (ed.) *Mathematics Teaching: the state of the art*, Falmer Press, UK.
Siemon, D. 1989, 'Knowing and believing is seeing: a constructivist's perspective of change', in N. Ellerton & M. Clements, *School Mathematics: the challenge to change*, Deakin University, Geelong.

D4.2 Theory into practice

Detailed procedure

This session involves an activity focusing on beliefs and theories, a review of what the participants have learnt from the course, and finishes with a discussion:

- **Beliefs, theories and implications** (15 min)
- **ANT: what has it been for you?** (45 min)
- **Closing the gap between theory and practice** (30 min)

Beliefs, theories and implications

1 Discuss the words we use/have used to talk about the beliefs that underlie theories ... *beliefs, values, assumptions, concepts, models...*

We all have beliefs, and beliefs form the basis of theories, though they may not have been elaborated into fully articulated theories. In that sense, we all have at least implicit theories.

2 We have been looking at three models of the teacher's role and their implications for practice. The organisation of such models is fairly arbitrary. Let's look at how someone else organises the information ... fairly similar, but not the same.

3 Get participants to work in small groups or pairs. Give them *Teaching: espoused role and implied beliefs and practices* [HO9], and the 12 prepared pieces from *Teaching: implied beliefs* [AS2]. Get participants to sort the pieces to fit appropriately on the chart. There may well be disagreement. The following chart is developed from how Paul Ernest (1989) organises the categories:

Teaching: espoused role and implied beliefs and practices

TEACHER AS ...	*view of maths*	*view of learning*	*pattern of use of curricular materials*	*intended outcome for the student*
INSTRUCTOR	accumulation of facts, rules and methods for some external purpose	student's compliant behaviour in mastering skills	the strict following of a text or scheme	skills mastery with correct performance
EXPLAINER	a consistent, connected and objective structure	the reception of knowledge	modification of the textbook approach, enriched with additional activities	conceptual understanding with unified knowledge

FACILITATOR	dynamically organised structure, in a social and cultural context	active construction of knowledge and even exploration and autonomous pursuit of own interests	teacher or school construction of the maths curriculum	confident problem-posing and problem solving

Constructed from Ernest, P. 1989, 'The impact of beliefs on the teaching of mathematics', in P. Ernest (ed.) *Mathematics Teaching: the state of the art*, Falmer Press, UK, pp. 250–252.

4 Discuss with participants whether the three models have much in common with the three positions explored by the course. Ask: *If this is a continuum of positions:*

- *Where would you place the second (explainer) model?*
- *Where would you place your experience as a learner? As a teacher?*
- *Where would you like to be?*

ANT: what has it been for you?

1 Individually, in pairs or small groups, get participants to summarise what the ANT course has been about for them, what has been important, what will make a difference to their practice as teachers, what assumptions have been confirmed or challenged, what they would like to do as teachers. They can do this by way of:

- a mindmap
- drawing a picture or diagram
- telling the experience as a soap opera

2 Get participants to share their stories, maps and/or pictures.

Theory into practice: closing the gap

1 Ask: *OK, we've talked a lot about theory and its implications for practice but we all know that practice does not always match theory. Why not?*

Get participants to brainstorm reasons for the mismatch.

The article by Dianne Siemon, 'Knowing and Believing is Seeing: A constructivist's perspective of change', may be referred to here and her ideas of *personal* and *public* theories of mathematics education and the conflict between them discussed.

The metaphor of the early Australian painters is one that is valuable in this discussion.

2 Then get them to work in small groups to read *The impact of beliefs on teaching* [P13], and to discuss the question:

What can we do to narrow the gap between what we believe and what we do?

3 Report back to the Group, and collect suggestions on board.

D5 And so what is numeracy?

D5.1 The chance of numeracy

Brief description

Participants engage in a variety of activities involving probability.

Rationale/aims

This session gives the participants a chance to review the course through a reflective engagement with a range of different activities focusing on ideas of chance. The activities give participants an introduction to ideas of probability, and also encourage them to analyse the range of teaching strategies and approaches used.

Preparation

Presenter
Read
Lovitt & Clarke (1988)
Prepare probability stations [AS3 to AS9] for the group.
ANT Lotto Cards [AS10]
ANT Lotto barrel—a device for selecting six numbers, e.g. six numbered balls or cubes, six cards, etc. Also you could bring along some prizes for the winners of the Lotto games—maths tickers, 10 sided dice or good old minties, jelly beans, etc.

Time: 1.5 hours

Materials needed

Concrete materials
Betting away
Materials related to betting and gambling—Tattslotto tickets, scratchie tickets; racing betting slips (TAB or on-course), racing pages from newspapers; pack of cards, dice; newspaper or magazine articles about gambling issues; other sport gambling information; etc.
Collecting data
Three coins, two dice, pens and copies of the instruction sheet—enlarge to A3 size.
Language for chance
Newspapers, cards for writing words on, textas/pens, a probability axis ranging from 0 (0%) to 1 (100%) or an enlargement of the one provided on AS5
Handouts/OHTs/paper
ANT Lotto winning table [HO11]
Maths and Lotto [HO12]

References

Ascher, M. 1991, *Ethnomathematics: a multicultural view of mathematical ideas*, Brooks/Cole Publishing Co, California.

Jacobs, H. 1982, *Mathematics: a human endeavor*, W.H. Freeman and Co., New York (or Australian Edition, 1977, published by Lloyd O'Neil, Victoria)

Lovitt, C. & Clarke, D. 1988, 'Maths and Lotto', in *Mathematics Curriculum and Teaching Program (MCTP) Activity bank*, Vol. 1, pp. 112–115.

Nelson, D., Joseph, G.G. & Williams, J. 1993, *Multicultural Maths: teaching mathematics from a global perspective*, Oxford University Press, Oxford.

D5 And so what is numeracy?

D5.1 So what's the chance of numeracy?

Detailed procedure

This session consists of two activities:

– **Probability fair**	(45 min)
– **Maths and Lotto**	(45 min)

Probability fair

1 Set up the probability stations [AS3 to AS9]. The idea is to enable each participant to spend time on at least three of the stations. Some activities, such as *Collecting data* [AS4], only take a few minutes whereas other activities may take up to 15 minutes.

2 Ask participants to move around the room and take time to complete at least three of the activities.

One idea behind this session is to also model a way of working with a group of students on collecting data for probability/statistics activities. Often students are asked, either individually or even in pairs, to sit and toss coins or roll dice to collect data for drawing conclusions. However this is often very time consuming and boring, and therefore has a very narrow focus. The idea of stations is that you can have a range of activities and experiments as stations around the room and ask students to move around and spend only 5–10 minutes at each. This gives everyone an opportunity to see and experience a number of different experiments whilst collecting enough data for the class as a whole to analyse. In this actual activity some of the stations are of a different nature and are introducing other concepts and ideas.

3 After about 40 minutes ask participants to return to their spots and spend some time discussing each of the activities and their possible use with ABE students.

Some relevant points for each of the activities are listed below.

Betting away

- The idea here was to allow participants to look at a range of everyday gambling materials and reflect on the maths involved.
- It is also one of the activities where it should be possible for participants to think of ways that they can use chance and probability in a critical social sense.

Collecting data

- These two activities are examples of stations that demonstrate how you can give students an opportunity to see and experience a number of different experiments whilst collecting enough data for a class as a whole to analyse, rather than have students all doing the same, repetitive activity.
- These are a way of collecting raw data to analyse chance type activities using experimental data before introducing the 'theory' behind them. Other stations attempt to look at the same events in a more theoretical way.

Language for chance

- This station provides an opportunity to analyse how we use words and statements in everyday situations that are using an expression about chance and probability.
- The debate about where different words or expressions fit on the probability number line allows for interesting and valuable discussions about chance and probability.

Tree, and other, diagrams

- This station introduces participants to diagrammatic representations for probability.
- It looks at the same two events as the station, *Collecting data*, but enables probabilities to be worked out using visual representation and the concept of equally likely outcomes.

Take your chances

- These statements give participants a chance to reflect on how probability gets used in different situations, and they can react to them in differing ways. You can highlight how probabilities are used and what value and significance is given to them. Do they actually represent true statements or not?

Cultural/historical derivation of probability.

- A reading piece to give participants some background to the historical and cultural perspectives of probability.

Black and yellow

- This station is taken from a Year 10 school text book and is fairly typical of how probability can be treated in schools and in a transmission mode.

4 To sum up return to our two and a half theories and reflect on each of the activities and decide whether they fitted one of the theories more than the others. Which ones could be extended or adapted to fit our critical constructivist model?

Maths and Lotto

This activity is adapted from Maths and Lotto, Mathematics Curriculum and Teaching Program (MCTP) Activity Bank, Vol. 1, pp.112–115.

1 Ask participants what they know about Lotto and their expectations of winning.
Who knows what Tattslotto or Lotto is? How do you play it?
If you bought one standard twelve game ticket each week of your life, how often do you think you might win first prize?

2 Introduce that you are going to investigate the mathematics behind Lotto by playing a simplified game. Give out some ANT Lotto playing cards to each participant and show the group your lotto barrel, and say:

- *you are to choose just two numbers out of six possible numbers, 1 to 6*
- *you need to guess both numbers correctly to win.*
- *mark one of your ANT lotto cards with game 1 and circle your two choices.*

3 Ask: (but make sure people guess—no maths expected at this stage)
How many winners do you expect from our group in each game?
If we were to play 100 times, how many times do you think you might win?
Write everyone's guesses on the board. Keep these figures on the board for later

4 Draw out 2 numbers from your ANT Lotto barrel, and see how many winners there are. You could give out a prize to the winners each time!
Ask each participant to construct a winning table like the one below, or use *ANT Lotto winning table* [HO11] to record the results, as shown below:

Game	My numbers	Winning numbers	Number of winners in group
1	2,6	1,6	1
2	1,5	2,6	0
3			
4			
5			

5 Repeat the process and play the game at least ten times.
Continue recording the results in the table.

6 **Now undertake an analysis of the maths involved. Remember that many participants may feel insecure about maths associated with probability and chance, so don't let any maths teachers take over too quickly.**

Use people's intuition and their own data which they have just recorded. Don't get side-tracked into a mathematical analysis of the results or the related probabilities.

Start by summing up how many winners the group had out of the total number of games played. That is, if you had fifteen participants each having played 10 games with 9 winners then the group had 9 winners out of 150 games played. Ask:

Do you think we were lucky or particularly unlucky?

Or do you think the results are about right?

7 From first principles work out all the possible pairs of results.

It could proceed something like this:

- *We need to know the chances of winning one game.*
- *You win if your pair of numbers is drawn, so how many pairs could we draw out?*
- *For instance there will be 1 with 2, 1 with 3, 1 with 4, 1 with 5 and 1 with 6. Write these on the board as you go:*

1,2 1,3 1,4 1,5 1,6

Ask: *Can you work out how many possibilities there are altogether?*

You should be able to get someone to tell you that there are 15 possible pairs.

Make sure that everyone can see all the possibilities.

A couple of issues to point out here include:

- This is an example of sampling *without replacement.* You could pose what happens when you can replace the numbered ball or counter each time, i.e. have 1,1; 2,2; etc? Do participants know of games where replacement is allowed, i.e. where you can have the same outcomes repeated?
- The issue of *independence* of events is also important. That is, the outcomes of each game are independent of any consequent game. Winning (or losing) in game 1 does not decrease (or increase) your chances of winning the next game. A nice way to illustrate this issue of independence is to note that the dice, or coin, or numbered ball, has no memory and therefore can't remember what number it showed previously (we think!).
- *Combinations* versus *permutations*, i.e. whether order is important or not in working out the number of possible outcomes.

You could quickly discuss situations where order *is* important (permutations) such as telephone numbers, car number plates, horse racing versus situations where order *is not* important (combinations) such as Lotto, or composition of people in a committee.

8 **Conclude that this means that:**
Since there are 15 possible pairs, your chances each time are 1 in 15, or that you can expect to win one game in every 15 games.

9 Now go back to the number of winners the group had. Ask:
What does this mean in terms of how many winners our group should have expected?
Was our group lucky or unlucky?
Here you need to analyse the expected number of winners by working out what multiple or fraction your number of players was of the possible number of winning pairs (15). That is, if you had a group of 15 players then you would have expected 1 winner per game, whereas if you had only 10 players you would expect less than one winner per game ($\frac{10}{15} = 0.667$ winners per game). If you had 20 players then you would expect more than one winner per game ($\frac{20}{15} = 1.333$ winners per game).

Calculate this for the whole Group and compare the expected result with the actual result. Return to the question *Was our group lucky or unlucky?* and answer it as a whole Group.

10 Now look back at everyone's guesses at the start about how many times they thought that they'd each win out of 100 games. Work out together that the answer from the maths analysis is that one winner out of every fifteen games means that there would be expected to be only 6.7 winners per 100 games.
Compare this with the initial guesses. Normally people will have overestimated the expected number of winners. See if this was the case, and ask questions such as:
It is interesting that most of you overestimated? Why?
What does this mean in terms of real Tattslotto?

- *for the player?*
- *for the operators of the games?*
- *why does it happen?*

Some issues:

- For the operator having a game that looks easier and attractive than it really is, is good news.
- For the players, the difference between reality and perception is an area that often leaves them open to disadvantage and exploitation.

11 Compare the 2 from 6 ANT Lotto game with the real 6 from 45 (or 6 from 40) Lotto game. The following is a possible explanation without going into detailed mathematics:

Lets look at Tattslotto where you select six from 45. You win first division if your six numbers are drawn from the barrel. So, how many groups of six are there in 45?

You could do it like this, just as you did for the 2 from 6 game:

1, 2, 3, 4, 5, 6
1, 2, 3, 4, 5, 7
1, 2, 3, 4, 5, 8
1, 2, 3, 4, 5, 9
⇓
1, 2, 3, 4, 5, 43
1, 2, 3, 4, 5, 44
1, 2, 3, 4, 5, 45 etc.

You can see that there would be a very large number of combinations.

It's actually 8 145 060 different groups of six.

That means your chances are one out of eight million for a single game.

In reality, many people fill out a ticket with twelve games which costs about $4.

So you can expect to win once in every 678 755 weeks, that is once every 13 053 years.

You could find out how many million games are played and hence find out how much is spent on Lotto in your state or territory using current prices in a week, etc.
Then you could work out how many first division winners you would expect each week, or how long on average you would expect a winner. Keep a check on how many winners there are each week. See how near it is to the mathematical predictions.

This section is the critical jump from a simple model to the real game. The experience with the simple game and the gap between perception and reality provides a base for an appreciation of the large numbers, and the low chances of winning the real game.
By showing how small the chances of winning are, hopefully, these figures will have some sort of shock value to participants unfamiliar with the probabilities.

12 If brought up by a maths participant and you feel the group could get something from the maths, you could introduce the formal maths method for working this out using combinations: choosing six items from a group of 45 where order is not important is ${}^{45}C_6 = 8\ 145\ 060$. This could be compared with the way of working out permutations.

13 Finish the activity by discussing how much of the activity you could use with ABE students. Give out *Maths and Lotto* [HO12] as background material to the activity they have just undertaken.

D5.2 Towards numeracy

Brief description

Participants look at the area of probability and chance, and use this to address the issue of meaning making in mathematics and the nature of numeracy.

Rationale/aims

This final session in the Adult Numeracy teaching course focuses on the question of the nature of numeracy and reflects on the course and its impacts on the participants and their teaching. The course finishes by looking at the question of what is numeracy by asking participants to reflect on what they have learnt from the course and on considering that numeracy is not less than mathematics but more.

Preparation

Participants
Read
Critical numeracy? [P14]

Time: 1 hour

Materials needed

Handouts/OHTs/paper
Critical numeracy? [P14]
Five strands of numeracy [HO13]
Numeracy definitions [HO14].
Five strands of meaning [OHT6]

References

Beazley, K. 1984, *Education in Western Australia*, Education Department of WA, Perth.
Cockroft, W. 1982, *Mathematics Counts*, HMSO, London.
Girling, M. 1978, 'Towards a definition of basic numeracy', in *Mathematics Teaching*, No. 81.
Johnston, B. 1994, 'Critical numeracy?' in *Fine Print*, Vol. 16, No. 4, pp. 32–36.
Marr, B. & Helme, S. 1991, *Breaking the Maths Barrier*, DEET, Canberra.
Thiering, J. & Barbaro, R. 1992, *Numeracy and How We Learn: a professional development program*, TNSDC, Sydney, pp. 48–51.
Willis, S.(ed.), 1990, *Being Numerate: what counts?*, ACER, Melbourne.

D5.2 Towards numeracy

Detailed procedure

This last session consists of two sections:

• **Five strands of meaning**	(45 min)
• **And, so what is numeracy?**	(45 min)

Five strands of meaning

1 Participants will have read the article *Critical numeracy?* [P14].

Discuss the article and especially the five strands of meaning making. These five strands are summarised in *5 strands of meaning* [OHT6].

Say to participants that as this is the end of the course it is a chance to reflect on the meaning of numeracy and the outcomes of the course.

2 Use it to discuss whether and how the probability activities of Section D5.1 match the five different strands of meaning-making referred to in the article.

Give out *Five strands of numeracy* [HO13] to participants in small groups of 2 or 3.

Ask them to discuss each of the probability activities and see how many of the strands the activities covered (or could have covered if easily extended).

After about 15 minutes ask each group to report back on each of the activities and as a group discuss which strands matched which activities, and which ones could easily be extended to cover a wider range of the strands.

3 Ask:

Do the five strands match our three or two and a half theories of maths learning and teaching?

Discuss as a group, and use it to lead into the final part of the course.

And, so what is numeracy?

1 Ask participants to jot down some comments, reactions and thoughts in their journal on what the ANT course has meant to them.

Some stimulus questions to pose:

How has it changed their ideas about adult numeracy?

Will it change, or has it changed, how they will teach numeracy?

What part did the coverage of the theories play in the impact of the course on them as teachers of numeracy?

Refer participants to *References for Module D* [HO15].

2 **Ask participants also to write down some comments on how they might define adult numeracy—not a formal definition, but some personal ideas and meanings.**

Give out a selection of numeracy definitions, *Numeracy definitions* [HO14].

Look at them in terms of historical development and changes, and also ask whether they agree with them.

Issues to discuss include:

- maths versus numeracy
- changing definitions
- incorporation in literacy definitions
- quantitative literacy
- the critical aspect of numeracy.

3 Use the earlier discussions as the basis for the question; 'So what is numeracy?'

Ask:

So what is numeracy?

Who is it for? Who does it benefit?

Who benefits from inadequate numeracy?

What is the relationship of numeracy to maths?

4 The aim of the course is to have participants to conclude:

that numeracy is not less than maths but more.

Look at the statement in *Critical numeracy?* [P14]:

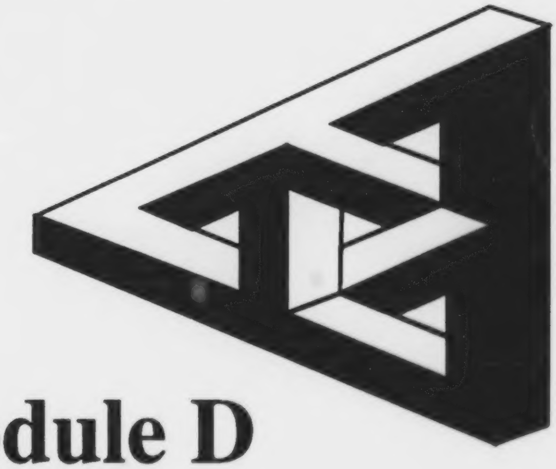
> *If you don't understand it, it's not numeracy,*
> *if it's not in context, it's not numeracy, and*
> *if it's not political, it's not numeracy.*

5 So what does it mean to be numerate?

- for them?
- for their students?

How has the course changed their position or view on this?

6 Celebrate!



Module D
Naming theories: implications for practice

Resources

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The following readings are not supplied and must be obtained before they are needed for copying and distribution to participants. Alternatively, direct participants to obtain copies themselves.

P1 Johnston, B. 1993, 'Mathematics for empowerment: what is critical mathematics education?' in S. McConnell & A. Treloar (eds) *Voices of Experience: a professional development package for adult and workplace literacy*, Book 4, 'Reframing Mathematics', DEET, Canberra, pp. 10–15.

P6 Shuard, H. 1986, Primary *mathematics today and tomorrow*, Longman, for the School Curriculum Development Committee, London, pp. 44–49.

P14 Johnston, B. 1994, 'Critical numeracy?' in *Fine Print*, Vol. 16, No. 4, pp. 32–36.

The following reading is not supplied as it is available in *Numeracy in Focus*, No. 1. It was also used as reading P3 for Module A.

P7 Johnston, B. 1995, 'Which map shall I use?' *Numeracy in Focus*, No. 1, pp. 33–37.

Two (and a half) contrasting views of knowledge

HO1
OHT1

Two (and a half) contrasting views of knowledge and their implications for learning and teaching

	positivism	***constructivism***	***critical constructivism***
Knowledge as	– external reality – objective – above human – true – God-given – unproblematic	– internal reality – subjective – human activity – viable – socially negotiated – problematic	as for constructivism + – situated – political
Learning as	– reception of information – absorption of facts – reproduction	– constructing – dealing with perturbation – reconstructing	as for constructivism + – purposeful – critically reflective – challenging power relations
Teaching as	– transmission – expert – concern for product	– questioning, provoking – collaborating, facilitating – concern for process	as for constructivism + – awareness of power relations – negotiation of power relations

Adapted from Jane Johnston 1990, paper given at MERGA conference

Constructivism in education

P2

page 1

From von Glaserfeld E. 1988, 'Constructivism in Education', in T. Husen and N. Postlethwaite (eds),
International Encyclopaedia of Education: supplement vol. 1, Pergamon, Oxford

Constructivism is a theory of knowledge with roots in philosophy, psychology, and cybernetics. It asserts two main principles whose application has far-reaching consequences for the study of cognitive development and learning as well as for the practice of teaching, psychotherapy, and interpersonal management in general. The two principles are:
(a) knowledge is not passively received but actively built up by the cognising subject;
(b) the function of cognition is adaptive and serves the organisation of the experiential world, not the discovery of ontological reality.

To accept only the first principle is considered *trivial constructivism* by those who accept both, because that principle has been known since Socrates and, without the help of the second, runs into all the perennial problems of Western epistemology.

The present flourishing of constructivism owes much to the doubts about the accessibility of an *objective* reality in modern physics and philosophy of science (Hanson, Kuhn, Lakatos, Barnes) and the concomitant interest in the sceptical core of eighteenth-century empiricism. Constructivism has as yet only an implicit relation with the constructivist approach to the foundations of mathematics (Lorenzen, Brouwer, Heyting).

• • • • • • • • • • • • • • • • •

The revolutionary aspect of constructivism lies in the assertion that knowledge cannot and need not be 'true' in the sense that it *matches* ontological reality it only has to be 'viable' in the sense that it fits within the experiential constraints that limit the cognising organism's possibilities of acting and thinking.

Cybernetics and control theory, being concerned with self-regulating systems, have developed a similar approach to cognition, according to which adaptation to the environment and a viable conception of the world must and can be constructed from data generated internally by 'trial and error' and required no input of 'information' (Maturana and Varela 1980, Foerster 1985).

Another source of constructivism was the analysis of communication and language stimulated by computer science. Shannon's mathematical theory (1948) confirmed that only directives of choice and combination could travel between communicators, but not the *meanings* that have to be selected and combined to interpret a message. Language users, therefore, build up their meanings on the basis of their individual experience, and the meanings remain subjective, no matter how much they become modified and homogenised through the subject's interactions with other language users. From the constructivist point of view, meanings are conceptual structures and, as such, to a large extent influence the individual's construction and organisation of his or her experiential reality.

At present the constructivist approach has had most impact on psychotherapy and the empirical study of literature. Among family therapists, for instance, the notion that every individual constructs his or her own experiential reality has led to the realisation that, in order to eliminate interactional conflicts, subjective constructs must be modified, rather than elements of an 'objective' situation (Elkaim 1984, Keeney 1983).

In literary studies, the realisation that *meanings* are not materially inherent in words or texts, but have to be supplied by readers from their individual stores of experiential abstractions, has drawn attention to the fact that interpretations are necessarily subjective and that the source of interpersonal agreement concerning an author's intentions must be found in the social construction of a consensual domain (Schmidt 1983).

The students' subjective interpretation of texts and teachers' discourse, and thus the subjective view of linguistically presented problems, is increasingly being taken into account in educational practice and research. Such a constructivist perspective has noteworthy consequences (Glasersfeld 1983). There will be a radical separation between educational procedures that aim at generating *understanding* ('teaching') and those that merely aim at the repetition of behaviours ('training'). The researcher's and to some extent also the educator's interest will be focused on what can be inferred to be going on inside the student's head, rather than on over 'responses'. The teacher will realise that knowledge cannot be *transferred* to the students by linguistic communication but that language can be used as a tool in a process of guiding the student's construction. The teacher will try to maintain the view that students are attempting to *make sense* in their experiential world. Hence he or she will be interested in students' 'errors' and, indeed in every instance where students deviate from the teacher's expected path; because it is these deviations that throw light on *how* the students, at that point in their development, are organising their experiential world. This last point is crucial also for educational research and has led to the development of the Teaching Experiment (Steffe 1983), an extension of Piaget's clinical method, that aims not only at inferring the student's conceptual structures and operations but also at finding ways and means of modifying them.

Different theories, different questions

OHT2

For the transmission view of teaching, breakdowns in communication are explained by means of an at least implicit reference to the successful case, because failure is the anomaly ... The constructivist on the other hand has to explain situations in which the students do understand what the teacher is saying, or do ‘see’ the mathematical structure. The key move in accounting for these situations is to see the teacher's and students' successful communication as cases in which they are continually not miscommunicating.

Cobb 1988, p. 175

That is,
the question for transmission teaching is to account for failure ... why do people ever fail?

For the constructivist teacher, the question is to account for success ... how come people ever construct the same knowledge?

Get rich quick

OHT3

You have a job working every day for four weeks (28 days straight). Your boss gives you a choice about your wages.

> You can be paid $1000 a day
> or
> you can be paid 1c the first day, 2c the second, 4c the third, 8c the fourth and so on, doubling each day.

Which method of payment will you choose?

How much do you think you would earn the second way? … which of the following would be the closest? … guess!

$1 $10 $100 $1000
$10 000 $100 000 $1 000 000 ???????

How numbers grow

HO2
page 1

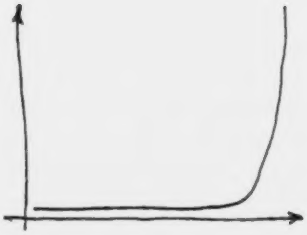
If you earned 1c the first day, 2c the second day, 4c the next, 8c the next day and so on, doubling the amount each day, how many dollars would you have earned after 4 weeks?

$1, $10, $100, $1000, $10 000, $100 000, $1 000 000 ... Which is closest?

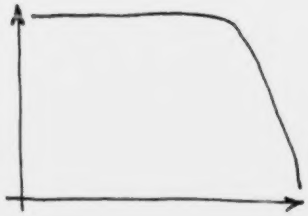
1, 2, 4, 8, 16, 32, 64c: one whole week and on the seventh day all you get is 64c! Not even a dollar! Let's have a look at the next week: $1.28, $2.56, $5.12, $10.24, $20.48, $40.96, $81.92; better, but still fairly pathetic. Why would you accept such a job?

Because when you work it out, you find that the wage for the twenty-eighth day alone is over $1 million. How does such vast change take place so quickly, when it seemed so slow to begin with? When you double a very very little number, you still get a very little number, and when you double that very little number, you still get quite a small number. But when you double a big number it gets very big. It took a week to earn the first dollar, but only another day to earn the second, and in another week you would earn over $100. In the last week alone you would earn more than $2 million.

This is exponential growth; it looks like the top graph on the right.



The bottom graph shows how much I owe the bank on my mortgage over the next twenty years. It is a similar shape, but upside down. It shows a similar slow start. I pay interest on my loan, and the bank calculates a repayment rate that makes the difference between repayment and interest so small, that for years my debt hardly decreases. When gradually the debt does become slightly less, the interest will become slightly less, and the difference between interest and repayment will be greater and so the debt will at last decrease a bit more and so on ...



To whose advantage would it be to start with slightly higher repayments? How is it possible to find yourself owing more than you borrowed? Would it help to pay back a lump sum early?

...and keep on growing...

And after finding out how to become a millionaire, you might like to see what David Suzuki, Canadian scientist and broadcaster, had to say about population and exponential growth in an address to young people at the ANZAAS Conference in Sydney in May, 1988.

> Now let me show you the really pernicious aspect of growth by giving you an example. I'm going to give you a test-tube and it's full of food for bacteria. I'm going to inoculate that test-tube with one bacterial cell, and that cell is going to undergo exponential growth; it's going to double every minute. So at time zero you've got one cell, at one minute you've got 2 cells, at two minutes you've got 4 cells, at three minutes you've got 8 cells—that's exponential growth. And at sixty minutes the test-tube is going to be completely full and there will be no food left. And the question that's asked is: when is the test-tube half-full, or half-empty? Anybody got an answer? Yes, at fifty-nine minutes … and at fifty-eight minutes you're twenty-five per cent full … at fifty-five minutes you're three per cent full.
>
> Now if a bacterium at fifty-five minutes turned around and said to one of its mates, 'Hey guys, I think we've got a population problem, I think we're running out of food and space,' the other cells would look at him and say, 'You're nuts—97% of the test-tube is empty, we've got vast reserves,' and they'd be five minutes away from filling .... Now if the bacteria are like humans and they only shape up when their backs are against the wall, then at fifty-nine minutes they finally wake up and say, 'We've got a problem.' So they jump out of the tube, and they roar around using the latest scientific gear and they discover three test-tubes full of food with no cells in ... So they run back to the test-tube and say 'Hallelujah! We're saved! We've got four times as much space and food.'
>
> How long will it last? At sixty minutes the first test-tube is full, at sixty-one minutes the second test-tube is full, and at sixty-two minutes all four are full. By quadrupling the amount of food and space you have bought two extra minutes. That is the consequence of exponential growth, and there are many people, including myself, who believe that we are long past fifty-five minutes.

Target

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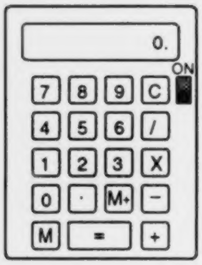
TARGET 100

- *Find another player and a calculator.*
- *Player 1 enters any number into the calculator.*
- *Player 2 then has to multiply the first number by another number, trying to get an answer with 100 as the first three digits.*
- *Player 1 then multiplies this new answer, trying to get even closer to 100.*
- *The players take turns until one player hits the target of '100.*****' on the calculator display.*



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Target 100: a sample game

player	*keys pressed*	*display shows*	*thoughts!*
1	*27*	*27*	
2	*x 4*	*108*	*– too big! but how can you multiply and make it smaller?!*
1	*x 0.9*	*97.2*	*– not bad*
2	*x 0.1*	*9.72*	*– hopeless! ... what was I thinking?*
1	*x 10.1*	*99.99*	*– wow!*
2	*x 1.01*	*100.9899*	*– gotcha!*

Variations

- Extend the target number to '100.00***' as you improve.
- Try playing using only division, or adding five before you multiply.
- A simpler version is found in *Mathematics: A New Beginning* where you specify a starting number (say 23) and a target number (say 100). The aim is to find what number is required to multiply with the starting number to give the target number as the answer.

Example

trial	answer	
23 x 6	138	
23 x 4	92	
23 x 5	115	
23 x 4.5	103.5	
23 x 4.3	98.9	etc

Four-in-a-line

HO4

A game for two players ...
you will each need a set of counters of one colour.

1 *Take turns to choose two numbers from the selection board.*

2 *Multiply the numbers on your calculator.*

3 *Cover the answer on the answer grid with a counter of your colour.*

4 *The first player with four in a line wins.*

selection board

7	27	56
11	31	67
16	46	71

ANSWER GRID

506	4757	77	1917	616	469
1426	1809	496	736	322	1512
217	1242	3752	737	896	1736
837	3082	432	189	781	2077
1136	341	176	2576	3976	392
3266	112	2201	1072	497	297

Variation

To help students reinforce tables knowledge, make another version using smaller numbers (e.g. 1, 3, 5, 6, 7, 8, 9, 11 and 13) for the selection board.

Adapted from The Shell Centre for Mathematical Education, Nottingham,
Calculators in the Mathematics Classroom: a day course for teachers.

Decimal patterns

HO5

Multiplication

A. Use your calculator to find the products:

7.3 x 18

0.73 x 180

73 x 18

730 x 0.18

0.73 x 0.018

7.3 x 0.18

0.073 x 18

What do you notice about where the decimal point is placed in the answer?

If the pattern isn't clear, make up some more combinations using pairs of digits: 73 and 18 with decimal points in different places.

B. How many decimal places in each answer?

8.2 x 7.124

0.01 x 0.03

Can you state a rule for placing the decimal point in each answer?

C. Now try these on your calculator...

2.44 x 0.35

0.315 x 0.2

3.65 x 17.25

76 x 1.85

90 x 0.423

3.60 x 0.30

Does your rule still work?

What's wrong?

Adapted from material from The Shell Centre for Mathematical Education, Nottingham.
Calculators in the Mathematics Classroom: a day course for teachers.

Calculating claims ✣ AS1

As a numeracy teacher, how would you respond to these claims?

'If we allow students to use calculators we might as well pack up and go home.'

'Numeracy is not just doing arithmetic, but this is ALL that calculators can do.'

'If students use calculators they will become lazy and mathematics will just become button pushing.'

'Numeracy just means being able to use a 4-function calculator sensibly.'

'Now that we have calculators, paper and pencil methods of calculation need not be taught at all.'

The case for calculators

P3

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From The Shell Centre for Mathematical Education n.d., Calculators *in the Mathematics Classroom: a day course for teachers.* The Shell Centre for Mathematical Education, Nottingham.

Ammunition for arguments

Teachers who advocate the use of calculators may have to justify their positions to parents or employers, or even to their colleagues. (Some parents and employers ask, 'Why doesn't this school use calculators *more* or *at all*?') Here are some suggestions to help deal with polemical opponents.

They won't be able to do arithmetic.
They will—and probably do it better if we teach them with the calculator. Over 50 research studies have been done and *none of them have shown any damage to the learning or personal arithmetic skills* from using the calculator; studies where the calculator was sensibly built into the teaching showed pupils own arithmetic progressing faster than one without. This is because the calculator is a powerful teaching and learning aid as well as a calculating aid.

Of course, you may not want to teach them as much arithmetic in a world with calculators—do we really still want log tables, or even long division?—but if you do, you can do it better if they have calculators.

How can that be? I don't believe it!
There are a lot of ways in which the calculator can contribute. You may know some like 'Forecast and Check' where the children check the answers to their arithmetic with the calculator—they don't cheat because they enjoy it and they also know they won't have a calculator in the test. But others are more important—exploring numbers gives them fluency.

You are only allowed to press these keys: $\boxed{3}$ $\boxed{4}$ $\boxed{-}$ $\boxed{x}$ $\boxed{=}$
Now make each of the following numbers in turn, using your calculator: 9, 2, 5, 6, 7, 10, 11.

The calculator is also good at revealing misunderstandings of basic principles: the pupil's finger hovering between the $\boxed{x}$ and $\boxed{+}$ buttons for example, suggests a diagnosis.

These are only a few of the ways in which the calculator can help.

They'll become lazy! Mathematics will just become button pushing.
Mathematics is not just doing arithmetic. It is just as important to know whether to multiply or to add than to do either of them—and children find that hard. Teachers won't be short of things to do. You can start to use mathematics more as well—most so called applications now have very simple numbers in so children don't get lost in the arithmetic. With a calculator, they can handle real numbers, even ones coming from their own measurements and observations.

'Mathematics is not just doing arithmetic'—but this is all calculators can do.
They can also be used to teach a lot of basic principles, reinforce concepts and stimulate weaker pupils, as well as the things I have already mentioned. Here's another example ... (choose something simple out of the material you have).

Employers want basic skills.
In most situations where calculations have to be done, employers provide calculators—they cannot afford to rely on possibly inaccurate pencil and paper arithmetic. They do still appear in quite a lot of employers' selection tests (they are one way to cut down the number of applicants to a short list); while this remains so, we had better use our calculators to help children develop their own arithmetic skills faster.

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page 2

People need arithmetic for everyday life.
We still need to retain basic skills, but we need to be more realistic about what they are. When did you last do long division, or work out something like 134 x 237 or 17.3 + 37? We still have to find out what really basic skills will have a payoff—there is no virtue in not using a calculator, anymore than there is in not using a ball point pen. It will depend on the ability, and interest, of the individual. For most people it will probably be worth knowing that 6 + 7 = 13, and knowing your multiplication series, being able to estimate the order of size of the answer and do some commonsense checking.

They are not allowed in exams.
They are allow in most GCE Maths exams, (though not in CSE maths with some Boards). They are often allowed in exams in other subjects. We have already said how using calculators in class can help children perform better in tests without calculators, and the sooner there are lots of calculators in our school the sooner the CSE Boards will allow them; one of their great concerns is being fair to all candidates.

I could do logs/trig/long division ... at their age.
They can do things you could not do. (the world is changing Mr Van Winkle). There is plenty that needs to be done in school without doing things that are neither useful nor interesting.

I spent half my professional life teaching children to do arithmetic: are you now saying this is no longer a marketable skill?
For the moment it still is, but it is done better with the calculator. However it is pretty clear that as time goes on there will be less emphasis on doing arithmetic and more on learning mathematics and using it to solve problems, practical or otherwise.

But we use difficult 'mechanical' arithmetic as part of our selection procedure for apprentices.
They are mostly not very good at it and I am sure your firm can't afford to rely on their pencil and paper skills, or to let them waste their time doing things by hand, so if you really need the arithmetic done you will already have got calculators for them. Do have a look on the shop floor at whether they really do do the arithmetic; there are lots of things in employers' training programs that are never used now.
You mainly have the selection test to cut down the number of applicants in a defendable way. Hard arithmetic can do that, but so can many more interesting or more useful bits of mathematics. Or you could try Latin.

Log tables are cheaper.
When next your wife asks for a new washing machine or Hoover, suggest she makes do with a cheaper dolly tub or Ewbank. Calculators are cheaper than football boots.

I won't let children use the calculator until they have mastered the basic pencil and paper algorithms. It's more important they understand.
Even if they are 15 years old? There are plenty of children who can use arithmetic without being able to do the calculations accurately, and plenty of others who can perform the calculations without understanding.

How can you be sure there won't be harmful effects?
Research shows that calculators do not reduce basic arithmetic skills; if anything, they improve. They seem almost universally to increase the motivation for doing mathematics. Complaints about inadequate skills in arithmetic have been with us for a very long time—they were just as common and just the same in 1900, when there was almost nothing else in the mathematics curriculum … Only a small proportion of children could ever do the long operations accurately if you tested them a few weeks after they had been taught and practised in them. Simple arithmetic isn't simple for most people, and that's not a consequence of modern maths. Calculators can help both in teaching arithmetic and reducing the need for it.

The impact of technology P4

From Hilary Shuard 1986, *Primary Mathematics Today and Tomorrow*, Longman
for the School Curriculum Development Committee, London, p. 135

Another major issue in curriculum development is the impact of new technology on primary mathematics. Calculators have become everyday tools in adult life, and the pencil-and-paper algorithms for the 'four rules' are disappearing fast; people now calculate mentally or use a calculator. In school, the pencil-and-paper algorithms are one of the most important bastions of 'transmission' teaching; children have had to be taught the correct ways of doing these algorithms, even though there is evidence that many children are remarkably resistant to learning them, and prefer their own methods. Now, the algorithms are no longer needed as useful life skills—a calculator is always available for computation which a person cannot do in his or her head. This new context for primary mathematics needs to provoke a complete reassessment of the curriculum in the field of number; at present, calculators are largely being used to support the traditional number curriculum, whose major effort was focused on the pencil-and-paper algorithms.

The removal of the need for children to learn the pencil-and-paper algorithms as tools for use will enable number work to focus on understanding of number system, and on using it for problem solving, both within mathematics and wherever else in the curriculum problems occur. In this way, it will become easier for the style of teaching in primary mathematics to move from the 'transmission' model to a 'constructivist' model. However, the pencil-and-paper algorithms for the four rules have been a major, and necessary, element of primary arithmetic for the last hundred years, and it will not be easy for many teachers to grasp that these algorithms are no longer necessary for use, nor that a valid, useful and exciting primary mathematics curriculum is available without them. The new technology of calculation provides the biggest challenge to the content of school mathematics throughout the whole history of compulsory schooling in this country. It is a challenge that must be tackled if the primary mathematics curriculum is not to become 'the sabre-toothed curriculum'—activities that are only done in school, and which have nothing to do with life.

The calculator is a cheap, personal, portable tool which mechanises one type of mathematical skill. The same cannot yet be said of the computer. Outside school, it is still largely a desk-top machine, and within school it is still an expensive resource that has to be shared between many children. however, this may change within the next ten years. Portable computers the size of an A4 file are already available, although the size of the display limits their use at present, and the cost is still prohibitive for individual use by most people. It is possible, however, that the next generation of primary children may own portable computers in the way that the present generation own calculators.

The social shaping of technology

P5

From Richard Noss 1991, 'The social shaping of computing in mathematics education', in D. Pimm & E. Love (eds) *The Teaching and Learning of School Mathematics,* © Hodder & Stoughton, London, pp. 208–21.

I will remain a little longer at the general level of technology, before I attempt to draw the implications for mathematics pedagogy in the next section, and consider the balance between living and dead mathematical labour 'within' computers. This balance is not arbitrarily defined and so I need first to consider how technologies are shaped, how social forces interact with their evolution.

I begin with an example. While writing this article, I discovered to my cost (literally) that monochrome film now costs almost twice as much to be developed commercially as colour. When I asked for an explanation I was told that this was because 'the machine has to be set up specially'. This is despite the considerably simpler process and cheaper chemicals involved in monochrome processing. Presumably this technological change signals the end of mass black-and-white photography. Moral: the ways in which technologies develop are comparatively little to do with the inner logic of that technology and much more related to the social forces underpinning the production (and distribution) process.

But that is not all: such changes reinforce the view that photography is 'about' reproducing reality as 'faithfully' as possible (I do not discuss the truth of this assertion: only that it is one among many possible views of photography). From this perspective, colour is *better* because it is closer to reality, a more faithful representation of what is photographed. And so there is a dialectical relationship between the way in which the technology evolves and the ideas (or ideologies) which it gives rise to (and, recursively, which shape the technology). The social and economic spheres shape the development of the technology, and *simultaneously* legitimise particular views of what the technology is about.

This perspective is variant of what has become known as the 'social and shaping of technology' perspective. This approach focuses on the construction, design and development of technologies from the perspective of the network of social forces and interactions on which design is based, and which crucially shape the invention/production process.

There are numerous debates and issues which arise from this perspective (see for example, the collection by Mackenzie and Wajcman, 1985, and the helpful review by Mackay and Gillespie, 1989). From my point of view, the most important element of the argument is the possibility it opens for *alternatives.* Monochrome photography does not *have* to go out of existence; computers in schools do not *have* to become tools for developing managerial skills in the few; mathematics education is not doomed to an endless diet of increasingly (un-) intelligent tutorial systems (in various guises) or well-intentioned 'investigations' which result in the further fission of an already fragmented learning domain. Of course, changing such patterns of development involves all kinds of difficult social, economic and political dimensions which are beyond the scope of this paper. Nevertheless it is to the potential for change that I now turn.

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Constructivism in a straitjacket

From G. FitzSimons & P. Sullivan 1993, 'Constructivism in a straitjacket,' in J. Malone & P. Taylor (eds) *Constructivist Interpretations of Teaching and Learning Mathematics*, National Key Centre for School Science and Mathematics, Curtin University of Technology, Perth, pp. 100–103.

The authors were involved in teaching the mathematics strand of a Victorian TAFE vocational course *Advanced Certificate of Applied Science (Laboratory Technology)* .

The Implemented Program: Constructivism In Practice

The context of this mathematics course has been described, as have the thinking and principles which guided the teaching. The following shows details of the implementation of this approach.

The students, especially the women returning to study, are nervous at first. To overcome their anxieties, strategies such as the re-arrangement of seating to encourage interaction, introducing cooperative groups, and seeking alternative forms of assessment have been implemented. A mathematics anxiety Bill of Rights is also discussed early in the course, along with a video of the contrast between school mathematics and the mathematics used by children and adults in the everyday world (Ministry of Education Victoria, 1985). This video has humorous as well as thought-provoking moments, highlighting the importance of context in mathematics, as well as the diversity of approaches that adults adopt in solving real problems. Viewing it allows students time to calm their initial feelings of apprehension, and provides the opportunity for discussion of educational philosophy, and recent changes in approach to the teaching of mathematics.

Seating students in groups of about four at a diagonal arrangement of tables allows them to converse readily with one another while being able to see the board. Cooperative logic puzzles (e.g., Erickson, 1989) allow students to get to know one another, and encourage them to make conjectures and discuss mathematical ideas. The idea of working cooperatively instead of competitively is new to some students, and values need to be made explicit through discussion following the puzzles. The idea to being able to work cooperatively also provides further assurance to those who are feeling anxious.

The following are examples, taken from the first author's personal journal, of approaches taken to three separate topics illustrating in practical terms the approach taken.

Introduction to arithmetic. Early in the course it is necessary to review arithmetic processes. Beginning a class with students of diverse backgrounds (junior secondary to tertiary and non-science or non-English speaking backgrounds) it is difficult to know where to begin. In this case the students were asked to construct their own questions on whole numbers, decimals, fractions, powers, percentages and integers, after being given the answers and some restrictions. On stipulation was that everyone in the group had to be confident in using the processes. Results were to be checked using calculators. At intervals group results were compared as a whole class activity, and conflicts were exposed, or 'what if...?' questions were asked to extend concept formation. For example:

> If 0.4 x 0.2 = 0.08, how could you make an answer of 0.008?
>
> How could you use a three-digit decimal as one of the multipliers to make 0.08?
>
> What is the square root of 0.4? Why is it not 0.2?

Students were encouraged to keep regular journals for reflection on their own learning and to provide feedback for the teacher.

Scientific errors. Another key topic was errors. We began with a discussion of the distinction between mistakes make by humans, equipment that was faulty in operation or calibration, and the errors that arise due to the limitations of measuring instrument. In order to provide a shared common experience the students took measurements of length, time, capacity, pressure, temperature and mass. The measurements thus collected were used in calculations related to absolute, fractional and percentage errors.

The most important part was the discussion which followed the measuring. The stopwatch completed a revolution after only 30 seconds yet several students timed their activity as lasting one minute, seemingly unaware of the discrepancy. we had a discussion on the greatest length for which it was feasible to use a one-metre ruler, and, if two were joined together over a long distance, would their individual errors still need to be added? What is air pressure, and why is it measured in millimetres? What are millibars? One student who is an experienced laboratory technician explained a discrepancy between two thermometers as being due to one using mercury and the other suing alcohol. She claimed that, at high temperatures, such as the boiling point of water, the alcohol thermometer was inaccurate. how could you tell? Of the two top-loading balances, one had readings to two decimal places, and the other had only one decimal place. The former, however, was affected by the vibration of the building, and the last point fluctuated: how does this affect the error allowance? Why can't you read a temperature of 37.5 degrees when there are only whole degrees marked? Or can you? What about 7 millilitres when the markings are graduated in two's? On several occasions it was necessary to step outside of the traditional role of mathematics teacher, and encourage the students to by-pass the mathematical rules intended by the syllabus writers, in order to have the students think out the issues. Normally this lesson would have been completely comprised of rule giving and rule following. Application questions relating to the laboratory would have ignored the context, picked out the numbers, manipulated them, and checked the answer at the back of the book. These students have been urged to pursue some of the concepts with their physics, chemistry and biology teachers.

Metric conversions. This topic was practically based. past experience has shown that students have difficulty in converting between metric units. For example, to convert square centimetres to square millimetres they want to square the dimensions as well as the units, or else they forget to square the units, multiplying by 10 only. In this class they were asked to measure and draw a square centimetre on graph paper, then construct a rectangular shape of six square centimetres, and finally to convert the area of both shapes to square millimetres, justifying their answers.

The second part of the lesson was to draw a net and construct a cube with sides of length one centimetre. One Cambodian-born student made an open cube which held together perfectly without any tape or glue—not quite what was expected, but a meticulous piece of work. An origami expert constructed a closed cube using a different net from the traditional one found in textbooks. Students then were asked to predict and calculate the surface area, volume, mass, and capacity when their cube was filled with water. They were also able to explain why their measured mass was not exactly the same as their prediction. This part of the lesson was a particularly memorable one, and frequently referred to in later lessons when making calculations. next the students worked together to construct a cube of ten centimetres in length, performing similar calculations as for the one centimetre cube. In previous years students have attempted to weigh a litre of water, enclosed in a very soggy paper bag held together with sticky tape, on a small top-loading balance—with near disastrous results! many students in the past have been unable to give a sensible answer to the question of how many decilitres there were in four litres. yet when handling, looking at, or even visualising a measuring cylinder, they were able to ascertain the number of decilitres in one litre, how many millilitres were in one decilitre, and give meaningful answers to questions like the one above. Those who did not know the meaning of 'deci-' were helped by supportive discussion with their peers.

The most interesting part was when the students worked in groups to create a poster showing the meaning of important metric prefixes, and conversions between them. At first they were a little daunted by the openness of the project, although they had been supplied with a standard SI units brochure. Gradually the level of conversation grew from a gentle hum to a buzz, with much merriment in an all-male group. It was pleasing to hear so much technical conversation being meaningful to its users, instead of the usual 'How did you get that?'.

The group with the male students was most creative in that it took alcohol consumption as its theme, and utilised the various prefixes right through to the portions that a cell with its body would need to 'have a drink'—complete with illustrations! A representative from each group was asked to explain the poster and answer any questions, and then the posters were displayed on the wall.

The last part of the lesson was to have the students construct and answer their own questions on conversion between metric units. In the past this is the section of the course where students had complained most vociferously about it being difficult and/or boring, so the posters and the questions were an attempt to turn this around. Even so, constructing questions still proved a difficult task for some, but because they were used to working in groups they were able to have meaningful conversations when difficulties arose. As an aside, it is interesting that the students actually requested that they be given exercises to consolidate their skills. For homework students were given a measurement chart showing how various lengths, such as the diameter of an amoeba and the height of a tree, were located in the logarithmic scale, and asked to add any lengths that were significant to them.

The quantitative evaluation showed that there were no significant differences between the performance on skill and application exercises by these students and those from previous years. However the qualitative evaluation showed that these students had developed more positive attitudes to learning, they were more confident and were more willing to contribute their won ideas and to feel in control of their own learning.

Outcomes

According to von Glaserfeld (1987:324), 'the primary goal of mathematics instruction has to be the students conscious understanding of what he or she is doing and why it is being done.' In the teaching examples listed above, students conscious understanding of 'what he or she is doing and why it is being done.' In the teaching examples listed above, students were receiving first-hand experience, negotiating meaning through interaction with others, and justifying their answers. Symbols were a shorthand notation for the recording of experiences, not the first stage of exploration. Building a firm foundation, students were able to add several layers of abstraction.

Because they were relaxed and confident in themselves as learners, and comfortable in a conjecturing environment, those who persisted with the course as a whole successfully completed the requirements of Laboratory computations. Even those who were initially anxious overcame their fears, and continued on with a full statistics subject run along similar lines. Two people, returning to study after many years, with limited backgrounds, initially among the most anxious, even asked what mathematics they could study next!

As educators we are aware that 'knowledge and competence are products of an individual's conceptual organisation of the individual's experience' (von Glaserfeld, 1987: 330), and that our role is to guide and help the student in certain areas of experience. At the same time it is through discussion with and between students that we have come to know more about what they are thinking, to assist us in our role as facilitators of learning.

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The school of Barbiana

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The School of Barbiana 1969, *Letter to a Teacher*, translated by
N. Rossi and T. Cole, © Penguin, London, pp. 34–37

A striking example of a critical use of statistics can be found in the classic little book Letter to a Teacher *by the School of Barbiana (extracts below). The School of Barbiana was set up by a priest in a church hall to give some education to the many young people who were failing in the state system. There were few teachers, and few resources. One of the projects that eight of the boys chose to do was to write about how they failed in the system, and how the system failed them.* ***Letter to a Teacher*** *is exactly that: an extraordinarily powerful letter to the teachers they all had, the teachers who failed them, posing questions and describing uncomfortable patterns of behaviour.*

There are many unusual and impressive features of the book, but the one we are focusing on here is its use both of eloquent, simple language and of powerful mathematics, in its analysis of the Italian education system. Students describe their own experiences of failure in the system, and then say: but you could argue that we had just had bad luck...this is not so, and we will show you with the use of numbers that we are not a special case. And they do so—so effectively, that their book won a prestigious prize from the Italian Physics Society for its use of statistics. And all this done by school drop-outs!

Can we teach like this? Can we help our students use numbers to argue causes which affect their lives?

Disarmed The poorest among the parents don't do a thing. They don't even suspect what is going on. Instead, they feel quite moved. In their time up in the country they left school at nine.

If things are not going so well, it must be that their child is not cut out for studying. 'Even the teacher said so. A real gentleman. He asked me to sit down. He showed me the record book. And a test all covered with red marks. I suppose we just weren't blessed with an intelligent boy. He will go to work in the fields, like us.'

Statistics

At the national level Here you might object that we happened to take our examinations in particularly bad schools. Also, that whatever reports we receive from elsewhere all happen to be sad. You can say that you know a lot of other examples, as true as ours, but leading to the opposite conclusions.

So, let us drop, all of us, a position that has become too emotional and let us stand on scientific ground.

Let us start all over this time with numbers.

Unfit for studying Giancarlo took on himself a job of compiling statistics. He is fifteen years old. He is another of those country boys pronounced by you to be unfit for studying.

With us he runs smoothly. He has been engulfed in these figures for four months now. Even maths has stopped being dry for him.

The educational miracle we have performed on him comes out of a very clear prescription.

We offered him the chance to study for a noble aim: to feel himself a brother to 1,031,000 who were failed,[18] as he was, and to taste the joys of revenge for himself and for all of them.

The cocksure teacher Scores of statistical compendia, scores of visits to schools or inquiries by letter, and trips to the Ministry of Education and to ISTAT[19] to gather further data, and whole days spent at the calculating machine.

Others may have done similar research before us. They must be the kind of people who can't translate their findings into plain language.

We haven't read their findings. Neither have you teachers.

And so none of you has a clear idea of what really goes on inside the schools.

We mentioned this to a teacher visiting our place. He was mortally offended: 'I have been teaching for thirteen years. I have met thousands of children and parents. You see things from the outside. You don't have a deep knowledge of the problems in a school.'

Then it is *he* who has a deep knowledge—*he*, who has only known pre-selected boys. The more of them he knows, the more he goes off the track.

Gianni means millions Schools have a single problem. The children they lose. The Giannis.

Your 'compulsory school' loses 462,000 children per year.[20] This being the case, the only incompetents in the matter of school are you who lose so many and don't go back to find them. Not we: we find them in the fields and factories and we know them at close range.

Gianni's mother, who doesn't know how to read, can see what the problems of the school are. And so will anybody who knows the pain endured by a child when he fails, and who has enough patience to look through statistics.

Then these figures will begin to scream in your face. They say that the Giannis run into millions and that you are either stupid or evil.*

The pyramid Since the statistical tables may be hard to digest, we have put them into appendixes. Here in the text we cut them down to a human measure. To fit into a classroom that can be embraced with one loving glance.[21]

We have decided to keep the pyramid diagram here.[22] It is a symbol that leaves an impression on the eye.

It looks as if it is chopped out by hatchet blows. Every blow, from the elementary years up, is a creature going off to work before being equal.

Tracing the class of 1951 But the pyramid does have the defect of putting students from age six to age thirty on the same sheet of paper—failures old and new.

Let's try to follow one class of children throughout their eight years of compulsory schooling.

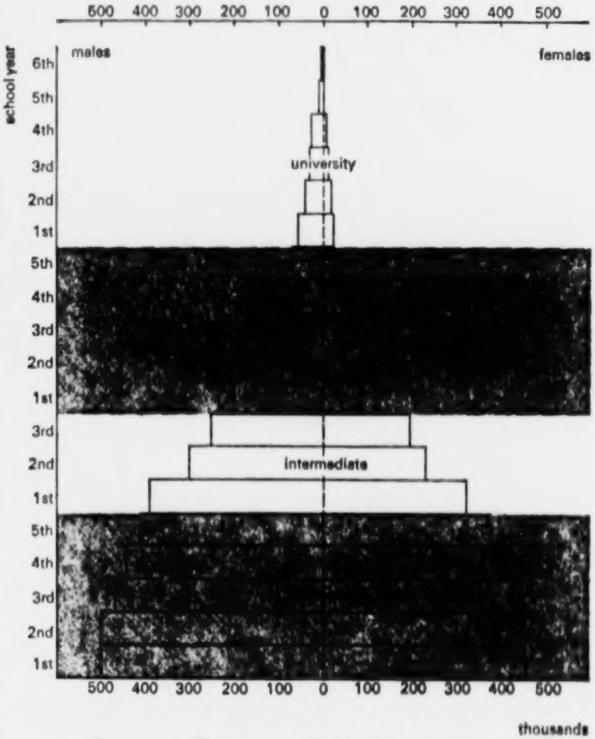

FIGURE 1 Children enrolled in the year 1963–4

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*Here, from pages 37 to 54, follows a statistical analysis of the failure and dropout patterns in the Italian schools, demonstrating a powerful discrimination against the children of the working of farming classes. These are specifically Italian problems. However, the British reader may still be interested in these analyses and calculations as a sample of the way the students at Barbiana were taught always to make their point and base their findings on solid statistical foundations. Because of this serious effort on the part of these children, the Italian Physical Society gave a prize (generally given to promising physicists) to the school of Barbiana after the publication of this book [Translators' note].

18. Failed from the compulsory school during the school year 1963–64 …

19. ISTAT: Istituto Centrale di Statistiche [Central Institute of Statistics].

20. This figure is taken from Table A (pages 118–19) and from the procedure in Table C, pages 124–6.

21. We have imagined a first year of 1957–8 with thirty-two students. That is 29,900 times smaller than the actual number that year. Anyone preferring the actual figures can find them in Table C (for 1951) on pages 124–6.

22. All data used to draw the pyramid are taken from the *Annuario Statistico Dell' Illstruzione Italiana* 1965 [Yearbook of Statistics on Italian Education 1965].

BEST COPY AVAILABLE

Activity theory: a summary P10

Adapted from S. Mellin-Olsen 1987, *The Politics of Mathematics Education*.
D. Reidel Publishing Company, Dordrecht, pp. 30 & 56–57.

I have developed a concept of *Activity* which is related to man's capacity to take care of his own life-situation, be responsible for it, and make decisions for it, together with others. According to Soviet psychology of the thirties *tools* are vital for Activities, originally tools for the hand, later communicative tools and thinking tools, such as language and mathematics. Tools are meaningful if and only if they can be related to Activity. In this case we say they are *functional* for the individual. When pupils cease to learn mathematics this will be related to their failure to relate the thinking tools to what they can recognise as an Activity.

According to Vygotskian research and simple observations of children's play, we realise that they possess important thinking tools in the form of sign-systems. These tools are functional for children's play. Where Piaget tends to say 'the child has not', Vygotsky tends to say 'the child has, look how she uses her thinking tools.' Where the followers of Piaget, more or less in the spirit of his theory, construct tasks which direct them away from their daily Activities, Vygotsky would ask us to start there, and connect school Activities to them, having regard to the historical and social dimensions of the Activity.

For the purposes of education, Activity theory as developed by Soviet psychology during the thirties is too limited. From the experiences reported in my Introduction, we see that the pupils are in a position where they can refuse to participate in school learning. Furthermore, Freire has demonstrated clearly how silence and passiveness occur among people when they are denied access to Activities. We have to develop Activity theory so that it can include such phenomena. As people of today's societies have different ideologies for their Activities, the political nature of Activity has to be analysed and explored.

It is implicit in the concept of Activity that we, as educators, have to include the pupils as decision-makers, planners and organisers of Activities. Moreover we have to look for methods, in particular from social anthropology, which can narrow the gap between what school by its history and current practice regards as sensible tasks for its pupils and what pupils themselves regard as Activities.

Cups of birdseed

P11
page 1

From J. Mousley 1993, 'Constructing meaning in mathematics classrooms: text and context',
in J. Malone & P. Taylor (eds), *Constructivist Interpretations of Teaching and Learning Mathematics,*
National Key Centre for School Science and Mathematics,
Curtin University of Technology, Perth, pp. 128–131.

Years 5 and 6 teachers in this account had agreed to carry out a particular activity with their classes. Instructions for the activity were: 'From a given piece of cardboard, make a regular cup of birdseed which holds one cup of birdseed. Make a similar shape which is twice as big.'

Defining the Task

Some teachers thought the given task too vague and realised that it might not lead to the discovery of the 'mathematical knowledge' they felt was bound within the task. One teacher (Teacher H, Year 6), for instance introduced the lesson by writing on the chalkboard. 'Make a box which holds on cup of seed. Make another box which holds twice as much.'

At a later date, the researcher (R) interviewed this teacher (TH):

R I am interested, first, to know why you thought the shape should be a box. Had you thought about the possibility of making other shapes?
TH Yes, but I wanted to build on this lesson to give them an understanding of volume.
R Good. So they will do that with box shapes?
TH Yes. They have to learn length by width by height and ... well, they couldn't do that with other shapes ... Oh, I guess they could, but, like cones and other shapes—I didn't want shapes where they couldn't measure length and width and height.

In planning the activity around a particular learning objective—a formal rule—Teacher H clearly demonstrated that he expected all students to take a relatively directed path of 'discovery'. His pupils later demonstrated an acceptance of this role context, clearly displaying characteristics of students waiting to be led.

Rob But it has to be a box. Not a box. He said a cube. That's the same all around. The same size—this way, this way, this way. Ask him how big. Darren, ask him how big to make it.
Darren How big would fit? You've got the cardboard. How big could we make it? It has to hold a cup.
Rob Just ask him, Daz. He knows.
Darren Okay. He knows. (Inaudible) Mr H ....

Similarly, Teacher N (Year 5) thought that the activity could be used for children to discover what happens when all three dimensions of a cube are doubled. She first told the children to 'Make a cube 5cm by 5cm by 5cm.' When they had all finished, she asked them to 'Now make one measuring 10 by 10 by 10' and to use the birdseed to compare the volumes of shapes constructed. After a short discussion which led to the teacher expressing the generalisation that 'the big one holds eight times as much', she proceeded to give a six-minute explanation of why this is so. Words such as *third dimension, multiply, multiples* and *comparative volume* were used. When asked later if she thought that all of the children would have understood, she claimed

TN Yes. That is why it is important to have the hands-on work first. Yes. They had seen it with their own eyes. That is why children need to do real things in mathematics—so they understand why the mathematics works.

Traditionally, such explanations by teachers have played an important part in the teaching of mathematics: so much so that it is hard for us to imagine learning mathematics without having it explained to us by a teacher. However, as Wittgenstein (1956) acknowledges, explanation fails when students lack knowledge to interpret it with understanding. This teacher did not explore the individual understandings arising from the task before attempting to explain the 'real' mathematics she saw in the task. Such explanation does not allow for a process of gradually adapting new knowledge to fit prior cognitive structures. It assumes that understandings are transmitted through oral discourse rather than created and negotiated by students.

Our teachers recognised that the task could be interpreted by students in different ways and saw this as opening up opportunities for people to work at their own levels. When Teacher M (Year 5/6) first saw the activity, for instance, she said

TM	It will be interesting to see what they make of it. I'm not going to explain what *regular* means. I wonder if they will ignore the word.... Seeing the different shapes will be fun. And their reactions when they double different aspects.

The groups in her class made various shapes, and some surprisingly difficult explorations were the result. Learning, in this case could be classified as constructivist because it involved an unrestricted interplay between reality and possibility (Inhelder & Piaget, 1958). Space does not allow a detailed reporting of the resulting group discussions, but the following is a fairly representative example.

A group of three girls had built a square pyramid, left open at the apex so seed could be poured in using a small funnel. They had found that their shape held more than one cup of seed, so traced it onto another piece of card and trimmed the triangular parts of the net gradually until a pyramid of the right size was formed.

Silvia	Don't take too much off. Remember...(inaudible)... you are not just taking that bit. That long bit. You are taking it four times. No, eight times. There and there and (etc.)
Binny	And it's not just thinner. The shape. Look. It gets shorter so you are losing this bit. The top bit every time. The whole pyramid loses the top bit. (Some trial and error followed).
Silvia	Good. That's it. One cup. I'll tape it up... Now. Two cups. Now for two cups.
Rachel	Two cups. Yes. Or twice as big? Two cups isn't twice as big.
Binny	(Inaudible, then laughed.)
Silvia	Yes it is. But I know. But...I know what you think. Like...like two times the edges. Make the bottom twice as big. The sides too?
Binny	Not the sides.
Silvia	Why?

Such excerpts show the negotiation of meaning necessary for the completion of the task as well as the development of complex mathematical ideas. These did not arise only through external discourse but also through abstract reasoning and expression of predictions. Each participant in the latter discussion contributed to its movement towards clarification of the task and the creation of possible solutions to the problems posed along the way. Individuals noted when others had not followed their reasoning and used alternative text forms (sketches, hand movements, pointing to and handling the pyramid) to help others construe their meanings in order that shared understandings could be constructed. Such cooperation was not necessary in the earlier examples because the teacher remained in control of instructional functions. These passages support the claim of Candy (1989) that learners will not attain full and undisputed ownership over the learning situation while they construe the instructor as still exerting residual authority. The markedly different atmospheres in the classrooms studied indicates that students attend to the way that schooling has been constructed within their classrooms.

Presentation of a loosely-defined task seems to allow for some exciting, as well as some disconcerting, developments. Schratz (in press) claims that

> The more interpretive space a teacher leaves in the learning process, the more likely the chances will be for the students to use their own knowledge of the world and everyday reasoning to tackle education problems. However this process is also dependent on providing materials that can be subject to negotiation and multiple interpretations. (In press)

Whetley (1991) comments that with instructional processes which sanction natural instincts to construct meaning,

> students come to realise they are capable of problem solving and do not have to wait for the teacher to show them the procedure or give them the official answer. Students come to believe that learning is a process of meaning-making rather than the sterile academic game of figuring out what the teacher wants. (p.15)

However, recognising that knowledge is what is learned from action and experience challenges many of traditional pedagogy. The teacher is no longer passing on a codified body of knowledge, so s/he can no longer be thought of as in control of the learning process.

No doubt many teachers would find progress frustratingly slow. This group, for instance, took thirty-seven minutes to decide on and make their first shape. In the meantime, others had completed their two shapes and gone on with related activities of their won inventions (e.g., weighing the seed from each shape, writing about the activity or making new shapes). In such a set-up the traditional teaching role of controlling activity time is lost.

Control over discourse is also lost. Much of the group's discussion during this period was about their roles in the school play and about the shapes that other groups were making. There were periods of up to four minutes when one student of the three was not on task and most of the measuring and cutting was completed by Rachel. However, the end product for each of the students seems to be a furthered and complex understanding of how dimension is related to volume. This has yet to be formalised, but if this knowledge can be retained, one would imagine it will form a potentially useful basis for later learning.

Probably the most troublesome problem, though is the teacher's loss of control over what is learned (or at least what is assumed to be learned).

Implementation or critique? P12

From R. Noss 1990, 'Political dimensions, political axes', in The Proceedings of the Political Dimensions of Mathematics Education conference, London, p. 3.

I think there is one interpretation which might guide some of our thinking at the conference: and that is to reflect on the *self image* which mathematics education, conceived as a field of study, has of itself. Of course, the ways in which mathematics educators see themselves structures and is structured by the field in which they operate. But I think that it is not too much of a generalisation to say that, at least until recently, mathematics education has been concerned with implementation rather than critique: it has conceived its role as improving mathematical learning and teaching, while regarding as relatively unproblematic ontological questions about the nature of mathematics and its relationship to the society in which it is produced.

That is not to say that the field has not generated fierce controversy. For example, the debate between 'teacher effectiveness' researchers, and those motivated by a 'constructivist' model of learning, has formed an important arena for clarifying what kinds of intervention are appropriate within the pedagogical domain. The view of children and teachers as active meaning-makers, has challenged the underlying assumptions of those who prefer a view of the teacher–student relationship as essentially modelled on that of transmitter and receiver. Nevertheless, this kind of debate still takes place within a strictly psychological framework, where all the important elements are essentially taking place within the heads of individuals (or, occasionally, groups of individuals).

This kind of paradigm has essentially left us powerless to answer the criticisms levelled at the educational enterprise in general, and at mathematical education in particular. Even populist criticisms for a return to rote learning and computation are difficult to challenge without some understanding of why and how mathematical knowledge is produced, and some critical analysis of its function. Without such analysis, it is not difficult to yield to one of two temptations—either to settle for implementation rather than critique, or to choose self-marginalisation by a disengagement from the realities determining educational practices.

Gorbachev and gender

HO6

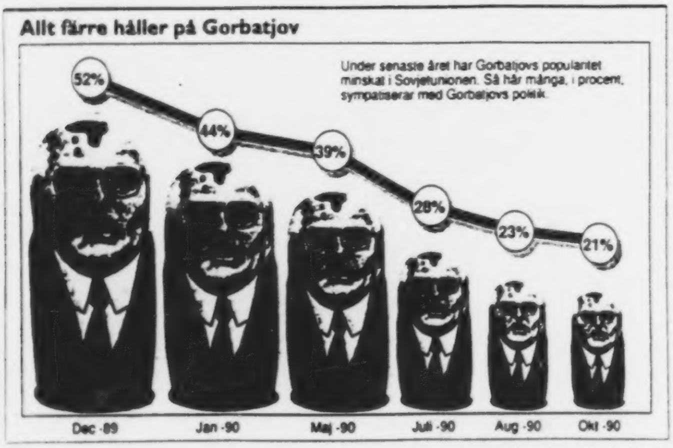
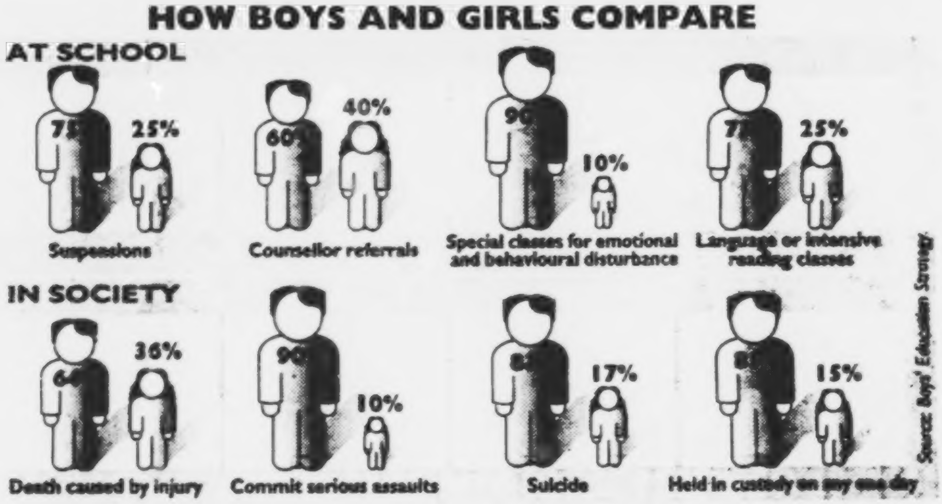


Diagram Lennart Lindgren

From Dunkels, A. 1994, 'Interweaving numbers, shapes, statistics and the real world in primary school and primary teacher education', in *Selected Lectures from the 7th International Congress on Mathematical Education*, Quebec.



From *Sydney Morning Herald*, April 1994.

The transmission model

OHT4

KNOWLEDGE AS
external reality, objective, above human, true, God-given, unproblematic

units of measurement are unproblematic, given
10 mm = 1 cm, 100 cm = 1 m, 1000 m = 1 km etc.

LEARNING AS
reception of information, absorption of facts, reproduction

- rote learning of relationships between mm/cm/km etc.
- learning algorithms for adding, subtracting, converting etc.

TEACHING AS
transmission, expert, concern for product

- instruction, telling, correcting
- getting students to practise

Planning sheet

HO7

constructivist	*critical constructivist*
Knowledge as: *internal reality, subjective, human activity, viable, socially negotiated, problematic*	Knowledge as also: *situated, political*
Learning as: *construction, perturbation, reconstruction*	Learning as also: *purposeful, critically reflective, challenging power relations*
Teaching as: *questioning, provocation, collaboration, concern for process*	Teaching as also: *aware of power relations, negotiating power relations*

Assessment issues

▤ OHT5

– Do you assess?

– Why do we assess?

– Who do we assess for?
Who are we accountable to?

– What do we want to assess?

Two (and a half) views of assessment

HO8

Two (and a half) contrasting views of knowledge and their implications for assessment

Earlier we discussed three views of knowledge and considered the implications for teaching and learning. Can you extend the three columns of the table we looked at then to include what might be the implications for assessment?

	positivism	***constructivism***	***critical constructivism***
Knowledge as	- external reality - objective - above human - true - God-given - unproblematic	- internal reality - subjective - human activity - viable - socially negotiated - problematic	as for constructivism + - situated - political
Learning as	- reception of information - absorption of facts - reproduction	- constructing - dealing with perturbation - reconstructing	as for constructivism + - purposeful - critically reflective - challenging power relations
Teaching as	- transmission - expert - concern for product	- questioning, provoking - collaborating, facilitating - concern for process	as for constructivism + - awareness of power relations - negotiation of power relations
Assessment as			

Teaching: implied beliefs and practices

✋ HO9

teacher as	*view of maths*	*view of learning*	*pattern of use of curricular materials*	*intended outcome for the student*
Instructor				
Explainer				
Facilitator				

Constructed from Ernest P. 1989, 'The impact of beliefs on the teaching of mathematics', in Ernest P. (ed.) *Mathematics Teaching: the state of the art*, Falmer Press, Lewes, pp. 250–252.

431

Teaching: implied beliefs ✣ AS2

Photocopy, onto coloured card if possible, enough sets for each pair of participants to have one set, and cut into sets of 12 pieces. Put pieces into labelled envelope.

accumulation of facts, rules and methods for some external purpose

student's compliant behaviour in mastering skills

the strict following of a text or scheme

skills mastery with correct performance

a consistent, connected and objective structure

the reception of knowledge

modification of the textbook approach enriched with additional activities

conceptual understanding with unified knowledge

dynamically organised structure, in a social and cultural context

active construction of knowledge, even exploration and autonomous pursuit of own interests

teacher or school construction of the maths curriculum

confident problem posing and problem solving

The impact of beliefs on teaching

P13

page 1

From Ernest P. 1989, 'The impact of beliefs on the teaching of mathematics', in P. Ernest (ed.) *Mathematics teaching: the state of the art*, © The Falmer Press, Lewes, pp. 252–3.

Official reports such as NCTM (1980) *Agenda for Action*, and the Cockcroft Report (1982) recommend the adoption of a problem-solving approach to the teaching mathematics. Such reforms depend to a large extent on institutional reform: changes in the overall mathematics curriculum. They depend even more essentially on individual teachers changing their approaches to the teaching of mathematics. However, the required changes are unlike those of a skilled machine operative, who can be trained to upgrade to a more advanced lathe, for example. A shift to a problem-solving approach to teaching requires deeper changes. It depends fundamentally on the teacher's system of beliefs, and in particular on the teacher's conception of the nature of mathematics and mental models of teaching and learning mathematics. Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change. Furthermore, these changes in beliefs are associated with increased reflection and autonomy on the part of the mathematics teacher. Thus the practice of teaching mathematics depends on a number of key elements, most notably:

the teacher's mental contents or schemes, particularly the system of beliefs concerning mathematics and its teaching and learning;
the social context of the teaching situation, particularly the constraints and opportunities it provides; and the teacher's level of thought processes and reflection.

These factors determine the autonomy of the mathematics teacher, and hence also the outcome of teaching innovations—like problem-solving—which depend on teacher autonomy for their successful implementation.

The mathematics teacher's mental contents or schemes include knowledge of mathematics, beliefs concerning mathematics and its teaching and learning, and other factors. Knowledge is important, but it alone is not enough to account for the differences among mathematics teachers. Two teachers can have similar knowledge, but while one teaches mathematics with a problem-solving orientation, the other has a more didactic approach. For this reason the emphasis below is placed on beliefs. The key belief components of the mathematics teacher are the teacher's:

view or conception of the nature of mathematics,
model or view of the nature of mathematics teaching,
model or view of the process of learning mathematics.

The teacher's conception of the nature of mathematics is his or her belief system concerning the nature of mathematics as a whole. Such views form the basis of the philosophy of mathematics, although some teachers' views may not have been elaborated into fully articulated philosophies. Teachers' conceptions of the nature of mathematics by no means have to be consciously held views; rather they may be implicitly held philosophies. The importance for teaching of such views of subject matter has been noted both across a range of subjects and for mathematics in particular (Thom, 1973). Three philosophies are distinguished here because of their observed occurrence in the teaching of mathematics (Thompson, 1984), as well as in the philosophy of mathematics and science.

First of all, there is the instrumentalist view that mathematics is an accumulation of facts, rules and skills to be used in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts. Secondly, there is the Platonist view of mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created. Thirdly, there is the problem-solving view of mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of inquiry and coming to know, not a finished product, for its results remain open to revision.

These three philosophies of mathematics, as psychological systems of belief, can be conjectured to form a hierarchy. Instrumentalism is at the lowest level, involving knowledge of mathematical facts, rules and methods as separate entities. At the next level is the Platonist view of mathematics, involving a global understanding of mathematics as a consistent, connected and objective structure. At the highest level the problem-solving view sees mathematics as a dynamically organised structure located in a social and cultural context.

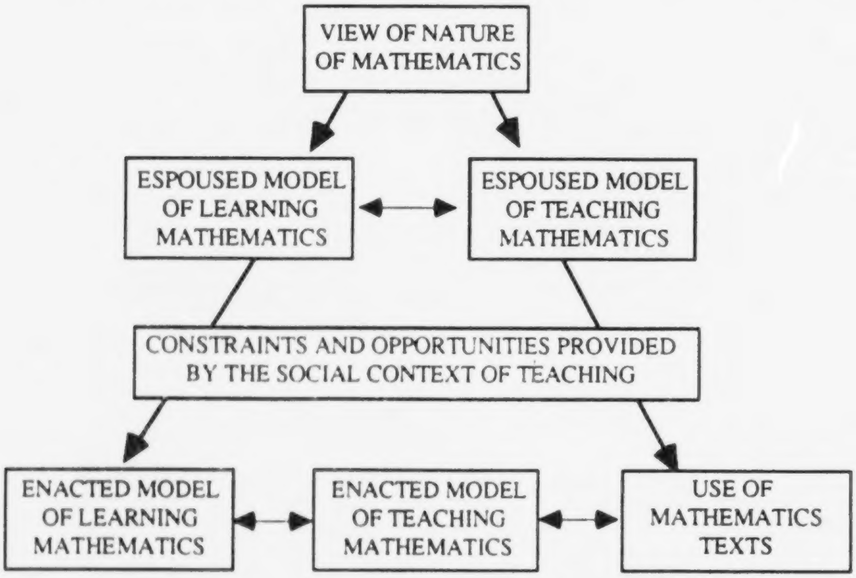
Relationships between Beliefs, and Their Impact on Practice

The relationships between teacher' views of the nature of mathematics and their models of its teaching and learning are illustrated in Figure 1. It shows how teachers' views of the nature of mathematics provide a basis for the teachers' mental models of the teaching and learning of mathematics as indicated by the downward arrows. For example, the instrumental view of mathematics is likely to be associated with the instructor model of teaching, and with the strict following of a text or scheme. It is also likely to be associated with the child's compliant behaviour and mastery of skills model of learning. Similar links can be made between other views and models, for example:

> mathematics as a Platonist unified body of knowledge—the teacher as explainer—learning as the reception of knowledge;
> mathematics as problem-solving—the teacher as facilitator—learning as the active construction of understanding, possibly even as autonomous problem-posing and problem-solving.

These examples show the links between the teacher's mental models, represented by horizontal arrows in Figure 1.

Figure 1. Relationships between Beliefs, and Their Impact on Practice


VIEW OF NATURE OF MATHEMATICS
ESPOUSED MODEL OF LEARNING MATHEMATICS
ESPOUSED MODEL OF TEACHING MATHEMATICS
CONSTRAINTS AND OPPORTUNITIES PROVIDED BY THE SOCIAL CONTEXT OF TEACHING
ENACTED MODEL OF LEARNING MATHEMATICS
ENACTED MODEL OF TEACHING MATHEMATICS
USE OF MATHEMATICS TEXTS

The teacher's mental or espoused models of teaching and learning mathematics, subject to the constraints and contingencies of the school context, are transformed into classroom practices. These are the enacted(as opposed to espoused) model of teaching mathematics, the use of mathematics texts or materials, and the enacted (as opposed to espoused) model of learning mathematics. The espoused–enacted distinction is necessary because case studies have shown that there can be a great disparity between a teacher's espoused and enacted models of teaching and learning mathematics (for example, Cooney, 1985). Two key causes of the mismatch between beliefs and practices are as follows.

First of all, there is a the powerful influence of the social context. This results from the expectations of others including students, parents, peers (fellow teachers) and superiors. It also results from the institutionalised curriculum: the adopted text or curricular scheme, the system of assessment and the overall national system of schooling. These sources lead the teacher to internalise a powerful set of constraints affecting the enactment of the models of teaching and learning mathematics. The socialisation effect of the context is so powerful that despite having differing beliefs about mathematics and its teaching, teachers in the same school are often observed to adopt similar classroom practices.

Secondly, there is the teacher's level of consciousness of his or her own beliefs, and the extent to which the teacher reflects on his or her practice of teaching mathematics. Some of the key elements in the teacher's thinking—and its relationship to practice—are the following:

awareness of having adopted specific views and assumptions as to the nature of mathematics and its teaching and learning;
the ability to justify these views and assumptions;
awareness of the existence of viable alternatives;
context sensitivity in choosing and implementing situationally appropriate teaching and learning strategies in accordance with his or her own views and models;
reflexivity—being concerned to reconcile and integrate classroom practices with beliefs, and the reconcile conflicting beliefs themselves.

These elements of teachers' thinking are likely to be associated with some of the beliefs outlined above, at least in part. For example, the adoption of the role of facilitator in a problem-solving classroom requires reflection on the roles of the teacher and learner, on the context suitability of the model, and probably also on the match between beliefs and practices. The instrumental view and the associated models of teaching and learning, on the other hand, require little self-consciousness and reflection, or awareness of the existence of viable alternatives.

Mathematics teachers' beliefs have a powerful impact on the practice of teaching. During their transformation into practice two factors affect these beliefs: the constraints and opportunities of the social context of teaching, and the level of the teacher's thought. Higher level thought enables the teacher to reflect on the gap between beliefs and practice, and to narrow it. The autonomy of the mathematics teacher depends on all three factors: beliefs, social context, and level of thought. Beliefs can determine, for example, whether a mathematics text is used uncritically or not, one of the key indicators of autonomy. The social context clearly constrains the teacher's freedom of choice and action, restricting the ambit of the teacher's autonomy. Higher level thought, such as self-evaluation with regard to putting beliefs into practice, is a key element of autonomy in teaching. Only by considering all three factors can we begin to do justice to the complex notion of the autonomous mathematics teacher.

Probability fair Betting away

✣ AS3

Use the materials provided to examine the different ways that chance and probability are used in gambling. Introduce other forms of gambling not included in the materials available, if you know of them.

What language and types of chance are used in the different forms and materials?

How are the numbers used and written? What types of numbers are used?

Where and how are these types of gambling used? What social issues are involved in the different areas of gambling you have looked at?

Using these materials and resources what types of activities could you undertake with students? What resources could you use?

A side track—do you know how odds work?

Probabilities are often expressed, especially in gambling, in terms of 'odds'.
Odds are defined as:

$$\text{odds in favour of an event} = \frac{\text{number of favourable ways}}{\text{number of unfavourable ways}}$$

Example:
The odds in favour of getting a 3 in one roll of a dice would be 1 to 5, since there is one favourable way compared to 5 unfavourable ways. So a probability of 1 in 6 is the same as odds of 1 to 5 (often written 1:5).

Odds of 1:1 are often called 'evens'—that is, when the probability is $\frac{1}{2}$ which means there is supposed to be an equal chance of winning or losing.

Horse racing odds are written slightly differently. Find someone who knows about horse racing odds and get them to explain to you terms like '4 to 1 against' (probability of $\frac{1}{5}$ of winning) and '3 to 2 on' (probability of $\frac{3}{5}$ of winning).

Probability fair
Collecting data

✣ AS4

page 1

A: Two dice games

What are the chances of rolling different numbers when you roll two six-sided dice? This activity is a way of collecting data to analyse this occurrence.

What games do you know use two 6-sided dice? Add to the list below:

....

....

....

....

1 Roll the two dice.
2 Add the two numbers uppermost.
3 Record the total by shading in a square in the correct column of the table.
4 Do this 4 or 5 times.

2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Total of two dice										

Probability fair
Collecting data

❖ AS4

page 2

B: Tossing three coins

If you toss three coins, what are the possible numbers of heads you can get uppermost?

1 Toss three coins.
2 Count the number of heads uppermost.
3 Record the total by shading in a square in the correct column of the table below.
4 Do this 3 or 4 times.

0	1	2	3
No. of heads			

Probability fair
Language for chance

✣ AS5

page 1

Use the pages of the newspaper. -

Scan the text for words and phrases related to chance, likelihood or probability, e.g.

- possibly
- probably
- Buckley's
- 100 percent certain.

Highlight at least three words and write them on the cards.

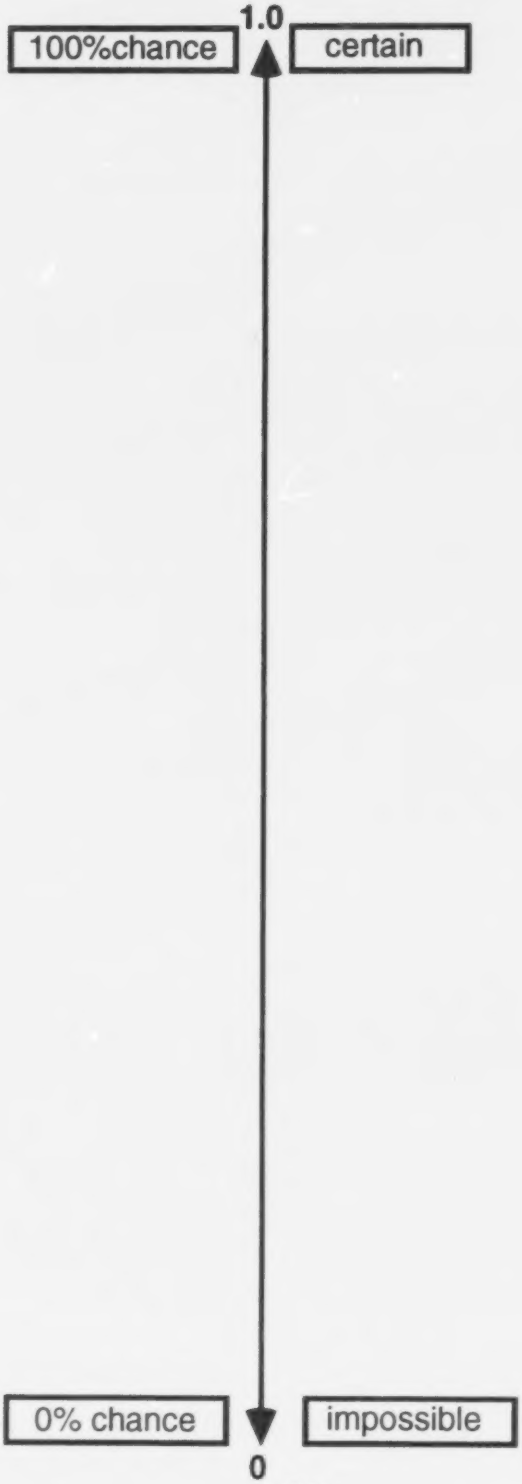
Place them in order on the probability axis from lowest probability to highest probability.

Buckley's??
No chance at all???
Probably???

Probability fair
Probability axis

AS5

page 2


1.0
100%chance
certain
0% chance
impossible
0

Probability fair
Tree, and other, diagrams

✣ AS6

page 1

You can use diagrams to visualise and represent probabilities.
Lets look at the two cases also analysed in Collecting data,
i.e. tossing three coins and rolling two dice.

A: Rolling two dice

If there are two operations involved in the event under analysis then you can use a tabular or graphical representation such as this number square:

You can add the pairs of numbers representing the two dice and get the total showing uppermost inside the square. Complete the table:

		dice 1					
		1	2	3	4	5	6
dice 2	1	2	3	**4**			
	2	3	**4**				
	3	**4**	5				
	4				8		
	5						
	6				10		

As there are 36 possible outcomes altogether you can work out the probability of getting a particular total. For example, the probability of getting a 4 will be 3 chances out of the total of 36 (see shaded cells in table), i.e.

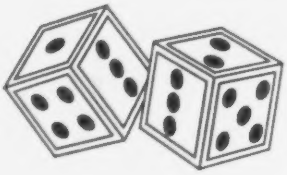
$$\frac{3}{36} = \frac{1}{12}$$

Can you work out the following chances?

a) the chances of throwing a 12?

b) the chances of throwing a 6?

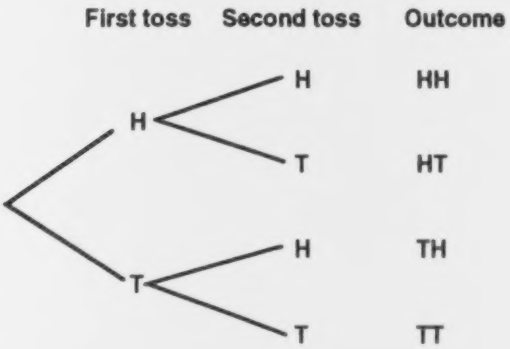
c) the probability of rolling a pair?



B: Tossing three coins

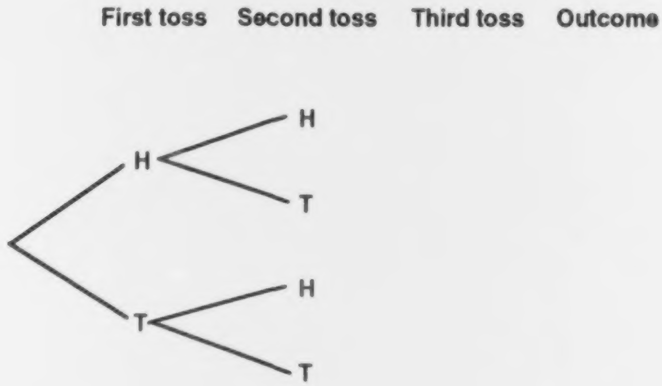
When there are three parts to the event, such as in tossing three coins, then a two dimensional diagram such as a table or graph won't work, so often what's called a tree diagram can be used.

Let's look at how this might work for the situation where you toss two coins:


First toss
Second toss
Outcome
H
H
HH
T
HT
T
H
TH
T
TT

How many possible outcomes are there for tossing two coins?
What is the chance of throwing two heads?

Can you now draw or extend the tree diagram for the third toss?


First toss
Second toss
Third toss
Outcome
H
H
T
T
H
T

Can you use the tree diagram to work out what the chance is for throwing:

a) three heads ………………

b) two heads ………………

c) only one head? ………………

Probability fair
Take your chances

✣ AS7

React to each of these statements using chance and probability, adding your comments/reactions in the space. Do they meaning anything to you? What do they mean?

There's a 50% chance that its going to rain tomorrow.

If you're male and 40, there's a 75% likelihood that you'll be dead by the age of 75.

If a couple is planning to have 4 children, then the probability that they will have 2 boys and 2 girls is a half.

My horse is odds on to win the next race.

The latest Gallup poll says that there's a 0.6 chance that the Liberals will win the next election.

The formula: $\Pr(A \cap C) = \Pr(A)\,\Pr(C|A)$

Probability fair
Cultural/historical picture

✣ AS8
page 1

Historically, and even in terms of modern day applications, probability has been mainly associated with and applied to gambling. The branch of mathematics called probability theory has its origin in the 16th Century, when an Italian physician and mathematician named Jerome Cardan wrote the first book on the subject, *The Book on Games of Chance*. For many years 'the mathematics of chance' was used primarily to solve problems associated with gambling. It has come a long way since then. Today the theory of probability is one of the major areas of mathematics.

Some of the first questions about probability had to do with games played with dice. An ordinary die (the singular of dice) is a cube whose faces are marked with from 1 to 6 spots. When it is rolled it is considered equally likely that it will land with any one of its six sides uppermost. Since there is one way out of six for a dice to turn up any one of its numbers, we say that the probability or chance of rolling that number, for example, a 3, is 1 in 6.

Jacobs 1977, p. 344

Games of chance have been deemed models of interaction with the supernatural and are often linked with religion. Just ask people today, in our culture, to what they attribute their good or bad luck of winning or losing games of chance, or why some think such games are sinful. Each culture, then, creates different games and embeds them differently. Nevertheless, games of strategy are evidence of the enjoyment of logical play and logical challenge, and in games of chance there is implicit involvement with concepts of probability.

Now we will look in detail at a simple but widespread Native American game of chance and then at a Maori game of strategy. [Then] we will discuss a ubiquitous logical puzzle found in Western mathematics recreation books, in Western folk culture, and in several variants in African culture.

A dish and some small flat disks are the objects used in a game of chance widespread among Native Americans. The name of the game varies but usually reflects the type of objects used: peach stones among the Cayuga … and butter beans among the Cherokee. Whatever the objects used, they are shaped and decorated or coloured so that each has two faces that are distinguishable from each other. The number of disks also varies from group to group but usually there are six or eight. There are two players. One of them places the disks in the dish and, by striking or shaking the dish, causes the disks to jump and resettle. The resulting assortment determines the number of points won and whether or not the player goes again or must pass the dish to his opponent. Just as the number of disks varies, so do the point values assigned … a set of sticks or beans serves as counters to keep track of the points won. We will concentrate on the version of this game found among some Iroquoian groups in what are now the north eastern United States and Ontario Province, Canada. Following some early European writings, we call it the game of Dish.

Among the Cayuga, the original inhabitants of the area in which I now live, the dish was a wooden bowl and the disks were six smoothed and flattened peach stones blackened by burning on one side. The auxiliary counters were beans. If the tossed peach stones landed with all six faces showing the same colour (six black or six neutral), the player scored five points. For five faces of the same colour (five black and one neutral of five neutral and one black), the player scored one point. In each of these cases, the player also earned another toss. For all other results, the player scored no points and had to pass the bowl to the opponent. Some pre-arranged total number of points, ranging from 40 to 100, determined the winner of the game.

Ascher 1991, © Brooks/Cole Publishing, California, pp. 87–88

Besides descriptive statistics and inference, the other fundamental idea is that of probability. It is fair to say that this subject was born when Fermat (1601–65) and Pascal (1623–62) corresponded over the 'problem of points' (eg. 'In a game of equal chance *A* needs 3 and *B* needs 2 points to win. The game is interrupted and abandoned. How should *A* and *B* divide the stake money?) And the topic of games of chance is well worth exploring for material.

If we look at games played elsewhere in the world, we find a wide variety of objects and scoring procedures are used. Amongst the objects are pyramidal dice, coins, two-sided dice (one side flat, one curved), and cowrie shells. On such dice, one face is marked or designated 'heads', a fixed number are thrown at each turn, and the score is based on the numbers of dice which fall showing 'heads' (or, in the case of cowrie shells, falling with their openings uppermost).

Table 3.8: Scoring for some board games in which four 'dice' are thrown

Result of throw (no. of 'heads')	**Score**			
	Nyout	Zohn Ahl	Pulic	Tablan
0	4	10	5	12
1	1	1	1	2
2	2	2	2	0
3	3	3	3	0
4	5	6	4	8

Nelson, Joseph & Williams 1993, © Oxford University Press, Oxford, pp. 81–83

Some questions

- What areas do you think you could investigate with your students?
- What maths would you be able to include?
- Do you know of other cultural applications of probability and chance?

Probability fair Black and yellow

✣ AS9

An urn contains 3 yellow and 7 black balls. Four balls are drawn at random from the urn, each ball being replaced before the next is drawn. This experiment is performed 50 times and each time the number of yellow balls in the sample is counted. The following data shows the number of yellow balls obtained in each of the 50 draws.

0	1	1	0	1	2	0	1	1	2
1	2	0	1	2	1	1	0	4	1
3	0	1	1	1	0	2	2	1	0
2	1	2	0	3	2	1	1	0	3
1	2	3	2	0	1	2	0	2	1

a) What is the maximum number of yellow balls in any sample of 4?

b) What is the minimum number of yellow balls in any sample of 4?

c) Fill in the frequency table below.

d) Construct a histogram to show the frequency of yellow balls in the sample of 4.

e) What information can be gained from the above data?

number of yellow balls	*frequency*

ANT Lotto cards ✣ AS10

ANT LOTTO Be in it to win it! Choose your six numbers.

Game number: ☐

1	2	3	4	5	6

ANT LOTTO Be in it to win it! Choose your six numbers.

Game number: ☐

1	2	3	4	5	6

ANT LOTTO Be in it to win it! Choose your six numbers.

Game number: ☐

1	2	3	4	5	6

ANT LOTTO Be in it to win it! Choose your six numbers.

Game number: ☐

1	2	3	4	5	6

ANT LOTTO Be in it to win it! Choose your six numbers.

Game number: ☐

1	2	3	4	5	6

ANT Lotto winning table

HO11

game	*my numbers*	*winning numbers*	*number of winners in group*
1	....	....	....
2	....	....	....
3	....	....	....
4	....	....	....
5	....	....	....
6	....	....	....
7	....	....	....
8	....	....	....
9	....	....	....
10	....	....	....
11	....	....	....
12	....	....	....
13	....	....	....
14	....	....	....
15	....	....	....
...	....	....	....
...	....	....	....
...	....	....	....
...	....	....	....
...	....	....	....

Maths and Lotto

HO12

Adapted from Lovitt & Clarke, pp. 112–115.

For many people, picking six numbers out of 40 or out of 45 doesn't seem too difficult —that's only about one seventh of the numbers. but as with all gambling games, the easier it looks, the more people are convinced they can win, the bigger profit the organisers make. Do people have realistic expectations about their chances? Should they? In this activity pupils play a simplified form of lotto, work out the odds, and discover that there is often a big difference between perception and reality.

Background

The nature of our society is that there are many schemes designed to part people from their money. Are Lotto games an issue of social concern? It is not hard to find evidence of tragedy and hardship caused by a few people over-indulging in such games. It is not hard to illustrate that most people have very little perception of their real chances. Is there a role (or a responsibility) for the mathematician in schools to address this issue?

An educator's axiom:
If a person pays to participate in any scheme they should know what to expect.

Lotto games are a social phenomenon, extracting millions of dollars from people's pockets each week. Do pupils, who are tomorrow's investors, (if not today's) have a realistic idea of their chances of winning and the probability of losses? Evidence from many classes suggests that pupils hopelessly over-estimate their chances of winning. Is such a sense of unreality healthy?

The weekly query, When *will your numbers come up?* is never answered, but it does have an answer—there is a mathematical expectation which can be taught.

The activity *Maths and Lotto* is an attempt to raise these issues in the classroom. It is not an attack on Lotto games—only an argument that they provide a real issue, worthy of the attention of mathematics teachers. It is to ensure that pupils have realistic expectations.

To be otherwise, is to be disadvantaged … *Fools and their money are soon parted* … Equally however … *You can't win if you're not in* … The message therefore becomes: *Buy a ticket by all means, but excessive entries straining financial resources and based on unreal expectations represent an unhealthy approach.*

Five strands of meaning

OHT6

- meaning through ritual
  a minimal strand where meaning is acquired through rote-learning of atomised content
- meaning through conceptual engagement
  where mathematical meaning is constructed through problem-solving, process and cognitive dissonance
- meaning through use
  where meaning is developed through use in everyday contexts
- meaning through historical and cultural understanding
  where meaning is enhanced by an understanding of the genesis and cultural use of specific mathematics
- meaning through critical engagement
  where meaning is generated by asking ‘in whose interest’ type questions and also questions about the appropriateness and limits of the maths model in the real situation

Five strands of numeracy

HO13

Using the five strands of meaning making: ritual; conceptual; use; cultural; and critical. Discuss each of the activities in the probability stations from the previous session and see which strands they covered or could easily be extended to cover.

Use the table below to indicate which strands they cover.

probability	***strands***				
stations and activities	*ritual*	*conceptual*	*use*	*cultural*	*critical*

The activities were:
  Betting away
  Collecting data
  Language for chance
  Tree, and other, diagrams
  Take your chances
  Cultural/historical derivation of probability
  Black and yellow and
  Maths and Lotto

Numeracy definitions

HO14

To be numerate is to function effectively mathematically in one's daily life, at home and at work. (Willis, 1990)

... the mathematics for effective functioning in one's group and community, and the capacity to use these skills to further one's own development and that of one's community. (Beazley, 1984)

We would wish the word 'numerate' to imply the possession of two attributes. The first is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enable an individual to cope with the practical demands of his everyday life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. … These simply imply that a numerate person should … be able to appreciate and understand some of the ways in which mathematics can be used as a means of communication ... Those who set out to make pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop skills of computation' (Cockcroft, 1982)

Numeracy is the understanding and application of the mathematics that a person requires for work, study and everyday life. Numeracy comprises survival skills and numerical skills and spatial abilities including the use of calculators, estimation, appreciation of shapes, size direction and measurement, problem solving and logical reasoning, and the interpretation and language of mathematical data and information. (Draft used as stimulus for national TAFE teacher survey, 1991)

To be numerate is more than being able to manipulate numbers, or even being able to 'succeed' in … mathematics. Numeracy is a critical awareness which builds bridges between mathematics and the real world, with all its diversity. Being numerate also carries with it a responsibility, of reflecting that critical awareness in one's social practice. So being numerate is being able to situate, interpret, critique, use and perhaps even create maths in context, taking into account all the mathematical as well as social and human messiness which comes with it ... Unlike mathematics, numeracy doesn't pretend to be objective and value-free' (Yasukawa & Johnston, 1994).

Being numerate is the ability to use a four-function electronic calculator *sensibly*. (Girling, 1978).

Literacy involves the integration of listening, speaking, reading, writing and critical thinking, it incorporates numeracy. It includes the cultural knowledge which enables a speaker, writer or reader to recognise and use language appropriate to different social situations. For an advanced technological society such as Australia, the goal is an active literacy which allows people to use language and to enhance their capacity to think, to create and question, in order to participate effectively in society. (Australian Council for Adult Literacy, 1990)

Literacy is the ability to read and use written information and to write appropriately, in a range of contexts. It is used to develop knowledge and understanding, to achieve personal growth and to function effectively in our society. Literacy also includes the recognition of numbers and basic mathematical signs and symbols within text. (Australian Language and Literacy Policy, 1991)

References for Module D

HO15
page 1

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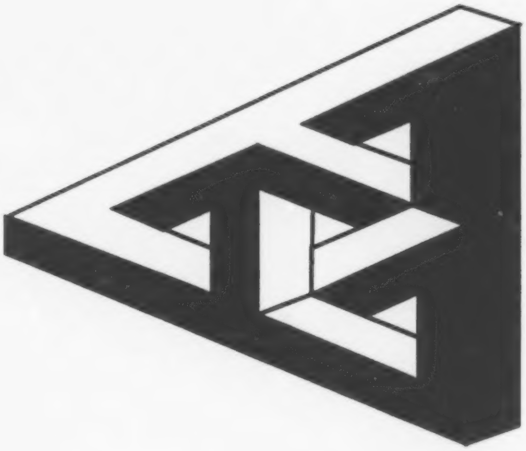

Curriculum Projects and Numeracy Journal

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Introduction

Throughout the course participants will be expected to:

- maintain a written Numeracy Journal reflecting on the content of the course and teaching implications (nine entries over the duration of the course);
- complete an action trial applying material from the course in an adult basic education context (Curriculum Project 1);
- investigate an aspect of mathematics either for their own mathematical development or for application in an adult basic education context (Curriculum Project 2);
- design and develop teaching or curriculum materials (Curriculum Project 3);
- design and develop an alternative assessment task (Curriculum Project 4); and
- generally contribute to course activities and discussions.

Time is given in the course for working on the four Curriculum Projects, including giving presentations to the whole Group. However, it is also expected that there will be work requirements outside of course session time, above this time allocation, and some activities will require participants to conduct a trial or make observations in a teaching situation.

It is expected that participants will complete the required nine journal entries and the two smaller Curriculum Projects (CP1 and CP4) adequately. Presenters should collect the journals from participants and give feedback. They are not to be graded, but the presenter should give back comments, reactions, reflections and questions to the participants. Presenters should make a point of noting what contributions participants make during discussions of CP1 and CP4.

The Curriculum Projects for Modules B and C (CP2 and CP3) are the major assessment tasks—these will need to be completed according to the contract agreed between the presenter and participant, and assessed accordingly. Participants should be encouraged to work in small groups or teams for completing the two major projects. One of the major projects needs to be written formally and linked to the theoretical framework of the course, mainly in order to satisfy entry requirements for postgraduate study. Although it is recommended that only one needs to be formally written up, some participants may decide to write both up in a formal way. The project written up should total approximately 2500 words. The link to the theoretical framework should be demonstrated by:

- evidence of wide reading
- evidence of good use of other resources
- understanding of issues involved in the social construction of maths (CP2)
- understanding of issues involved in critical numeracy (CP3)
- understanding of the pedagogical issues involved.

The written up project may well need to be handed in after completion of course. It is expected that the presenter only assess projects as 'satisfactory' or 'incomplete', asking for the project to be resubmitted if it has not reached the standard agreed to in the contract.

Details follow concerning each of the four Curriculum Projects and the Numeracy Journal.

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List of readings

The presenter must ensure that the readings listed below are available for use and/or distribution at appropriate stages of the course, as indicated in the Presenter's Notes. Reading CP4.4 can be obtained from Objective Learning Materials. PO Box 120, Glen Waverley, Vic., 3150 – phone (03) 9803 2822, fax (03) 9887 0405; CP4.5, CP4.6, CP4.7 and CP4.8 are found in fairly recent publications of the ACER, Melbourne.

CP3.4	**Poverty traps** Barnes, M. 1994, 'Casting light on poverty traps', *Numeracy in Focus*, No. 1, pp. 43–46.	available in *Numeracy in Focus*
CP4.2	**Thinking about assessment** Marr, B. 1994, 'Thinking about assessment', *Numeracy in Focus*, No. 1, pp. 10–14.	available in *Numeracy in Focus*
CP4.3	**Assessing students in numeracy programs** McRae, A. 1994, 'Assessing students in numeracy programs', *Numeracy in Focus*, No. 1, pp. 15–19.	available in *Numeracy in Focus*
CP4.4	**Assessment alternatives** Use a copy of Stenmark J.K. (ed.) 1989, *Assessment Alternatives in Mathematics: an overview of assessment techniques that promote learning*, EQUALS, Lawrence Hall of Science, University of California.	encourage all participants to borrow or purchase a copy
CP4.5	**Reshaping assessment practices** Stephens, M. 1991, foreword, in M. Stephens & J. Izard (eds) *Reshaping Assessment Practices: assessment in the mathematical sciences under challenge*, ACER, Melbourne, pp. vi–xii.	copy and distribute
CP4.6	**Who assesses whom ...?** Burton, L. 1991, 'Who assess whom and to what purpose?' in M. Stephens & J. Izard (eds) *Reshaping Assessment Practices: assessment in the mathematical sciences under challenge*, ACER, Melbourne, pp. 1–18.	copy and distribute
CP4.7	**A socio-constructivist approach ...** Yackel, E., Cobb, P. & Wood, T. 1992, 'Instructional development and assessment from a socioconstructivist perspective', in G. Leder (ed.) *Assessment and Learning of Mathematics*, ACER, Melbourne, pp. 63–82.	copy and distribute
CP4.8	**The role of assessment ...** Clarke, D. 1992, 'The role of assessment in determining mathematics performance', in G. Leder (ed.) *Assessment and Learning of Mathematics*, ACER, Melbourne, pp. 145–168.	copy and distribute

The Numeracy Journal

in which the participants write about their journey through
Terra Mathematica
where they leap chasms, slay dragons, joust with windmills,
rescue persons in distress and emerge (hopefully)
from the Slough of Despond into the
World of Numeracy.

Rationale/aims

During this course participants will be expected to keep a regular journal which will form part of the assessment work for the course. Journals have different purposes and the purpose of this one is to provide participants with opportunities to reflect on what is happening during the course, in an effort to narrow the gap between their theory and practice as numeracy teachers. The goals are analysis, communication, and making connections among issues, theories, mathematics and experience. The presenter may also get participants to use the Numeracy Journal at times during course sessions. The complete set of journal entries will be a record of participants' experience during the course.

Organisation

Number of entries:	8
Length of each entry:	about 400 words
Form:	on loose-leaf paper to go in a binder, or in a separate small exercise book
Due date:	as negotiated, but reminders will be needed

Assessment

The Numeracy Journal will not be graded, but *satisfactory completion is essential.* Journal entries should be collected from participants at regular intervals and feedback should be given. They are not to be graded, but the presenter should give comments, reactions, reflections and questions back to the participants. Participants may need a few reminders throughout the course of how many journal entries they have completed and how many are still outstanding, rather than leaving too many of them to be hurriedly completed at the end of the course. Feedback is time-consuming but essential if the Numeracy Journal is to be worthwhile.

The Numeracy Journal entries

A few topics (e.g. the first entry, finishing the case-study *Looking at boundaries*) are set during the teaching sessions of the course. Participants are also given a list of topics to choose from for their journal. Others may be added during the time of the course. All modules should be given some attention—at least two entries per module—and entries should be in response to a wide variety of issues.

Materials

The Numeracy Journal [NJ1] is given to participants as part of Section A1.1.

Curriculum Projects

Brief description

During the course participants will be involved in four Curriculum Projects—two short ones (each three hours of course time) and two longer ones (each nine hours of course time). The four Curriculum Projects are:

1 *CP1 – Making meaning*: in which the participants observe students and themselves;
2 *CP2 – Exploring maths*: in which participants investigate a mathematics topic they are interested in;
3 *CP3 – Developing a critical view*: in which participants choose a topic raised in the Second Journey, or earlier, and develop teaching materials to include a critical perspective of both the social and mathematical contexts;
4 *CP4 – Assessment tool* in which the participants explore a range of alternatives, and develop relevant tasks.

Rationale/aims

Participants spend almost one third of the course on the Curriculum Projects which, with the Numeracy Journal, are woven through the more content-based remainder of the course, giving them opportunities to work together to explore maths and to develop materials for use in teaching. Within guidelines, the content of the longer projects is negotiated. Group projects are preferable. The projects involve the participants in observing and listening to students, exploring maths and its sources, developing contextual materials for teaching numeracy, and assessing performance.

By the end of the course participants should have developed:

- skills in observing other people's ways of making meaning;
- an appreciation of what it means, for themselves, to be involved in their own mathematical quest;
- understanding, and experience, of a range of assessment strategies;
- skills in analysing the numeracy requirements of a given context and an understanding that those skills are highly contextualised; and
- strategies for developing numeracy teaching materials which take account of both mathematical and social contexts.

Materials needed

Assessment guidelines [HO1] is given to participants as part of Section A1.1.

Detailed organisation

Curriculum Projects 1, 2 and 3 should be timetabled so that they weave more or less through Modules A, B and C respectively. Curriculum Project 4 occurs in Module D. Each Curriculum Project should be spread over several weeks for full benefit. The last three hours of Curriculum Projects 2 and 3 are spent presenting the projects to the whole Group.

The short Curriculum Projects (CP1 and CP4) do not involve negotiation, but the two longer ones do (CP2 and CP3). It is important that presenter and participant agree from the start about what would make a satisfactory achievement. This is probably best done using a contract form of the sort included below (CP2.4 and CP3.2).

Assessment

The two smaller Curriculum Projects (CP1 and CP4) should be completed to the presenter's satisfaction, but not assessed.

CP2 and CP3 are the major assessment tasks—these will need to be completed according to the contract agreed between the presenter and each participant, and assessed accordingly. One of these two projects needs to be written formally and linked to the theoretical framework of the course. More detailed guidelines concerning assessment criteria for each of the projects is included with the project descriptions on the following pages.

Presentations

As a way of sharing ideas and materials developed by the participants, it has been built into the course that individual participants or teams of participants give a presentation of their projects to the whole Group. There are only three hours available for presentations of each of the two major projects. This could be done as one complete session for each project or be spread out over a number of weeks. Presentations need to be kept to 10 to 15 minutes each. This is often very difficult to achieve especially if there is a largish group, and other options may need to be considered. One possible option is to use poster sessions where participants prepare on butcher's paper some of their materials and findings and display them to the whole Group. Participants could make a presentation of one of their two major projects, and prepare a poster on the other one. This may be a good way to finish the course—having participants bring along a poster of the project on which they have not given a presentation.

Curriculum Project 1 – Making meaning

Module A: Constructing meaning

Time: 3 hours over 2 or 3 weeks

Brief description

Participants explore the meanings that they and their students have for the four basic operations.

Rationale/aims

'Making meaning' is a short project, giving participants an opportunity to analyse how they and their students use language and models to make meaning. By the end of the project participants should have developed some activities for use in their own teaching situation, and should have an extended repertoire of calculation methods, an appreciation of the validity of different methods and familiarity with a variety of materials and language that can model different situations.

Preparation

Presenter
Become familiar with Curriculum Project 1.

Time 1 hour initially, followed by two more hours in later sessions

Materials needed

Concrete materials
(for each small group of 4 or 5)
30 counters (15 of each of 2 colours)
a small set of Cuisenaire rods
a metric tape measure (as a number line)
money

Handouts
Curriculum Project 1 – Making meaning [CP1.1]
The four operations [CP1.2]

OHTs
Making meaning [OHT2] from Module A

References

Fuys, David & Tischler, Rosamond 1979, *Teaching Mathematics in the Elementary School*, Little, Brown and Co., Toronto.

McIntosh, Alistair 1990, 'Becoming numerate: developing number sense', in Willis, Sue (ed.) *Being Numerate: what counts?* ACER, Victoria.

Zaslavsky, Claudia 1973, *Africa Counts: number and pattern in African culture*, PWS Publishers, Boston.

Detailed procedure

In order for participants to talk to their students, there needs to be a gap of at least a few days between Part 1, the first hour, and Parts 2 and 3. Parts 2 and 3 could be programmed on the same day, but it would probably be more useful if they were also separated by some days.

Part 1
In pairs, participants explore the meanings they have for the four operations, using *Curriculum Project 1 – Making meaning* [CP1.1]. Display *Making meaning* [OHT2] while they are doing this. After about 20 minutes they join with other pairs to compare results. For the last 15 or 20 minutes a Group discussion can bring out points raised.

Preparation for Parts 2 & 3 is outlined in participants' notes.

Part 2
In small groups participants share the results of their observations of students. They discuss language and models used and how they combined to give meanings (or not). Remind them of the language/symbol/model diagram. Get them to read the *Four operations* [CP1.2] and to work together to complete the examples. Ask if any of the examples in the extract did not emerge from the observations of students?

Hand out materials to small groups or pairs so that they can work with counters, a number line, money and Cuisenaire rods to model the four operations. The emphasis at this point is still on small, mostly single digit numbers. The presenter, or a participant, may need to demonstrate how the Cuisenaire rods work. The rods 1–10 can also be used with the tape measure, to model a number line.

Not all the materials make good models for all operations. Pose the question, *'Which are best for which operations and why?'* If it is impossible to use all four types of materials, make sure that *at least* the rods and counters are used as they bring out different models well. Make sure that the differences are addressed, either by talking with or listening to all the small groups, or through a Group discussion at the end.

Part 3
Collect on whiteboard or butchers' paper as many different ways of doing the four operations as possible. Include methods for larger as well as single digit numerals. After the groups have pooled the results, participants choose one or two new ways that take their fancy and work in pairs to develop an outline of how they would use these methods in their teaching activities or worksheets, if appropriate. Remind them to use the 'Making connections' diagram. If there is time at the end, some of these plans and activities could be shared.

Discussion

Part 1
The presenter could use the 'Making meaning' diagram [OHT2] to give a framework for discussion: *What real situation, what model of it, what language is connected to the symbols we are trying to explain? Does the language fit the model?*

During the Group discussion of subtraction, for instance, the points might include:

- the language
  For subtraction, what words arose? ... 'take away', 'difference', 'minus', 'subtract', 'add...to make' (i.e. what do I have to 'add' to 4 'to make' 6?)
- the meaning, the real situation
  The story for 6 – 4 can be 'take 4 chairs away from this row of 6' or, quite differently, 'what is the age difference between your two children ... one is 6, the other 4 ?' In the first one I am not finding a difference between two groups; in the second I am not taking away one group from another.

- the model
  *If I try to model 6 – 4 by comparing a box 6 cm wide with one 4 cm wide and say, 'See, six take away four is two,' I will confuse the listener because I am using the **take-away** language with the **difference** model.*

Similar points could arise for the other three operations.

The two distinct models for division could cause trouble initially, but they can be revisited over the following week or two. It can be useful quickly to collect and analyse on the board the 'story' that each participant has for 12 ÷ 3. Usually the large majority is of the form: '12 *shared* amongst 3', with only a few being of the form: 'how many *lots of* 3 are there in 12?' The diagrams and models are quite different. So *sharing* is the commonsense understanding of division, but the language we use for formal short and long division is almost always derived from the *lots of* model. Is it possible to use the *sharing* language instead? (Yes) Would it help? Try doing and speaking 42 ÷ 3 in both ways.

Part 2

Before working with the materials, participants could share any unexpected or otherwise interesting observations with the Group. In discussing *The four operations* [CP1.2], for instance, the *lots of* model of division may not emerge from the discussions with students. What is the implication of this for teaching? Possibly …

- it may not be a good place to start with many students
- perhaps we shouldn't teach it at all—it may depend on how often students meet the situation that *lots of* models.

The materials: good and not so good …

(NB this chart is for the presenter only, as a basis for discussion and argument.)

		counters/money *	***number line***	***Cuisenaire rods***
good	+	– grouping	– repeating patterns (3+5, 13+5, 23+5..)	– combinations (4+3 = 5+2 = 6+1 …) – commutative law
	–	– take-away – complementary addition	– difference – complementary addition	– difference – complementary addition
	x	– lots of (repeated addition) – array – commutative law	– lots of (repeated addition)	– lots of (repeated addition) – array – commutative law
	÷	– lots of (partitive?) – sharing (distributive?)	– lots of (partitive?)	– lots of (partitive?)
not so good	+			
	–	– difference	– take-away	
	x		– array	
	÷		– sharing (distributive?)	– sharing (distributive?)

* Money can be used later as a model for place value.
At this point it is probably useful basically in the same way as counters.

In a discussion of the materials participants may say, 'We (or our students) feel like children using these coloured blocks.' … How can we extend students' vision of maths beyond a purely pencil and paper activity—how can we convince them that we are not treating them as children?' … possibly through a slow introduction of materials, a sharing of our own experience of the power of such materials in helping understanding, a growing experience of their own.

Criteria for the use of materials in a teaching situation might include:

- the effectiveness of the materials for modelling the concept
- the cheapness and availability of the materials
- the closeness of the materials to everyday materials
- its (low) likelihood of the materials being perceived as toys.

Part 3

Include examples of 'in your head' processes as well as algorithms from different countries, backgrounds and generations. These may include:

- for addition: counting on, adding to make tens …;
- for subtraction: complementary addition, verbal or written;
- for multiplication: the lattice and African doubling methods; and
- for division: the South American algorithm,
  as well as the two more common Australian approaches.

A possible question arising here is, *'Isn't it confusing to learn more than one method of multiplication?'*

Curriculum Project 2 – Exploring mathematics

Module B: Mathematics as a human construction

Brief description
Participants investigate a mathematics topic they are interested in, following a detour from the first Mathematical Journey (Module B).

Rationale/aims
'Exploring mathematics' is a more substantial project than the first one, giving participants an opportunity to experience what it means to be involved in their own mathematical quest. They will be expected, preferably in groups, to investigate in some mathematical depth a topic of interest to themselves that has arisen out of the activities of the first Mathematical Journey. This kind of experience is necessary if they are to encourage similar curiosity and investigation in their own students. The project may also involve participants in developing, from their investigation, one or two activities suitable for use with their own students.

Preparation

Presenter
Read Brown 1984 and Lerman 1989

Participants
Spend some time reading the following and thinking about possible detours to explore …
Curriculum Project 2 – Exploring mathematics[CP2.1]
Notes on negotiation[CP2.2]
A negotiated contract [CP2.3]
Sample contract [CP2.4]
Contract form[CP2.5]

Time 9 hours over several weeks

Materials needed

Handouts/OHTs/paper
Bring a large number of suggested topics and reference books—see *Map of the first journey: a conducted tour for* some suggestions on both.

Curriculum Project 2 – Exploring mathematics[CP2.1]
Notes on negotiation[CP2.2]
A negotiated contract [CP2.3]
Sample contract [CP2.4]
Contract form[CP2.5]

References

Brown, S.I. 1984, 'The logic of problem generation: from morality and solving to deposing and rebellion', *For the Learning of Mathematics*, Vol. 4, No. 1, pp. 9–20.
Lerman, S. 1989, 'Investigations: where to now?' in P. Ernest (ed.) *Mathematics Teaching: the state of the art*, Falmer Press, East Sussex.

Detailed procedure

This project should be timed to start towards the end of the first Mathematical Journey and should be extended over at least three weeks.

The first quite difficult task is to negotiate contracts with the participants who may be insecure about making their own choices, in general, and particularly in relation to mathematics. If contracts are to be successful then the presenter must listen carefully, try to help develop the participant's interests into a topic with rich enough mathematical potential (without actually setting a topic) and suggest suitable resources. Since the adequacy of the project can be assessed only in terms of the negotiated criteria for assessment it is particularly important to make these clear, including such items as:

- careful development of a mathematical topic
- evidence of wide reading
- evidence of good use of other resources
- evidence of risk-taking
- choice of useful pictures, charts, formulas and statements that suggest either hypotheses or doubts and challenges to the question itself
- clear links with cultural and/or historical contexts
- and other more specific criteria.

The process of negotiation with all participants may well take an hour or two of the presenter's time to complete, but is worth doing well if good projects are to be completed. Time may be saved if participants can be given the appropriate notes to take home, say, the week before.

After the negotiation is finished, the presenter's task is to be a facilitator, resource person, questioner, organiser of presentations, and collator and distributor of teaching activities.

Curriculum Project 3 – Developing a critical view

Module C: Mathematics as a critical tool

Brief description

Participants choose a topic and develop teaching materials to include a critical perspective of both the social and mathematical contexts.

Rationale/aims

'Developing a critical view', like the second project, it is a more substantial project than CP1 and CP4. Participants develop teaching materials around a topic or context of their own choice. The focus chosen should be one that will allow them to generate activities that will help their own students to activate and develop mathematical knowledge and to become more critical of the role that maths plays in the given context.

Preparation

Presenter

Read Brown 1984 and Lerman 1989

Participants

Read

Curriculum Project 3 – Developing a critical view [CP3.1]

Developing a critical view: contract [CP3.2]

Mathematics through problem-posing [CP3.3]

User-friendly numeracy resources [P9] from Module C

Time 9 hours over several weeks

Materials needed

Handouts/OHTs/paper

Bring as many learning resources—including books, articles, newspapers, journals (e.g. *New Internationalist*), trade course outlines and manuals, craft books, concrete materials, mathematicians—as possible.

Curriculum Project 3 – Developing a critical view [CP3.1]

Contract form: Developing a critical view [CP3.2]

Mathematics through problem-posing [CP3.3]

User-friendly numeracy resources [P9] from Module C

Poverty traps [CP3.4]

References

Barnes, M. 1994, 'Exploring social issues through mathematics: casting light on poverty traps', *Numeracy in Focus*, No. 1, pp. 43–46.

Brown, S.I. 1984, 'The logic of problem generation: from morality and solving to deposing and rebellion', *For the Learning of Mathematics*, Vol 4., No. 1, pp. 9–20.

Lerman, S. 1989, 'Investigations: where to now?' in P. Ernest (ed.) *Mathematics Teaching: the state of the art*, Falmer Press, East Sussex.

Marr, B. & Helme, S. 1993, 'User friendly numeracy resources (or, how we overcame the tyranny of the traditional textbook)' in S. McConnell & A. Treloar 'Reframing mathematics', Book 4, *Voices of Experience: a professional development package for adult and workplace literacy teachers*, DEET, Canberra.

Detailed procedure

This project should be timed to start at the end of Module C, after the mathematical excavation, and should be extended over at least three weeks.

The first task is to discuss the aims of this project. The reading *Mathematics through problem-posing* [CP3.3] may be useful in setting the scene, and *Poverty traps* [CP3.4] may be useful as one example of how a class carried out an investigation.

The next task is to negotiate contracts with the participants. Participants will have had some experience of this in CP2. Since the adequacy of the project can be assessed only in terms of the negotiated criteria for assessment it is particularly important to make these clear, including such questions as:

- are the materials likely to help students excavate maths for themselves?
- do they encourage students to extend their mathematical understanding?
- will they help students develop a critical approach to maths in this context?
- is there evidence of good use of other resources?
- are there 'useful pictures, charts, formulas, … challenges …'?
- are the materials 'user-friendly'? (see *User-friendly numeracy resources* [P9] from Module C)

and other criteria more specific to the particular contract.

The process of negotiation with all participants may well take an hour or two to complete, but is worth doing well if good projects are to be completed. Time may be saved if participants can be given the appropriate notes to take home the week before.

After the negotiation is finished, the presenter's role will be as facilitator, resource person, questioner, organiser of presentations, and collator and distributor of teaching activities.

Curriculum Project 4 – Assessment tool

Module D: Naming theories: implications for practice

Brief description

Participants explore a range of assessment alternatives, and develop a collection of relevant tasks

Rationale/aims

'Assessment tool' is a short project, giving participants an opportunity to investigate a range of assessment alternatives for numeracy and to extend the criteria (developed in Session D3.1) for what makes a good assessment task. By the end of the project the participants should be aware of an extended repertoire of assessment methods and will have developed some tasks for assessment use in their own teaching situation.

Preparation

Presenter

The previous week, the presenter will give out to the Group 4 or 5 different readings [from CP4.2 to CP4.8]

Participants

Read *Assessment tool* [CP4.1] and the given readings from CP4.2 to CP4.8.

Time 3 hours over 2 or 3 weeks

Materials needed

Handouts/OHTs/paper

Assessment tool [CP4.1]
Thinking about assessment [CP4.2]
Assessing students in numeracy programs [CP4.3]
Assessment alternatives [CP4.4]
Reshaping assessment practices [CP4.5]
Who assesses whom ...? [CP4.6]
A socio-constructivist approach [CP4.7]
The role of assessment [CP4.8]

References

Burton, L. 1991, 'Who assess whom and to what purpose?' in M. Stephens & J. Izard (eds), *Reshaping Assessment Practices: assessment in the mathematical sciences under challenge*, ACER, Melbourne.

Clarke, D. 1992, 'The role of assessment in determining mathematics performance', in Leder, G. (ed.) *Assessment and Learning of Mathematics*, ACER, Melbourne.

Marr, B. 1994, 'Thinking about assessment', *Numeracy in Focus*, No. 1, pp. 10–14.

McRae, A. 1994, 'Assessing students in numeracy programs', *Numeracy in Focus*, No. 1, pp. 15–19.

Stenmark, J. K. (ed.) 1989, *Assessment Alternatives in Mathematics: an overview of assessment techniques that promote learning*, EQUALS, Lawrence Hall of Science, University of California.

Stenmark, J.K. (ed.) 1991, *Mathematics Assessment: myths, models, good questions and practical suggestions*, NCTM, Reston, VA.

Stephens, M 1991, Foreword, in M. Stephens & J. Izard (eds) *Reshaping Assessment Practices: assessment in the mathematical sciences under challenge*, ACER, Melbourne.

Yackel, E., Cobb, P. & Wood, T. 1992, 'Instructional development and assessment from a socioconstructivist perspective', in G. Leder (ed.) *Assessment and Learning of Mathematics*, ACER, Melbourne.

Detailed procedure

This project should be timed to start after Section D3.2 on assessment, and should be extended over at least two weeks. There are two parts to the project.

Part 1 – The readings	1.5 h
Part 2 – Comparing results: a range of alternatives	1.5 h

Preparation for first session

The previous week, the presenter will give out 4 or 5 different readings (from CP4.1 to CP4.8) so that each person has one reading to do for the following week, and at least two or three participants will have read the same reading. Ask them to read with this three-part question in mind:

What does this article say about

- *why assess?*
- *what makes a good assessment?*
- *what alternative assessment procedures we could use?*

Part 1 – The readings

Reporting on the readings

So that the whole Group becomes aware of what reading is available on this topic, at the beginning of the session the participants should gather in groups of those who have read the same reading and spend 10 minutes comparing answers and preparing to report to the Group on their reading about the three-part question above.

The presenter lists on the board criteria for good assessment and alternative procedures as they emerge. Participants can use this list to extend the guidelines for good practice in assessment from Section D3.1.

An alternative assessment

At the end of the discussion/reporting, participants choose one alternative form of assessment they would like to explore. Those choosing the same procedure should gather together, collect and read information on that procedure and develop several appropriate tasks for different levels or contexts that might be appropriate for students they are currently teaching.

Preparation for following session

Between sessions participants try out one of the tasks with a student, and write a one page summary to bring to the Group, outlining

- the alternative assessment procedure
- the particular task
- the situation … person … context …
- the results … issues and/or problems …

Part 2 – Comparing results: a range of alternatives

1. Participants return to the groups they were in at the end of the first session to compare results using previous criteria for good assessment, and taking into consideration the table *Two (and a half) views of assessment* [HO8] completed in Section D3.1. Decide what worked and what didn't and if possible, why. When they are clear about this, they prepare a presentation for the Group.

2. One or two people from each group discuss with the whole Group
    - the alternative assessment procedure chosen by their group
    - the tasks that were successful and those that weren't and
    - the possible reasons.

If the Group wishes, it may be a good idea to photocopy assessment ideas and materials that were developed and distribute them amongst the Group.

Assessment guidelines

✋ HO1
page 1

Throughout the course you will be expected to:

- maintain a written Numeracy Journal reflecting on the content of the course and teaching implications (8 entries over the duration of the course)
- complete four Curriculum Projects and
- generally contribute to course activities and discussions.

Numeracy Journal entries

You will be expected to hand in 8 entries over the duration of the course for comment by the presenter. These will not be graded in any way. Details are on the accompanying handout *The Numeracy Journal* [NJ1].

Curriculum Projects

Time is given in the course for working on the four Curriculum Projects, including giving presentations to the Group. However, it is also expected that there will be work requirements outside of session time, above this time allocation, and some activities will require you to conduct a trial or make observations in a teaching situation.

During the course you will be involved in four Curriculum Projects: two short ones (CP1 and CP4—each three hours of course time) and two longer ones (CP2 and CP3—each nine hours of course time). The four Curriculum Projects are:

1. *CP1 – Making meaning*: in which you observe students and yourselves;
2. *CP2 – Exploring maths*: in which you investigate a mathematics topic you are interested in;
3. *CP3 – Developing a critical view*: in which you choose a topic raised in the course and develop teaching materials which will include a critical perspective of both the social and mathematical contexts;
4. *CP4 – Assessment tool* in which you explore a range of alternative assessment tasks, and develop relevant tasks.

You are encouraged to work in small groups or teams to undertake the two major projects.

Rationale/aims

You will spend almost one third of the course on the Curriculum Projects, which are woven through the more content-based remainder of the course, giving you opportunities to work together explore maths and to develop materials for use in teaching. The content of the longer projects is negotiated, within guidelines, and group projects are preferable. The projects involve you in observing and listening to students, exploring maths and its sources and developing contextual materials for teaching numeracy.

Assessment

The two smaller Curriculum Projects (CP1 and CP4) are mainly done during course time and be used to contribute to class discussion. They are not assessed.

The major assessment tasks (CP2 and CP3) will need to be completed according to a contract agreed between you and the presenter, and they will be assessed accordingly.

One of the projects needs to be written formally and linked to the theoretical framework of the course, mainly in order to satisfy entry requirements for postgraduate study. Although it is recommended that only one needs to be formally written up, some participants may decide to write both up in a formal way. Each project which is written up should total approximately 2500 words. The link to the theoretical framework should be demonstrated by:

- evidence of wide reading
- evidence of good use of other resources
- understanding of issues involved in the social construction of maths (CP2)
- understanding of issues involved in critical numeracy (CP3)
- understanding of the pedagogical issues involved.

The written up project may well need to be handed in after completion of course.

Projects will only be given a satisfactory grading of 'satisfactory' or 'incomplete'; you may be asked to resubmit the project if the coordinator feels that it has not reached the standard agreed to in the contract.

Presentations

You will be asked to give a brief presentation (about 10–15 minutes) on at least one of the two major Curriculum Projects. This is so that ideas and materials developed by other participants can be shared by the whole Group. If there is insufficient time in the course for all participants to give presentations on both projects, you may be asked to be part of a Poster session where you prepare on butcher's paper some of the features of your materials and findings for a static display to the whole Group.

Organisation

Detailed notes about each of the projects will be given out to you as the course progresses. Some of the project work will be done as part of the 84 hours of the course but you will need to do some research and writing in your own time.

The Numeracy Journal

✷ NJ1
page 1

in which you write about your journey through
Terra Mathematica
where you leap chasms, slay dragons, joust with windmills,
rescue persons in distress
and (hopefully) emerge from the Slough of Despond
into the World of Numeracy.

Rationale

During this course, you will be expected to keep a regular journal which will form part of the assessment work for the course. Journals have different purposes and the purpose of this one is to provide you with opportunities to reflect on what is happening during the course, in an effort to narrow the gap between your theory and practice as a numeracy teacher. Topics for the Numeracy Journal are included below, and others may be suggested throughout the course. You will have a choice about which of these you respond to. You should not worry too much about the 'right' style. Use your own voice. If you have something to say, say it. If you have an emotional response, put it down. Use concrete examples to make your point. The goals are: analysis, communication, and making connections among issues, theories, maths and experience. The presenter may also get you to use the journal during course sessions. The complete set of journal entries will be a record of your experience during the course. For this reason, it may be a good idea to review your Numeracy Journal occasionally and write a short entry reflecting on changes you perceive.

Organisation

Number of entries	9
Length of each entry	about 400 words
Form	on loose-leaf paper to go in a binder, or in a separate small exercise book
Due date	as negotiated

Assessment

The Numeracy Journal will not be graded, but satisfactory completion is essential.

The Journal entries

Below are some topics and ideas to choose from for your Numeracy Journal. You may add others during the time of the course. Make sure that all Modules are given some attention, respond to at least two issues from each Module.

The first entry

During the second session of the course you will engage in the activity *Looking at boundaries*, as part of a case study of yourself in relation to mathematics and learning. The second part of the case-study, when you engage in the same activity with some other adult, will be the first of the nine entries in your journal.

Other Journal ideas

At any point during the course

- Reflect on the last session, teasing out what you learnt from it, what remains mystifying, and how it applies to your own practice. Be as specific as possible.
- Review a relevant article, relating it to work done in the course.

For Module A

- Examine some materials you have used for their use of language: does the model match the language? What messages—both mathematical and non-mathematical—are being given?
- Can you multiply using only Roman (or Mayne) numerals? Try it. What are the advantages/disadvantages compared with our system?
- Develop the metaphors activity further.

For Module B

- Build up a map, picture or metaphor … of 'what is mathematics?'
- What maths emerged from the last session? Did you see any old knowledge in a new light?
- How could you use ethnomathematics in your teaching?
- What is different about this way of learning maths from how you learnt it at school? How do you feel about this kind of learning environment?
- Follow up one of the suggestions for further mathematical exploration mentioned in the session, e.g. *Sink or swim.*

For Module C

- Use questions from C1.2 as a stimulus.
- Is this way of learning maths different from the previous one? How?
- Describe an example you have come across of the 'street maths, school maths' distinction. How is the distinction relevant to your teaching?
- To what extent can literacy and numeracy needs be differentiated?
- Describe an occasion (and the maths involved) where the use of maths could have, or did lead to greater empowerment.

For Module D

- Think about your use of calculators (or computers): how could you use them more creatively with your students?
- Describe an incident with your students, and analyse it in terms of the transmission/ constructivist/critical constructivist frameworks we have been using.
- Collect some 'good' assessment tasks, and try to say why they are good, if you think they are; alternatively, why they are not.
- What is 'good practice' in adult numeracy? Evolve some guidelines.

Curriculum Project 1 – Making meaning

CP1.1
page 1

Part 1

Working in pairs, one person takes on the role of explainer, the other of questioner. There are four expressions to explore, so work out a way of swapping roles.

The role of the questioner is to push aside her/his own understanding in an effort to hear and understand what the explainer is saying, and to ask questions to clarify meaning. It is important that the questioner tries to distinguish what she/he knows from what the explainer is saying. They should try to take as little as possible for granted.

The role of the explainer is to work from the assumption that the questioner has little prior understanding and to use a variety of ways of making clear the meaning conveyed by the symbols.

Below are four expressions, numerals combined with operating signs:

$5 + 7$
$6 - 4$
5×3
$12 \div 3$

Explore the meanings you have for these expressions. You can do this by talking about them to your partner, by making up stories that illustrate them, by drawing diagrams, by acting them out, by using objects to model them.

After about 20 minutes, join with one or two other pairs and compare what you have found out.

- the language
  What language is used with each of the four symbols for the operations? Is there a range of language? Is it common, everyday language or is it exclusively mathematical? Is it ambiguous?

- the model
  Does the model/diagram fit the language used?

- the meaning
  Did everyone come up with the same kind of story to illustrate the expression, or can the stories be quite different? What does 'same' and 'different' mean here?

CP1.1
page 2

Preparation for Parts 2 and 3

Spend some time with one or two adults, preferably from among your own students, exploring their meanings and ways of doing one or more of the four operations.

Collect as wide a variety as possible of ways of doing the four operations, with two or three digit numbers as well. Ask people from different generations and countries, look in old and new books.

See if you can understand the ways that they do addition, and the explanations they give for their methods. You might do this by finding out what they would like to learn and trying to teach it to them, you might ask them how they would explain the four expressions above to a child, you might be lucky enough to have a student from another country who learnt different methods for some of the operations and could try to teach you. Be aware of the language used, and whether the model employed fits that language or confuses it. Watch how meaning is made. Watch whether students expect to find/make meaning. Write brief notes.

Part 2

In small groups, discuss the notes from your student observations. Discuss language, models and how they combined to give meanings (or not). Look at *The four operations* [CP1.2] and complete the examples. Did most of these situations arise in your discussions with students?

Use counters, a number line, money, straws/matches and/or Cuisenaire rods to model the four operations. Keep the numbers small. Which material works best for which operation? Which would you use with your students and why?

Part 3

After the Group has pooled the results of its research on ways of carrying out the four operations, choose one or two new ways that take your fancy and work in pairs to develop an outline of how you would use these methods in your teaching activities or student worksheets, if appropriate.

The four operations

CP1.2

page 1

From Fuys & Tischler 1979, © Little, Brown & Co., Boston, pp. 199–203.

1 Addition

All of the stories and diagrams concern the addition 3 + 4 = ☐

On the line to the left of each story problem write the letter of the diagram which fits it best.

Which diagram?

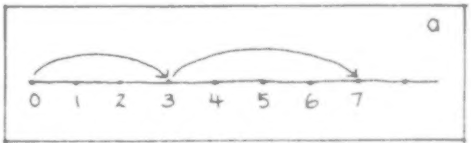
_______ There are 3 apples in a bowl. I put in 4 more. How many are there now?

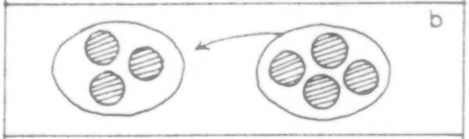
_______ I have 3 dogs and 4 cats. How many pets do I have?

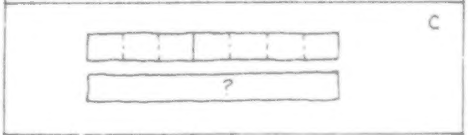
_______ I walk 3 blocks, buy a paper, and then walk another 4 blocks. How far have I walked?

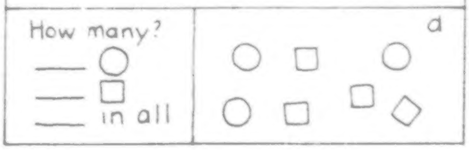
_______ This morning 3 centimetres of snow fell. This afternoon it snowed another 4 centimetres. How much snow fell today?

Sometimes addition is 'active', that is, the two sets of objects or lengths are physically combined or put together, and sometimes 'static' objects are classified or counted. Which problem above seems 'static'?

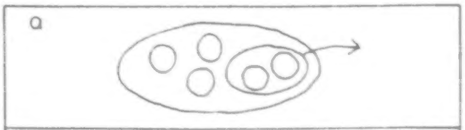


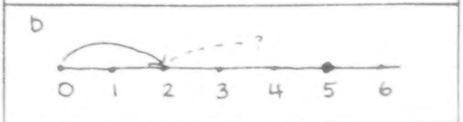


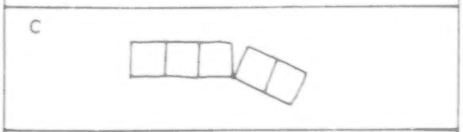


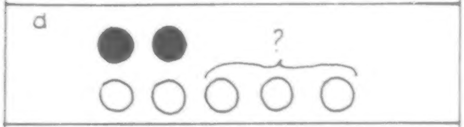


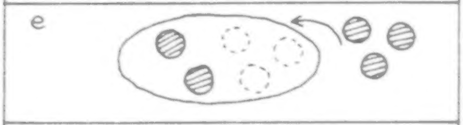
2 Subtraction

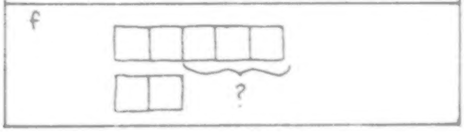












All of the stories and diagrams concern the subtraction 5 – 2 = ☐ Match the diagrams to the stories and also write your own in the blanks.

Which diagram?

Take-away Subtraction

_______ I have 5 inches of ribbon, and I cut off 2 inches. How much is left?

____

____

Missing addend subtraction

_______ I have 2 cups on a table and there are 5 children. How many more cups must I get?

____

____

Comparison subtraction

_______ Today 2 centimetres of snow fell and yesterday 5 centimetres fell. How much more did it snow yesterday than today?

____

____

3 Multiplication

All of the stories and diagrams involve the multiplication 3 x 4 = ☐

Which diagram?
Grouping

_______How many wheels do 3 cars have?

_______Three 4-inch pieces of ribbon will make what length?

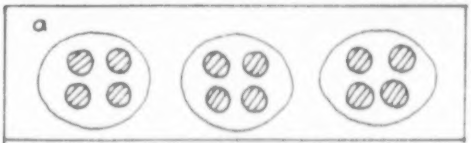
Array

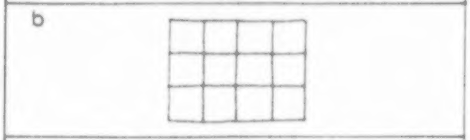
_______Children in a marching band line up in 3 rows with 4 in each row. How many children are in the band?

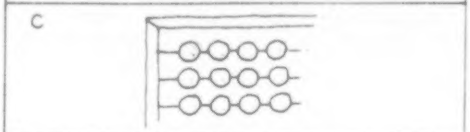
_______A table top is covered by 3 rows of tiles with 4 tiles fitting across the table top? How many tiles cover the table top?

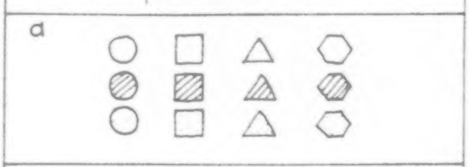
Combination (or Cartesian product)

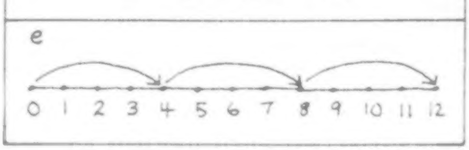
_______I am making cookies with 3 flavours: vanilla, chocolate, and raisin. Cookies will be made in 4 shapes. How many different cookies can I make?











4 Division

All of the stories and diagrams below involve the division 10 ÷ 2 = ☐

Which diagram?
Partitive or sharing division: Find two equal parts of 10, or share 10 things between 2.

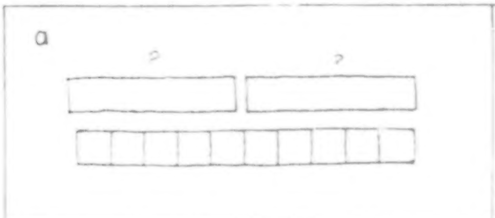
_______Ten cards are dealt to 2 children. How many cards does each child get?

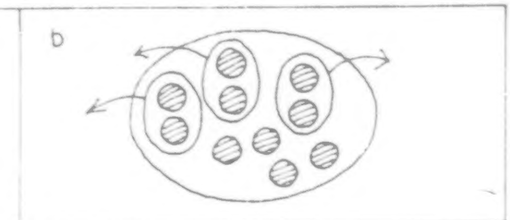
_______Two children want to share a 10-centimetre string of licorice. How much will each one get?

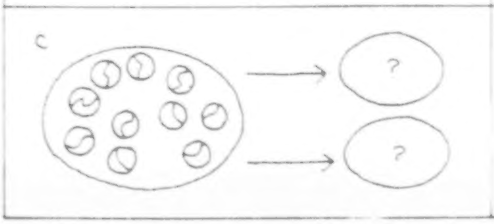
Measurement or repeated subtraction division: Measure 10 with units of 2, or ask how many 2's are in 10.

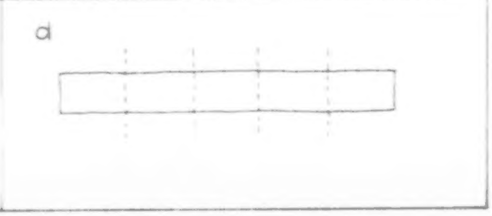
_______I have 10 slices of bread. How many sandwiches (2 slices each) can I make?

_______I have a piece of licorice 10 centimetres long. I cut off pieces 2 centimetres long for some friends. How many friends can I give a piece to?









Curriculum Project 2 – Exploring mathematics

CP2.1

page 1

Introduction

The main aim of *Curriculum Project 2 – Exploring mathematics* is to give you an opportunity to experience what it means to be involved in your own mathematical journey. For some participants of the course, this will be a familiar experience, and for you the main aim will be to explore more contextual links. Many participants however, have been deprived of experiences of this kind and for you the main aim is to build up some experience in a field which is fundamental if you are to encourage curiosity and investigation in your own students.

The content of the exploration

You will be expected to choose, investigate and report on a topic of mathematical interest arising out of the activities of the first Mathematical Journey (Module B)—see *Map of the first journey: a conducted tour* for some suggestions. In general through the project, you should be extending your knowledge of some area of mathematics that interests you and trying to see what links you can make with historical, cultural and/or natural contexts.

The process

The project has been allocated 9 hours of course time, the first six hours to be spent on negotiating, exploring and developing the topic and the last three hours or so being set aside for presentation of the various findings to the whole Group. You will need to spend some additional time on preparation and research.

Your work as a teacher can be greatly enhanced by input from colleagues. For this reason we would like you, if it is at all possible, to carry out this exploration with other colleagues on the course. If you plan to do this, then it is important that each member of the group has a chance to engage in exploration. Your joint contract form should be fairly explicit about the contributions planned by each member of your working group.

Having chosen your topic, preferably in collaboration with two or three other participants, your next task is to draft the learning contract. Read *Notes on negotiation* [CP2.2], the handout *A negotiated contract* [CP2.3], the *Sample contract* [CP2.4] and complete the *Contract form* [CP2.5] accordingly.

The product

The project will result in two, or possibly three, end products. The first, is the presentation (length to be negotiated) that you will prepare and give to the whole Group. Try to see this as a chance to engage your colleagues in any questions and journeys that you found intriguing.

The second product of your exploration will be one or two small activities that you develop from your exploration, suitable for use with your students if they are working at that level of mathematics, or if they are not, then they may be activities that you can use to illustrate your research with fellow colleagues in the ANT course and use in your presentation. Any suitable ABE student activities may be collected by the coordinator of the course, collated and distributed back to participants for their own use.

The possible third product would be an essay, the result of your choice of this project as the one to write up as part of your assessment for the course (see *Assessment guidelines* [HO1]).

CP2.1
page 2

The project written up should total approximately 2500 words. If you decide to choose this project for that purpose remember that you will need to link your exploration to the theoretical framework by:

- evidence of wide reading
- evidence of good use of other resources
- understanding of issues involved in the social construction of maths.

Summary

The exploration should include:

- choosing a relevant mathematical topic or question
- with the presenter, negotiating a suitable contract around your chosen topic
- tracing, unearthing and using appropriate resources to explore your topic—people, books, objects …
- making links with the cultural and/or historical contexts of the mathematical material
- taking risks and making errors
- spending time!
- developing a short presentation to share your findings in an engaging manner with the rest of the Group
- developing one or two small activities for use with your students (or your colleagues), in a form which can be collated with others into a resource for the whole Group.

Note:
One minor aim of this exploration is the production of teaching materials. Remember, however, that the main focus here is the mathematical investigation itself and the sharing of it with colleagues, and try to allocate time proportionately.

Notes on negotiation

CP2.2

Learning contracts take into account how adults learn best.

> One of the most significant findings from research about adult learning (e.g. Allen Tough's *The Adult's Learning Projects,* Ontario Institute for Studies in Education, Toronto, 1971) is that when adults go about learning something naturally (as contrasted with being taught something) they are highly self-directing. Evidence is beginning to accumulate, too, that when adults learn on their own initiative they learn more deeply and permanently than what they learn by being taught.
>
> While learning that is engaged in for purely personal development can perhaps be planned and carried out completely by an individual on his/her own terms and with only a loose structure, those kinds of learning that have as their purpose improving one's competence to perform in a job or in a profession must take into account the needs and expectations of organisations, professions and society. Learning contracts provide a means for negotiating a reconciliation between these external needs and expectations and the learner's needs and interests.
>
> Furthermore, in traditional education the learning activity is structured by the teacher and the institution. The learner is told what objectives he is to work toward, what resources he is to use and how (and when) he is to use them and how his accomplishment of the objectives will be evaluated. This imposed structure conflicts with an adult's need to be self-directing and may induce resistance, apathy or withdrawal. Learning contracts provide a vehicle for making the planning of learning experiences a mutual undertaking between the learner and an adviser, mentor, teacher and often, peers. By participating in the process of diagnosing his needs, formulating his objectives, identifying resources, choosing strategies and evaluating his accomplishments, the learner develops as sense of ownership of (and commitment to) the plan.
>
> Geoff Scott, University of Technology, Sydney (course notes)

In the light of these notes look at
- *A negotiated contract* [CP2.3]
- *Contract form* [CP2.5] and
- *Sample contract* [CP2.4] which has been filled in to show how you might have used the contract form if the activity *Looking at boundaries* had been a negotiable contract.

When you fill in your own form, try to address the items set out below in each of the columns of the contract.

Curriculum Project 2 – A negotiated contract

CP2.3

A guide to writing this negotiated learning contract

OBJECTIVES

What will you learn/gain from this investigation?

In this section you should state clearly:
- what mathematical question you aim to explore
  e.g. what 2-D polygons will tessellate ?
- what historical, cultural or natural contexts you might also explore
  e.g. look at tile patterns in Islamic art and architecture.

STRATEGIES/RESOURCES

How are you going to do it?

In this section you should include a project plan which shows:
- how the work will be allocated, if you are working in a group
- what books and readings you will refer to
- any materials you will use
- what interviews you might conduct with relevant people.

WHAT TO ASSESS

What are you going to produce?

In this contract, you will produce:
- a presentation for your colleagues in the whole Group
- a few related teaching materials/activities

and possibly, if this is to be your assessed contract:
- a report of some sort which will require further negotiation.

CRITERIA FOR EVALUATION

What criteria should be used for evaluation of this contract?
That is, how will we know that you've done it?

The criteria should basically be those that you have demonstrated achievement of what you set out in your Objectives, but could include:
- careful development of a mathematical topic
- evidence of wide reading
- evidence of good use of other resources
- evidence of risk-taking
- useful pictures, charts, formulas, statements that suggest either hypotheses or doubts, challenges to the question itself
- clear links with cultural/historical contexts

Curriculum Project 2 – Sample contract

CP2.4

Student Chris

Presenter/Coordinator Betty

OBJECTIVES
What will you learn/gain from this investigation?

- *an insight into the way people think mathematically.*
- *opportunity to have some of my own beliefs and assumptions surface about maths learning and teaching*
- *learning from another person about how they process and understand maths*
- *consideration of implications for ABE practice.*

STRATEGIES/RESOURCES
How are you going to do it?

Give the boundaries problem to a person I feel to be less mathematical. Watch and record as they work with the problem.
Encourage the person to articulate the process as much as possible. Also use own experience in a teaching exercise for reflection.

WHAT TO ASSESS
What are you going to produce?

A case study in which my own thoughts are recorded.
A summary of my own beliefs and assumptions about maths learning and teaching.

CRITERIA FOR EVALUATION
What criteria should be used for evaluation of this contract?
That is, how will we know that you've done it?

- *Evidence that the exercise has been done*
- *A sound summary*
- *A written critical reflection of my own assumptions and beliefs and implications for ABE practice.*

signed Chris (participant)
signed Betty (presenter/coordinator) date 16/11/95

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Curriculum Project 2 – Contract form

CP2.5

Student ______________________

Presenter/Coordinator ______________________

OBJECTIVES What will you learn/gain from this investigation?
STRATEGIES/RESOURCES How are you going to do it?
WHAT TO ASSESS What are you going to produce?
CRITERIA FOR EVALUATION What criteria should be used for evaluation of this contract? That is, how will we know that you've done it?

signed................................. (participant)

signed................................. (presenter/coordinator) date.......................

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Curriculum Project 3 – Developing a critical view

CP3.1

Introduction

The main aim of *Curriculum Project 3 – Developing a critical view* is to give you an opportunity to develop the mathematics excavated from some context, possibly the one you excavated in Section C5 into curriculum ideas and/or teaching materials that you can use with your students to help them to:

– excavate the maths embedded in a particular context
– extend their understanding of the mathematical concepts involved
– become more critically aware of the role that maths plays in the context involved
– experience and understand that 'doing' maths is a messy, human practice
– build up some strategies and resources that are useful in learning maths.

The content

Choose a context of potential mathematical interest. It can be one that you were involved in, or one that somebody else excavated or referred to. You can develop actual teaching materials or develop curriculum outlines/plans.

The process

The project has been allocated nine hours of course time, the first six hours to be spent on negotiating, and developing teaching materials and the last three hours or so being set aside for presentation of the materials to the whole Group. You will need to spend some additional time on preparation and research.

Collaboration with colleagues is one rich resource. For this reason we would like you, if at all possible, to carry out this project with other colleagues on the course. If you plan to do this, then it is important that each member of the group has a chance to contribute their own strengths and interests. Your joint contract form should be fairly explicit about the contributions planned by each member of your working group.

Having chosen your context, preferably in collaboration with one or two other participants, your next task is to draft the learning contract.

The end products

The project will result in two, or possibly three, end products. The first is the presentation (length to be negotiated) that you will prepare and give to the whole Group. Try to see this as a chance to try out with your colleagues a selection of the materials that you have prepared for your students.

The second product of your exploration will be a collection of teaching materials and/or a curriculum outline that you develop to explore your chosen context, suitable for use with your students, as described above. These could include lesson plans, student worksheets, outlines of investigations or activities, student reading materials. The materials should include a rationale and should be 'user friendly' (see *User-friendly numeracy resources* [P9] from Module C)—for both teacher and student. The materials will be collected by the coordinator of the course, collated where possible and distributed back to participants for their own use.

The possible third product would be an essay, the result of your choice of this project as the one to write up as part of your assessment for the course (see *Assessment guidelines* [HO1]).

The project written up should total approximately 2500 words. If you decide to choose this project for that purpose remember that you will need to link your excavation to the theoretical framework by:

- evidence of wide reading
- evidence of good use of other resources
- understanding of issues involved in critical numeracy
- understanding of the pedagogical issues involved.

Summary

The project should include:

- choosing a context with mathematical and social potential;
- negotiating with the coordinator or presenter a suitable contract around your chosen context;
- tracing, unearthing and using appropriate resources to explore your context—people, books and objects;
- developing materials that help students excavate, extend and become critically aware of the embedded mathematics;
- spending time!
- developing a short presentation to try out some of your materials with the rest of the whole Group;
- developing a substantial collection of materials for use with students, with at least some in a form that can be collated with others into a resource for the whole Group.

Curriculum Project 3 – Contract

CP3.2

Student ________________________________

Presenter/Coordinator ________________________________

OBJECTIVES
What will you learn/gain from this investigation?

STRATEGIES/RESOURCES
How are you going to do it?

WHAT TO ASSESS
What are you going to produce?

CRITERIA FOR EVALUATION
What criteria should be used for evaluation of this contract?
That is, how will we know that you've done it?

signed...................................... (participant)

signed...................................... (presenter/coordinator) date................

Mathematics through problem-posing

From Lerman, S. 1989, in Ernest, P. (ed.), Mathematics Teaching: the state of the art,
© The Falmer Press, East Sussex, pp 76-79.

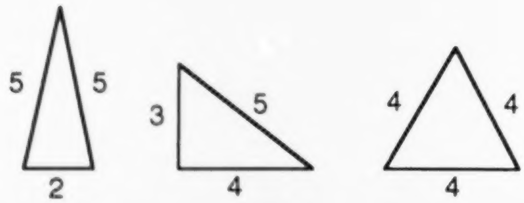
The assumptions for this part of the discussion are as follows. Since the context and the meaning are relative, and hence the engagement of the child is an individual response and not necessarily consequent upon the stimulus of the teacher, then offering the child an open situation, in which the child is encouraged to pose questions for her/himself, is the only way of enabling the child to advance conceptually. In a sense the child is doing this when s/he is learning, that is, providing the meaning and context that are 'meaningful' to that child. The problem is that most of what we do in the mathematics classroom is 'meaningless' to most children, and therefore is not learnt. Cobb (1986) describes the situation as follows:

> Self-generated mathematics is essentially individualistic. It is constructed either by a single child or a small group of children as they attempt to achieve particular goals. It is, in a sense, anarchistic mathematics. In contrast, academic mathematics embodies solutions to problems that arose in the history of the culture. Consequently, the young child has to learn to play the academic mathematics game when he or she is introduced to standard formalisms, typically in first grade. Unless the child intuitively realises that standard formalisms are an agreed-upon means of expressing and communicating mathematical thought they can only be construed as arbitrary dictates of an authority. Academic mathematics is then totalitarian mathematics.

Mathematical knowledge de-reified, seen as a social intervention, its truths, notions of proof, etc. relative to time and place, has to be seen as integrally involved with the doing of mathematics, and indeed cannot be separated from it. Mathematics is identified by the particular ways of thinking, conjecturing, searching for informal and formal contradictions, etc., not by the specific 'content'.

An example of a problem that may best be termed a 'situation', in that asking the question is left up to the student, provides an illustration for the discussion that follows. In a seminar with a group of postgraduate students, who were non-mathematicians. I presented the following investigations, taken from SMILE Investigations (1981): 'Consider triangles with integer sides. There are 3 triangles with perimeter 12 units. Investigate' (see Figure 1).

Figure 1: Diagram Accompanying Investigation
Source: SMILE, investigations, London, Smile Centre, 1981.



A number of groups were disconcerted, saying that they had no idea what to do, since there was no question being asked. Other groups worked on areas of the triangles, perimeters of the triangles, perimeters of rectangles, etc. All the students found this to be quite different from the other investigations they had tried before, and very challenging. Brown (1984) describes a number of other examples like this one, where students de-pose or re-frame the question as stated, and generate their own problems.

It can be suggested that there are more radical consequences in changing from a 'content' to a 'process' focus. There are political implications of the notion of problem-posing, as suggested by Cobb's use of the terms 'anarchistic' and 'totalitarian'. Freire (1972), in writing about his literacy work with the oppressed people of Brazil and elsewhere in South America, describes two rival conceptions of education. The traditional view of education is the 'banking concept', whereby pupils are seen as initially empty depositories, and the role of the teacher is to make the deposits. Thus the actions available to pupils are storing, filing, retrieving, etc. In this way, though, pupils are cut off from creativity, transformation, action and hence knowledge. The alternative view of education Freire describes as the 'problem-posing' concept. By this view knowledge is seen as coming about through the interaction of the individual with the world. 'Problem-posing' education responds to the essential features of the conscious person, intentionality and meta-cognition. Freire's discussion of opposing concepts of education is integrally tied with oppression and freedom.

We are working in education at a time when our students may be faced with a lifetime of unemployment and uncertainty; with threats to the ecology of the planet; an increase in disparity of wealth between rich and poor in society and between nations; and even the threat of total annihilation. Traditionally these have not been issues that have been thought of as having any relationship to mathematics. We have always rested safe in the knowledge that mathematics is value-free, non-political, objective and infallible. Freire's analysis, together with the doubts about such conceptions of the nature of mathematics and children's learning described above, suggest that this is not the case. Quite the contrary, we have a particular responsibility, since so many political, moral and other issues are decided using 'mathematical' techniques, to enable pupils to examine situations, make conjectures, pose problems, make deductions, draw conclusions, reflect on results, etc. These situations can be as in the investigation above, or the Fibonacci sequence (see Brown, 1984), information about expenditure on arms by the United States and the USSR, expenditure on education for different racial groups in South Africa, etc. There is plenty of 'content'. The last two situation would call on statistical and graphical techniques, for example. The difference is that engagement in the problem posed by the pupils puts mathematical knowledge in a different, and in my view appropriate, position. It is seen as a library of accumulated experience and just as any library is useless to someone who cannot read, so too this library is useless unless people have access to it. When a problem is generated which reveals the need for some of this knowledge, be it multiplying decimals, standard index form, complex numbers, or catastrophe theory, if the individual recognizes first that such help is needed, and secondly that it is available, the context, relevance and meaning of mathematical knowledge are established.

Enabling students to examine situations and to pose problems for themselves reflects the fallibilist or relativist view of mathematical knowledge, reflects the constructivist perspective of children's learning, and places a powerful tool in the hands of people to examine what is happening to their lives, and provide them with the possibility of changing it.

Curriculum Project 4 – Assessment tool

CP4.1

Introduction

'Assessment tool' is the last of your four Curriculum Projects. It is a short three hour project, giving you the opportunity to investigate a range of assessment alternatives for numeracy and to extend the criteria (developed in Section 3.6) for what makes a good assessment task. By the end of the project you should be aware of an extended repertoire of assessment methods and you should have developed and trialled some tasks for assessment use with students. There are three parts to the project:

Part 1 – The readings (1.5 h)
Part 2 – Comparing results: a range of alternatives (1.5 h)

Part 1 – The readings

You will read an article about assessment, and thought about this three-part question:
What does this article say about

- *why assess?*
- *what makes a good assessment?*
- *what alternative assessment procedures could we use?*

Reporting on the readings

To begin, people who have read different articles will report on them to the Group, so that everybody has some idea of the range of arguments and procedures currently being discussed.

An alternative assessment

At the end of the reporting and discussion, you will choose one alternative form of assessment you would like to explore. Those choosing the same procedure should gather together, collect and read information on that procedure, and together develop several tasks following that type of procedure for different levels or contexts that might be appropriate for students you are currently teaching.

Preparation for following session

Between this session and the next try out one of the tasks with a student, and write a one page summary to bring to the Group, outlining:

- the alternative assessment procedure
- the particular task
- the situation involved … person … context …
- the results … issues and/or problems …

Part 2 – Comparing results: a range of alternatives

1 When you return to the groups you were in at the end of the first session, compare results using previous criteria of what makes a good assessment, and taking into consideration the table Two (and a half) views of assessment [HO8], completed in Section D3.1. Decide what tasks worked and what did not, and if possible, why. When you are clear about this, prepare a presentation for the Group.

2 One or two people from each small group will then discuss with the whole Group:

- the alternative assessment procedure chosen by your group
- the tasks that were successful and those that weren't
- the possible reasons.